MTB Challenge – Winter Term 2017/2018

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1 Introduction

In many fields of human interest it is necessary to precisely estimate state of a system using some noisy sensors. There are more than one method of doing so according to system properties. It is extremely computationally challenging to solve arbitrary system with unknown stochastic parameters such as its Joint Probability Distribution ($P_{\mathbf{X}}$) or if it contain memory.

The stochastic systems are usually simplified as a Markov chain or a hidden Markov chain. This process is then called Markov process and it is assumed as a memoryless discrete system. The future state ($\boldsymbol{X}_{k+1}$) of this system is defined as a vector function of the actual state ($\boldsymbol{X}_k$) and the input of a system ($\mathbf{u}_{k+1}$) by:

$$\mathbf{X}_{k+1} = \mathbf{f}\left[\mathbf{X}_k, \mathbf{u}_{k+1}\right] + \mathbf{n}_{k+1}, \tag{1}$$

where $\mathbf{f}\left[.\,,.\right]$ is a process model and $\mathbf{n}_{k+1}$ is an additive process noise. We usually do not observe directly the state of a system but we use sensors which output is so called observation vector ($\mathbf{z}_{k+1}$) given by:

$$\mathbf{z}_{k+1} = \mathbf{h}\left[\mathbf{X}_{k+1}, \mathbf{u}_{k+1}\right] + \mathbf{w}_{k+1}, \tag{2}$$

where $\mathbf{h}\left[.\,,.\right]$ is a observation model and $\mathbf{w}_{k+1}$ is an additive measurement noise. The problem is to estimate system state and its error.

There are several filtering method how to obtain system state and error of the state estimate depending on the process and observation models. If the model is linear function of the process state and its input the Kalman Filter (KF) is optimal solution [1]. When the model is not a linear function then KF is not optimal and even may not converge. From the second half of 20–th century the Extended Kalman Filter (EKF) is mainly used for solution of nonlinear systems. The problem is, that EKF is only linearization of the process function by Taylor series of the first order by computation of the Jacobian of the process function. This method can be difficult because Jacobian may not be defined. Another problem is that if the process model is highly nonlinear the Jacobian is poor approximation and the error of estimate is high.

For this reason a new method was developed in 1990–ties. The method is based on particle filtration, but its main advantage is that for certain systems it gives better precision of the estimate than particle filter with far lower computational cost. This method is called Unscented Kalman Filter (UKF) and it is based on evolving only low number of correctly chosen particles called sigma points through nonlinear system function and approximation of the evolved sigma points by Gaussian distribution [2], [3]. Also, really handy is UKF tutorial by Cyrill Stachniss.

2 Competition setting

The task is to realize Unscented Kalman Filter for estimation of the position and heading of a four–wheel robot type differential drive which can be modelled as a unicycle by velocity (v) in heading direction and angular velocity (ω). Details about differential drive can be found here.

The problem is that unicycle model is not correct since the four–wheel robot movement is more "tank–like". This fact is causing nonlinearity in system model, because wheels are slipping while turning. At the same time when the set velocity is low and angular velocity is not set to zero robot is not able to overcome friction between wheels and surface and will not move at all.

Usually, wheeled robots are navigated using mainly Global Navigation Satellite System (GNSS) receiver and Inertial Measurement Unit (IMU). In this task the available sensors are Real Time Kinematics (RTK) Global Positioning System (GPS), hall sensors attached to motors and magnetometer. The RTK GPS is sending data using so–called National Marine Electronics Association (NMEA) protocol. For simplicity the GPS data are given as a vector of 2D Cartesian position, velocity and heading. Assume white Gaussian noise of the position with zero mean ($\mu_{xy} = 0$ m) and standard deviation $\sigma_{xy} = 0.3$ m. Statistical parameters of measured velocity and heading with GPS are more tricky because the precision of velocity decrease with decreasing velocity. Heading measurement depends on velocity, if two antenna solution is not used. For this reason the low pass filter is used and when velocity is lower than $v < 0.2$ ms^{-1} then the measuring of velocity and heading is stopped and value in NMEA is zero. Again, for simplicity assume Additive white Gaussian noise (AWGN) if $v \geq 0.2$ ms^{-1} with following parameters: $\mu_v = 0$ ms^{-1}, $\sigma_v = 0.03$ ms^{-1}, $\mu_H = 0$ rad, $\sigma_H = 0.01$ rad. Another problem is that GPS receiver sampling rate is 1 Hz, but the system is aimed for sampling period given by constant TS in *UKFdata.mat*. Also short outage can occur (seconds).

Hall sensors works with excellent precision at sub–millimetre level, but it cannot be properly used if wheels are slipping during turns following speed according to differential drive model. Distance between robot right and left wheel is L=0.44 m. Wheel radius is 0.13 m. Used Hall sensor measure distance in revolutions with 2000 pulses per revolution with $\sigma = 1$ pulse.

Finally, the magnetometer is used for measuring absolute heading value of the robot. Problem is that magnetic field measured by magnetometer can be easily influenced by near electromagnetic field produced by motors and by magnetic materials near the sensor. But it is possible to correct error and assume zero mean. High precision magnetometers can achieve heading precision around $0.5°$ RMS if the tilt is lover than $\pm 15°$. Assume low cost sensor with heading precision $\sigma = 0.1$ rad.

Assume independent observation ($\mathbf{z}_{k+1}$) and state ($\boldsymbol{X}_k$) vector. In another words every used covariant matrix (Σ) is diagonal. The system function is given by:

$$\begin{aligned} H_{k+1} &= H_k + T_S \omega_{k+1} + n_{H\,k+1} \\ x_{k+1} &= x_k + T_S v_{x\,k+1} \mathrm{e}^{-2T_S\omega_{k+1}^2} \left(\frac{1 - \left|\omega_{k+1}^{0.6}\right|}{\mathrm{e}^{5T_S v_{k+1}}} \right)^{10} + n_{x\,k+1} \\ y_{k+1} &= y_k + T_S v_{y\,k+1} \mathrm{e}^{-2T_S\omega_{k+1}^2} \left(\frac{1 - \left|\omega_{k+1}^{0.6}\right|}{\mathrm{e}^{5T_S v_{k+1}}} \right)^{10} + n_{y\,k+1}, \end{aligned} \tag{3}$$

where v and ω are inputs of the system ($\mathbf{u}$), T_S is time step between system samples in seconds, v_x and v_y are given by:

$$\begin{aligned} v_x &= v \cos H \\ v_y &= v \sin H. \end{aligned} \tag{4}$$

3 Criteria

- This project can be selected by unlimited number of students. However, no collaboration between students is expected.
- Project should be submitted including short documentation describing how the algorithm works.
- Like for regular projects, short presentation (couple of minutes) is expected.
- To be awarded with credits it is necessary to implement UKF to extrapolate observation to sampling period given by T_S.
- In competition the minimal quadratic error is judged and first n student achieving minimal quadratic error will be awarded with prices.
- No toolboxes or external codes and libraries (dll, mex) are allowed.
- It is possible to always withdraw from the competition and select of of regular projects. This decision should be discussed with lecturers and their approval is required.

4 List of Awards

References

[1] S. M. Kay. *Fundamentals of statistical signal processing, volume I: estimation theory.* Prentice Hall Signal Processing Series. Prentice-Hall PTR, 1993.

[2] S. J. Julier, J. K. Uhlmann, and H. F. Durrant-Whyte. A new approach for filtering nonlinear systems. In *American Control Conference, Proceedings of the 1995*, volume 3, pages 1628–1632 vol.3, Jun 1995.

[3] E. A. Wan and R. Van Der Merwe. The unscented kalman filter for nonlinear estimation. In *Proceedings of the IEEE 2000 Adaptive Systems for Signal Processing, Communications, and Control Symposium (Cat. No.00EX373)*, pages 153–158, 2000.