CYBERNETICS AND ARTIFICIAL INTELLIGENCE

Introduction to Cybernetics and System Dynamics, Introduction to Artificial Intelligence

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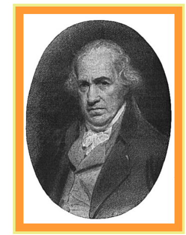
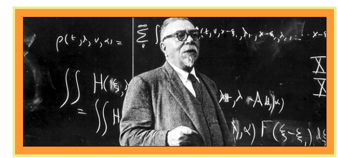
The subject

- What is the **purpose** of this subject?
  - To get *a general overview* about problems and methods in cybernetics and AI and to understand their nature.
  - To introduce *basic ideas and concepts* that are often used in very different contexts in various specialized 'cybernetics' subjects
    - Electric circuits, Systems and models, Systems and control, Theory of dynamical systems, Communication theory, Artificial Intelligence (Intro, I, II), Biocybernetics, ...
  - To draw attention to *connections* that cannot be explicitly stressed within these specific subjects.
- What is the **content** of this subject?
  - Cybernetics is a field of study with a long tradition (nearly a century).
  - There was a wide range of different subareas founded within C. during that period of time. Cybernetics has an extremely wide scope.
  - AI (and informatics as a whole) is one of the daughter fields of C.
  - This subject presents C. and AI as co-ordinate fields owing to the stress that we put on AI within our education program.

History of cybernetics

- Cybernetics is the study of complex **systems** and processes, their **modeling**, **control** and **communication**.
- **James Watt** (1736 - 1819)
  - *steam engine with* **regulatory feedback**
- **André-Marie Ampère** (1775 – 1836)
  - „Cybernetics" - *the sciences of government*
  - Kybernetes $(\kappa\upsilon\beta\epsilon\rho\nu\epsilon\tau\epsilon\varsigma)$ = governor or steersman
- **Norbert Wiener** (1894 – 1964)
  - functional similarity among living organisms, machines and (social) organizations, as well as their combinations
  - puts emphasis upon common features and methods of their description, namely the statistical ones
  - Cybernetics: or Control and Communication in the Animal and the Machine (1948)



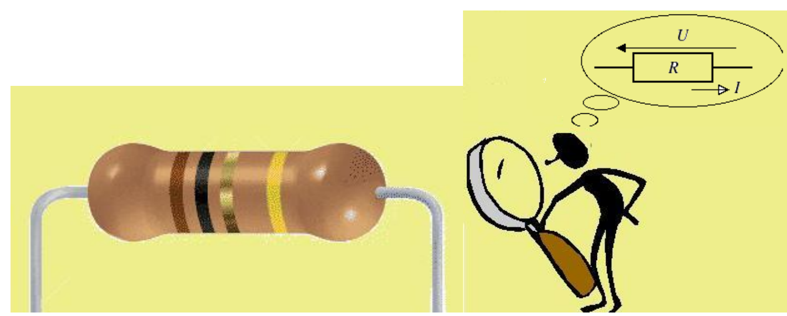


History of cybernetics II

- **Contemporary cybernetics**: a great variety of independent fields
  - Dynamical systems: *feedback control, state space, stochastic systems, control, ...*
  - Communication theory: *information entropy, communications channel and its capacity, ...*
  - **Artificial intelligence**: *perception and learning, multi-agent systems, robotics, ...*
  - Biocybernetics: *neural networks, connectionism, man-machine interaction, ...*
  - Decision theory, game theory, complexity theory, chaotic systems, etc.

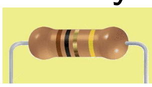
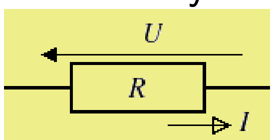
System, observer, model

- What do these fields of study have in common? They examine various aspects of (complex) systems.
- How to define a **system**?
- System is an assemblage of entities, real or abstract, comprising a whole with each and every component/element interacting or related to at least one other component/element. [Wikipedia.org]
- The definition is trivial . What is important are the systems originated by **abstraction** of real systems.
- **Observer** defines an abstract system by a determination of:
  - a list of crucial variables of a real system and their interaction
  - all the other variables/interactions represent the **environment** of the system
  - they can be ignored or influence the system **inputs** resp. be affected by its **outputs** (If input/output variables explicitly defined, the system is referred to as *oriented*.)



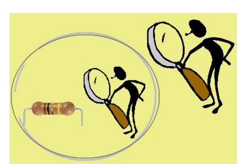
System, observer, model

- Simplification while keeping the key principals:

System	Entities	Interactions
Real system $\mathcal{S}$ 	∞: **voltage**, *colour*, *temperature*, **resistance**, *length*, **current**, *diameter*, ...	∞
Abstract system $\mathcal{S}'$ 	**voltage** U, **resistance** R, **current** I	$U = R \cdot I$

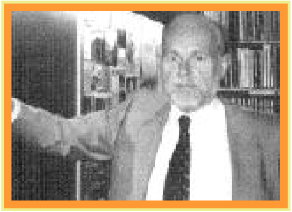
- Abstract system $\mathcal{S}'$ is a **model** of the physical system (object) $\mathcal{S}$. Model allows to *predict* behaviour of the real system.

- The result of the quantum theory: 
  - A physical system can never be observed (measured) without interfering with it.
  - *"Cybernetics of the 2nd order" (meta-cybernetics)* examines observer-system systems.



General systems theory

Ludwig von Bertalanffy
1901 Vienna - 1972 Binghamton, USA



George Klir
1932 Praha, now Binghamton, USA

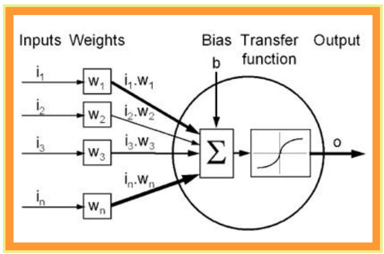
- GTS distinguishes systems by the descriptional level of detail
  - **Source system:** enumerates variables and their interacting subsets
    - e.g. variables: $\{U, R, I\}$, interactions: $\{\{U, R, I\}\}$
  - **Data system:** source system + empirical quantities of variables
    - např.

U	12V	10V	8V	...
R	1kΩ	1kΩ	1kΩ	...
I	12mA	10mA	8mA	...

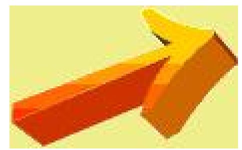
  - **Generative system:** variables + their relations. Enables to generate the data system.
    - $U = R \cdot I$
  - **Structural system:** distinguishes subsystems (e.g. describes their hierarchy)
    - e.g. electrical circuit split into its functional units
- Each system type carries **additional information** w.r.t its preceding type.

Emergence

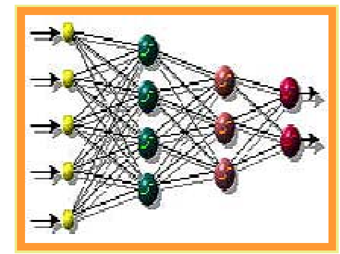
- **OBJECTION 1:** Why a special *system* science? Would *component-level* research do as well?
- *A system is "more" than just the sum of components*
- From simple interactions on the component level, intriguing system-level features can *emerge*.



Simple model of the **neuron**
Only calculates $\phi\left(\sum_{ij} w_{ij} x_i\right)$
(nonlin. function of weighted sum of inputs)



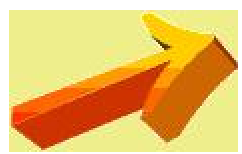
Interconnection of a large number of neurons



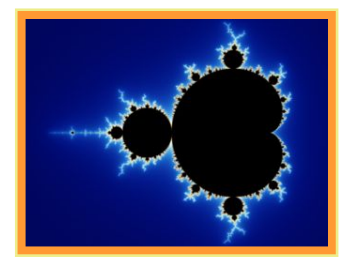
Artificial Neural Network
Can be **trained** to recognize images, simulate associative memory, ...

$$\boxed{f(z) = z^2 + c}$$

trivial relation btw. complex variables



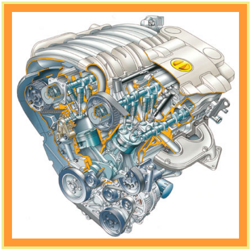
Color: rate of divergence $f(f(\ldots f(z)))$ for given c



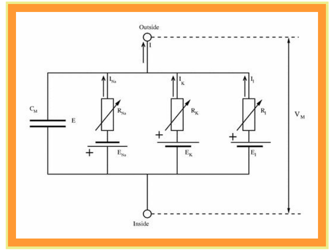
Generates an extremly complex **fractal** structure (self-similar for various zoom levels) in the Re $c\times$ Im c plane.

Examples of systems

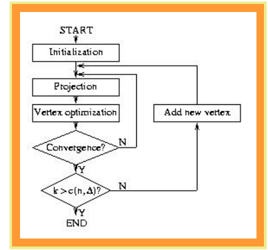
- Technical



combustion engine

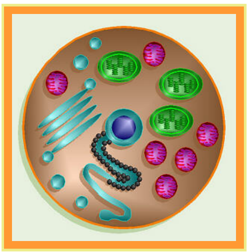


electrical circuit



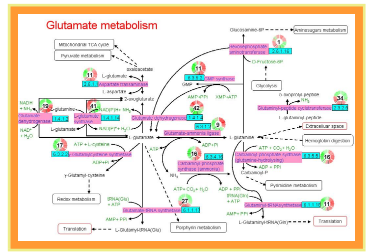
computer algorithms

- Biological



cell

brain



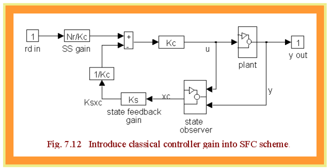
metabolic process

- Ecological (predator-prey oscillating populations), socio-economic, etc...
- Cybernetics studies systems of **very diverse characters**. Which implies .....

System Analogies

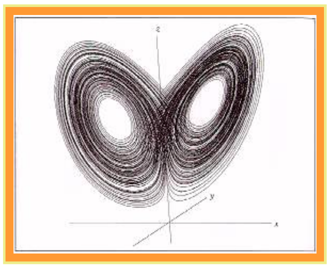
- **OBJECTION 2:** Why a general system science, if individual systems are studied in specialized fields (technology, biology, economy, ...)?
- Some cybernetic **concepts** are common to systems of diverse kinds. Related techniques can be exploited analogically. Examples:

feedback loop



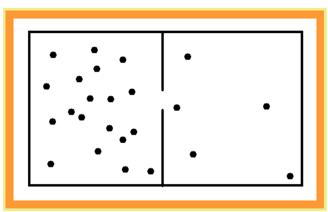
Omnipresent in nature (oceanic pH regulation, predator/prey population, stock markets, ...)
Largely exploited in technical systems

state space



Term introduced by Poincare for physical (thermodynamic) systems.
Now the most important technique for dynamical systems modeling.

entropy

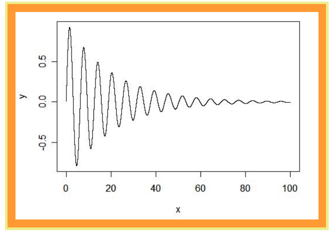


Originally introduced as a thermodynamic system property.
Now analogically in other systems (information entropy, algorithmic entropy).

System Analogies

- Some system **features** can be identified and studied for systems of diverse kinds. Examples:

harmonic time response



All linear dynamic systems
electrical, mechanical, hydraulic, ...

nondecreasing entropy

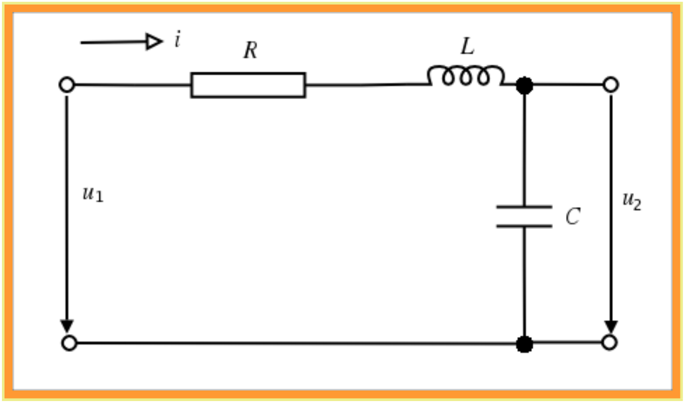
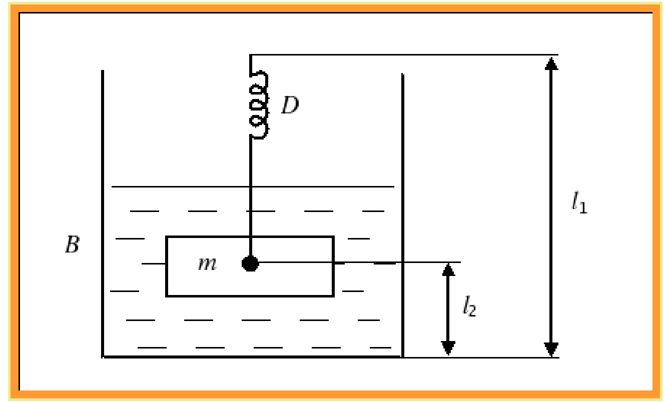
All isolated systems.
(with no energy supply)

fractal structures

Natural formations (costlines, mountains, plants)
State-space trajectories of chaotic dynamical systems.

System Analogies

- Some cybernetic **models** valid equally for different systems.

Electrical circuit	*Mechanical system*
	
$L\frac{\mathrm{d}^2 u_2}{\mathrm{d}\, t^2} + R\frac{\mathrm{d}\, u_2}{\mathrm{d}\, t} + \frac{1}{C}u_2 = \frac{1}{C}u_1$	$m\frac{\mathrm{d}^2 l_2}{\mathrm{d}\, t^2} + B\frac{\mathrm{d}\, l_2}{\mathrm{d}\, t} + Dl_2 = Dl_1$

inductance L	$\leftrightarrow$	mass m
resistance R	$\leftrightarrow$	damping force B
inverse capacity $1/C$	$\leftrightarrow$	spring constant D
voltage u_1	$\leftrightarrow$	displacement l_1
voltage u_2	$\leftrightarrow$	displacement l_2

- Same mathematical model (2nd order linear dif. eq.), only different variable names. Systems are said to be isomorphic (same up to naming). Each one is a model of the other.

System aspects related to Cybernetics and Robotics programme

- System aspects of interest in this course
- **Dynamics**
  - Linear and non-linear systems: from order to chaos.
- **Entropy and Information**
  - How to measure system disorder and quantify information using probability.
- **Information transmission**
  - How to transmit information. Communication channel, erroneous transmission, data compression.
- **Algorithmic entropy, decidability**
  - How to measure system complexity without using probability
    Decidability of problems.
- **Artificial intelligence**
  - Problem solving, decision making under uncertainty, recognition, learning, ...
- **Control**
  - External dynamics description, feedback, regulation of systems.

System dynamics

- Let $\vec{x} = [x_1, x_2, \ldots x_n]$ be a vector of system variables (not including *time*!).
- **Dynamics** of the system $=$ the unfolding of $\vec{x}$ in time.
- **Dynamical model** of the system: a rule that determines the unfolding
  - **discrete** model:

    $$\boxed{\vec{x}(k+1) = \vec{f}(\vec{x}(k))}$$

    $(k = 0, 1, 2, \ldots)$ The *subsequent state* determined by the current state.
  - **continuous** model:

    $$\boxed{\frac{\mathrm{d}}{\mathrm{d}\,t}\vec{x}(t) = \vec{f}(x)}$$

    $(0 \leq t \leq \infty)$ The *state change* determined by the current state.
- **Deterministic** dynamical system: $\vec{f}$ is a **function**
- **Stochastic** dynamical system: $\vec{f}$ determines the **probability distribution** of the next state (not on today's menu)
- Basic assumptions:
  - **Finite dimension** of the system: $n < \infty$. **Stationarity** (or 'time invariance'): $\vec{f}$ independent of k (resp. t).

System dynamics

- Suitability of the continuous / discrete model depends of the character of the modeled real system.
  - Continuous models pervade in physics (e.g. electrical circuit), discrete in economy (stock value at a date)
  - Discrete models often used as **approximation** of continuous ones (mainly in computer simulations). Then $t \equiv k \cdot \Delta\tau$ ($\Delta\tau$ - *sampling period*).
- **OBJECTION:** The model $\vec{x}(k+1) = \vec{f}(\vec{x}(k))$ oversimplifies, in real systems $\vec{x}(k+1)$ may depend on $\vec{x}(k-1)$, $\vec{x}(k-2)$, etc. as well
- Solution: simply introduce further variables acting as the system "memory". Example:

$$x_1(k+1) = x_1(k) + x_1(k-1)$$

$$\begin{aligned} x_1(k+1) &= x_1(k) + x_2(k) \\ x_2(k+1) &= x_1(k) \end{aligned}$$

  tj. $[x_1(k+1), x_2(k+1)] = \vec{x}(k+1)$ now depends on $\vec{x}(k)$ only.

- For continuous models analogically. To eliminate higher derivatives, simply introduce further variables as derivatives of the original ones (example in a while).
- System variables set this way together constitute the **state vector**. Its value at time t (resp. k) is the **system state** at time t (resp. k). The vector $\vec{x}$ from now dentoes the *state vector*.

Linear Oriented System

- A special type of dynamical systems with **huge** application: $\vec{f}$ is a **linear** mapping:

  discrete lin. system: $\vec{x}(k+1) = \mathbb{A}\vec{x}(k)$ continuous lin. system: $\frac{\mathrm{d}}{\mathrm{d}\,t}\vec{x} = \mathbb{A}x$

- Linear systems are easily mathematically analyzed. This enables to establish a more detailed, **oriented** linear model,

  discrete

$$\vec{x}(k+1) = \mathbb{A}\vec{x}(k) + \mathbb{B}\vec{v}(k)$$
$$\vec{y}(k) = \mathbb{C}\vec{x}(k) + \mathbb{D}\vec{v}(k)$$

  continuous

$$\frac{\mathrm{d}}{\mathrm{d}\,t}\vec{x}(t) = \mathbb{A}\vec{x}(k) + \mathbb{B}\vec{v}(t)$$
$$\vec{y}(t) = \mathbb{C}\vec{x}(t) + \mathbb{D}\vec{v}(t)$$

  in which one separates from the state $\vec{x}$:

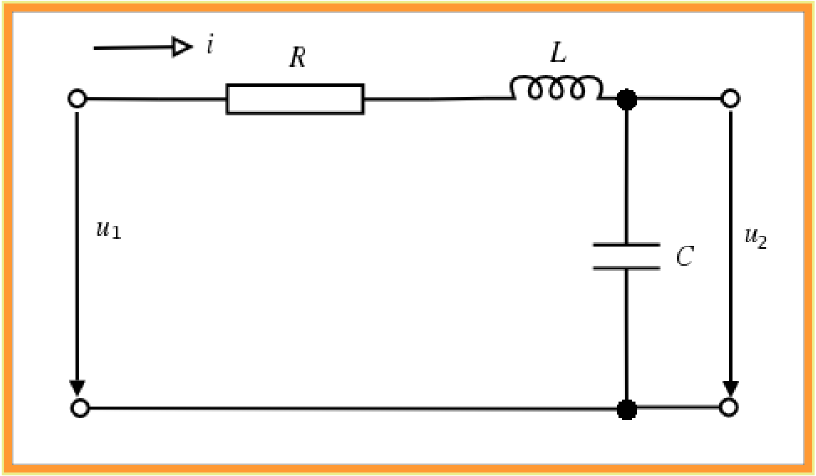
  - input variables $\vec{v}$ (not affected by the state)
  - output variables $\vec{y}$ (do not affect the state)

- The linear model **advantage**: unfolding of $\vec{x}(k)$ (resp. $\vec{x}(t)$) analytically derivable.
- **Disadvantage**: linear model often only approximation; real physical systems usually non-linear.

Example: continuous linear oriented system

Circuit equation:

$$L\ddot{u}_2 + R\dot{u}_2 + \frac{1}{C}u_2 = \frac{1}{C}u_1$$



Input:	$v := u_1$
State variables:	$x_1 := u_2$
	$x_2 := \dot{x}_1$ **(eliminating $\ddot{x}_1$)**
Output:	$y := x_1$

From the circuit equation:

$$\dot{x}_2 = -\frac{R}{L}x_2 - \frac{1}{LC}x_1 + \frac{1}{LC}v$$

State description:

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \underbrace{\begin{bmatrix} 0 & 1 \\ -\frac{1}{LC} & -\frac{R}{L} \end{bmatrix}}_{\mathbb{A}} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \underbrace{\begin{bmatrix} 0 \\ \frac{1}{LC} \end{bmatrix}}_{\mathbb{B}} [v]$$

$$[y] = \underbrace{\begin{bmatrix} 1 & 0 \end{bmatrix}}_{\mathbb{C}} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \underbrace{\begin{bmatrix} 0 \\ 0 \end{bmatrix}}_{\mathbb{D}} [v]$$

Matrix eigenvalues and eigenvectors

- Matrix $\mathbb{A}$ determines the fundamental properties of the linear dynamic system.
- To decrypt them we need the following notions: **eigenvalue** and **eigenvector** of a matrix.
  - $\vec{r}$ is an eigenvector, and λ is an eigenvalue of matrix $\mathbb{A}$ ifff

$$\boxed{\mathbb{A}\vec{r} = \lambda\vec{r}}$$

  - $\vec{r}$ are thus solution of the system of linear equations

$$\left(\mathbb{A} - \lambda\mathbb{I}\right)\vec{r} = 0 \tag{1}$$

    with parameter λ, where $\mathbb{I}$ is the *identity matrix* (1's on the main diagonal, 0's elsewhere).
  - This system has a non-trivial solution (non-zero $\vec{r}$) iff

$$\det\left(\mathbb{A} - \lambda\mathbb{I}\right) = 0$$

  - Solving this determinant equation we find all eigenvalues λ. Note: we search the solution in the *complex domain*.
  - For each λ we then solve the system 1, obtaining al eigenvectors $\vec{r}$.

Dynamical properties of the linear continuous system

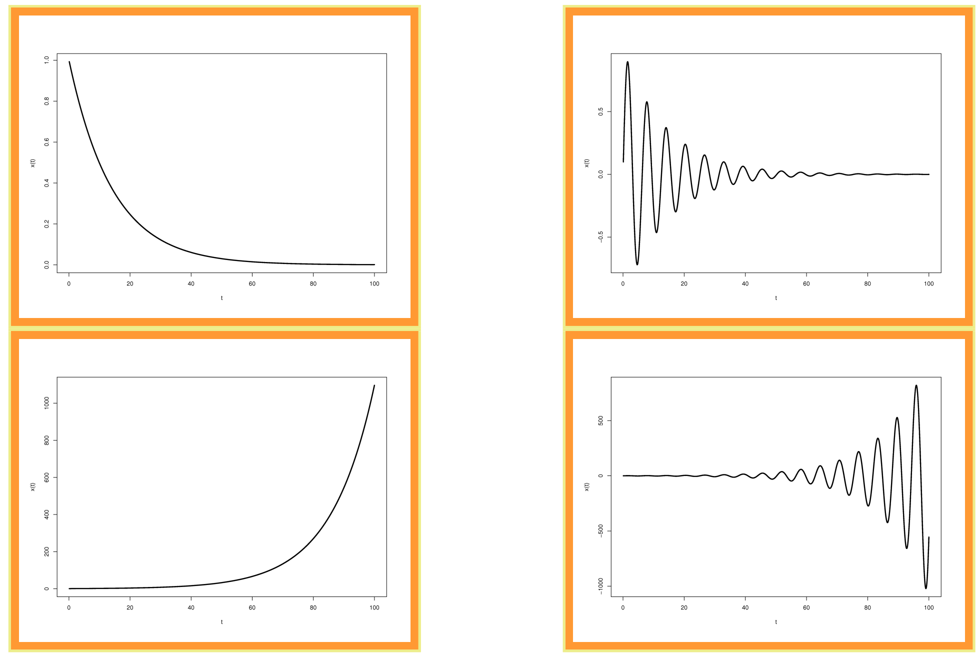
- When input decays at time t_0 ($t > t_0 \Rightarrow v(t) = 0$), **the general solution is** $\frac{\mathrm{d}}{\mathrm{d}\,t}\vec{x}(t) = \mathbb{A}\vec{x(t)}$:

$$\boxed{\vec{x}(t) = \sum_{i=1}^{n} k_i \vec{r}_i e^{\lambda_i t}}$$

  where $\vec{r}_i$ are the *eigenvectors* of $\mathbb{A}$, λ_i are the corresponding *eigenvalues* and k_i are constants depending on the initial condition ($\vec{x}(t_0)$ at time t_0 of input decay).

- Time unfolding examples: (examples for x_1)

non-oscillating ($\forall i \,\mathrm{Im}\,\lambda_i = 0$) | oscillating ($\exists i \,\mathrm{Im}\,\lambda_i \neq 0$)

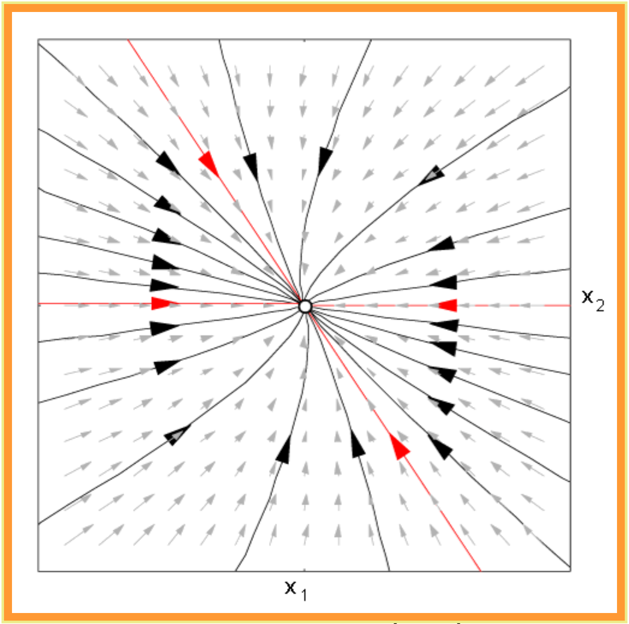


stable ($x(t) \rightarrow 0$ pro $t \rightarrow \infty$) if $\forall i \,\mathrm{Re}\,\lambda_i < 0$, i.e. all eigenvalues in **left complex half plane**.
Why does this imply stability?

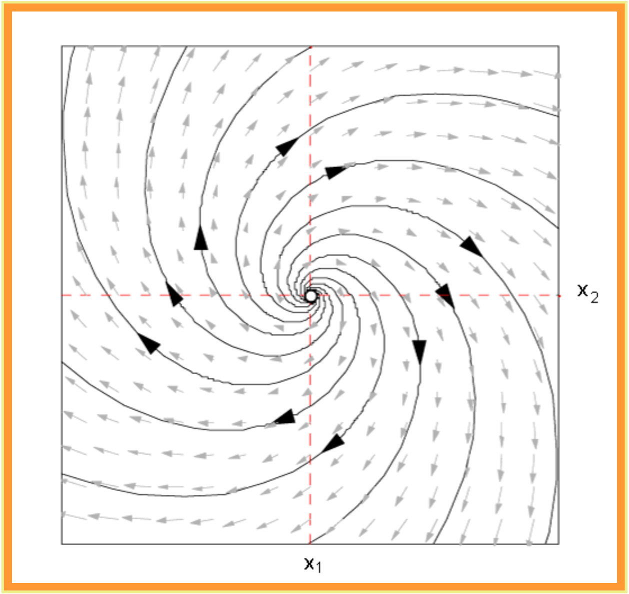
unstable ($x(t) \rightarrow \infty$ pro $t \rightarrow \infty$) if $\exists i \,\mathrm{Re}\,\lambda_i > 0$.
Not realizable physically if zero input signal, that is, if no energy supplied.

Linear Continuous System State Space

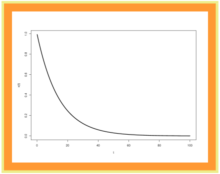
- Values of the n state variables = coordinates in the n-dimensional **state space**.
- In out example: $< x_1, x_2 = \dot{x}_1 >$
- Unfolding in time: trajectory in the state space. Examples:



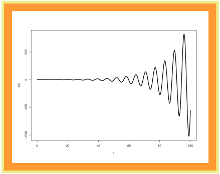
stable, non-oscillating. State (0,0) = **"attractor"**



unstable, oscillating



one of the trajectories: projection of x_1 in t



one of the trajectories: projection of x_1 in t

Dynamical properties of the linear **discrete** system

- As in the continuous case, dynamical properties of the linear discrete system can be easily mathematically derived.
- Assume the input signal decays at time k_0 ($k > k_0 \Rightarrow v(k) = 0$). We seek the solution of

  $$\vec{x}(k+1) = \mathbb{A}\vec{x}(k)$$

  for the initial condition $\vec{x}(k_0) = x_0$ at time k_0 of input decay. Evidently:

  $$\vec{x}(k) = \underbrace{\mathbb{A}\cdot\mathbb{A}\cdot\ldots\cdot\mathbb{A}}_{(k-k_0)\times}\vec{x}_0 = \mathbb{A}^{k-k_0}\vec{x}_0$$

  which can be written as

  $$\boxed{\vec{x}(k) = \sum_{i=1}^{n} \alpha_i \vec{r}_i \lambda_i^k}$$

  where $\vec{r}_i$ are the *eigenvectors* $\mathbb{A}$, λ_i the corresponding *eigenvalues* and α_i are constants depending on the initial condition.
- System is **stable** ($x(k) \rightarrow_{k\rightarrow\infty} 0$) if and only if $\forall i\ |\lambda_i| < 1$, i.e. all eigenvalues lie **inside the unit circle** in the complex plane.
- *Test your memory: where do the eigenvalues of $\mathbb{A}$ lie for a* **stable continuous** *linear system?*

Example: discrete non-linear system

- **Linear systems:** easy to model mathematically, behavior easy to determine from the state description of dynamics. **Non-linear systems:** situation much trickier.
- Example: population in time modeling. First "shot" at a discrete model:

$$x(k+1) = p \cdot x(k)$$

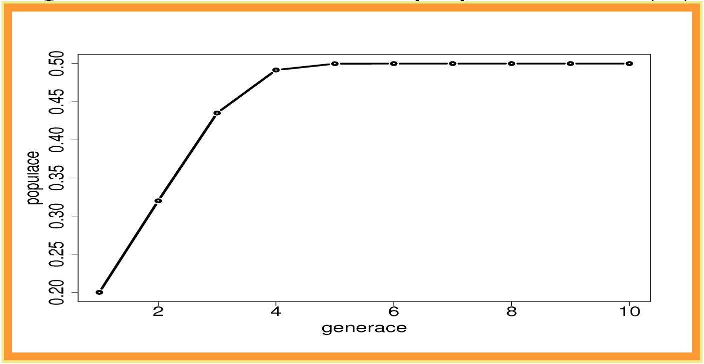
$0 \leq x(k) \leq 1$ population size in the k-th generation, p - growth parameter (breeding rate)

- This model is linear with solution $x(k) = p^k$, for $p > 1$ unstable ($x(k) \rightarrow_{k\rightarrow\infty} \infty$).
- Populations cannot grow to ∞ - will run out of food. Model must be refined by the food factor $(1 - x(k))$ decaying with population size. We obtain the **logistic model:**

$$\boxed{x(k+1) = p \cdot x(k) \cdot (1 - x(k))}$$

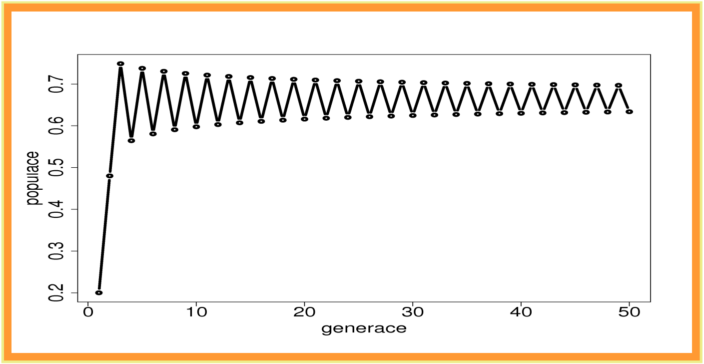
assuming a normalized population size: $0 \leq x(k) \leq 1$ for $\forall k$.

- Unlike in the linear model there is no analytical solution $x(k)$ now. Let us investigate numerically: e.g. for $p = 2$ and an initial population $x(0) = 0.2$.

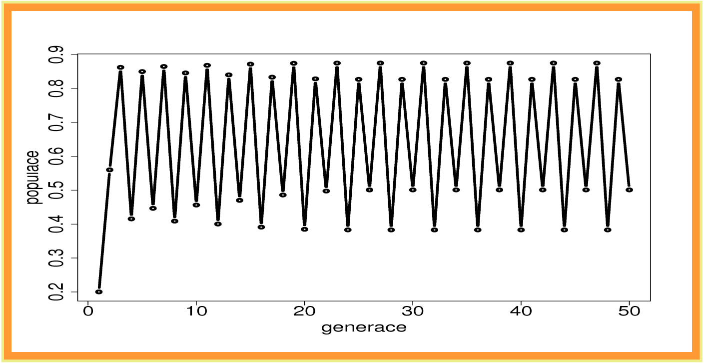


Pro $p = 2$: converges to a stable state.

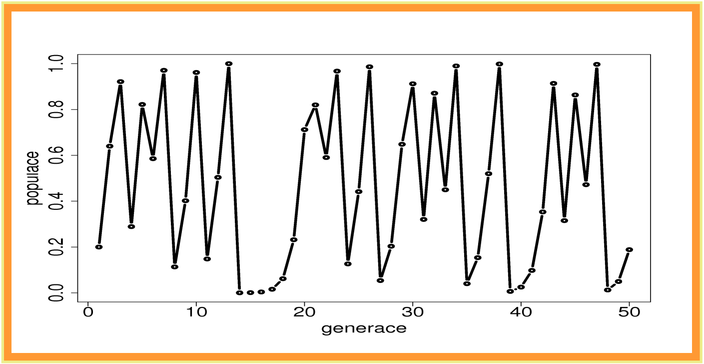
Example: discrete **non-linear** system

- 

  At $p \approx 3$, abrupt change:
  periodical behavior: a 2-generation period

- 

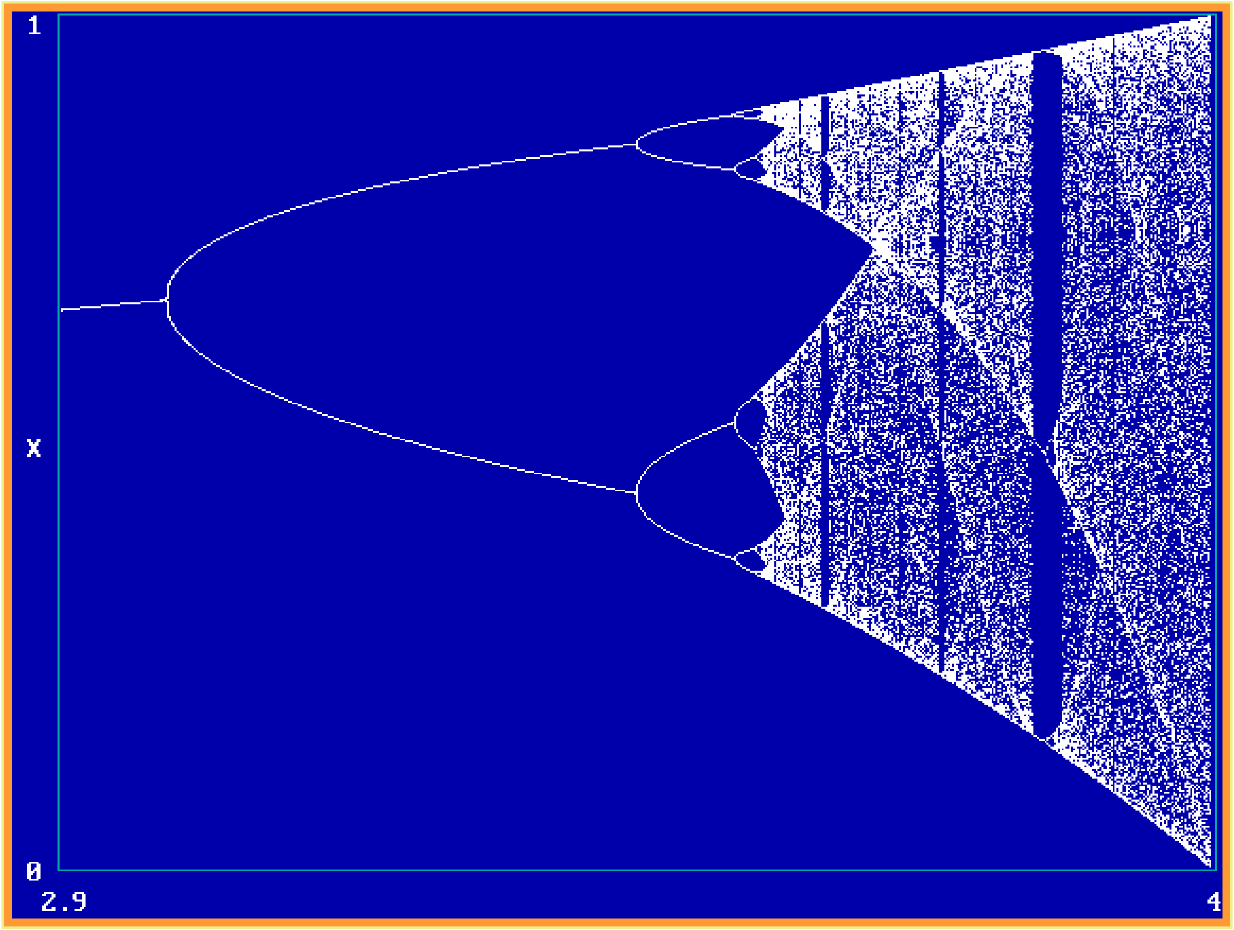
  At $p \approx 3.5$, abrupt change:
  period doubles to 4 generations.
  Further period doublings with further growing p.

- 

  At $p \approx 3.57$: Emergence of
  chaotic behavior
  non-periodic, $x(k)$ will eventually acquire **all** values in the interval $(0; 1)$ (although the system is discrete!).

Emergence of Chaos

- **Bifurcation diagram**.



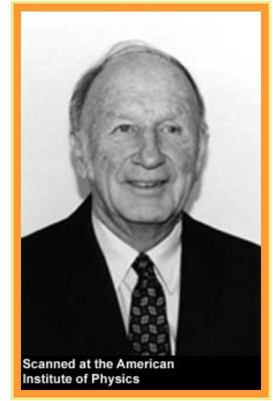
Horizontal: p value.
Vertical: all values acquired by $x(k)$ $(k = 0, 1, \ldots \infty)$ for a given p.

Chaos in continuous non-linear systems

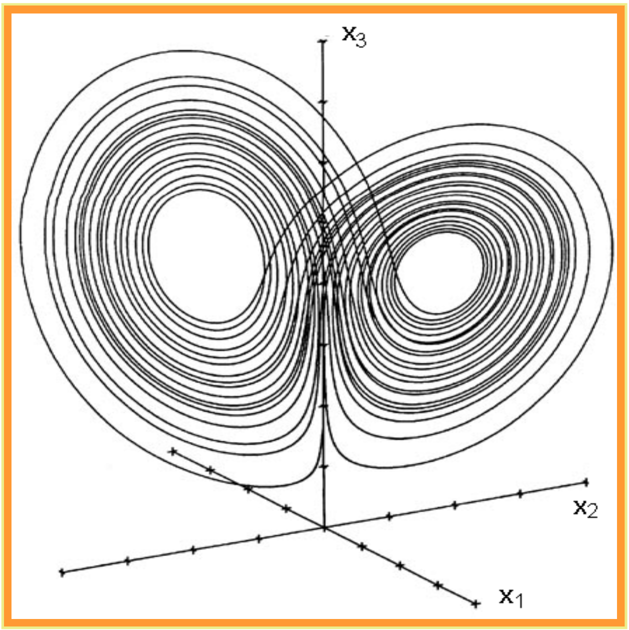
- **Edward Norton Lorenz** (1917 - 2008)
  Introduced a simple non-linear model of a meteorological phenomenon:

$$\dot{x}_1 = a(x_2 - x_1)$$
$$\dot{x}_2 = x_1(b - x_3) - x_2$$
$$\dot{x}_3 = x_1 x_2 - c x_3$$

  (a, b, c real constants).



- 3D trajectories in the **state space** $[x_1, x_2, x_3]$ (pro $a = 10, b = 28, c = 8/3$)



  - **“Strange attractor”**. Chaotic behavior:
  - Trajectory never intersects itself (consequence of non-periodicity).
  - Small difference in the initial condition $\Rightarrow$ Large difference after a small Δt. (‘butterfly effect’)
  - The first chaotic system “discovered”. (1963).

Artificial Intelligence

- Motto from Czech book Artificial Intelligence 1 *Natural intelligence will be soon surpassed by AI. But the natural stupidity won't be surpassed ever.*

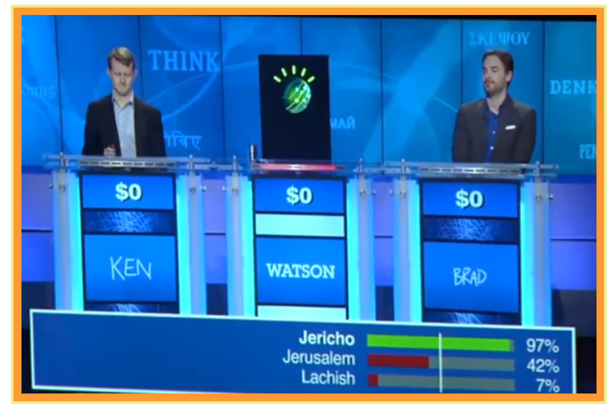
- Various definitions
  - Marvin Minsky, 1967 *Artificial intelligence is a science on creating the machines or system which, when solving certain problem, will be capable of using approach which used by a human would be considered as intelligent.*
  - Kotek a kol., 1983 *Artificial intelligence is the property of artificially created systems capable of recognition of objects, phenomena and situations, analysis of their mutual relations and therefore capable of creating inner models of the world the systems exist within and on such basis capable of making purposed decisions with capability to predict the consequences of those decisions and finding new laws among various models or their groups.*

Branches of AI

- Problem solving, knowledge representation, machine learning
- Pattern recognition, machine perception
- Neural networks, evolution algorithms
- Planning and scheduling
- Game theory
- Distributed and multi-agent systems
- Natural language processing
- Biocybernetics

Interesting state-of-the-art AI projects

- DARPA Grand Challenge/Urban Challenge
- IBM Jeopardy/Big Blue
- Rat brain robot
- Robotic projects (Boston dynamics)
- Computer Game AI (Starcraft, poker)



Summary

- Cybernetics is a science about non-trivial **systems** and **processes**, their **modeling** and **control** and **information** transmission.
- Investigates the aspects common to **diverse kinds of systems** (technical, biological, socio-economical, ecological, ...).
- One of the aspects of systems is **dynamics** (state unfolding in time).
- Dynamics easy to model for **linear systems**.
- Basic system dynamics model: **state description**.
- From a linear model state description one **easily derives** important **asymptotic properties** (mainly stability), and generally the **time response**, which is always a linear combination of
  - complex exponential functions (for continuous systems)
  - complex power functions (for discrete systems)
- For **nonlinear systems**, unfolding in time may be much more **complex** and there is in general no way to derive it mathematically.
- Even simply described non-linear systems may unfold in an extremely complicated manner - **chaotically**.
- **Next time:** Neural networks.