

Homework assignment

Use the template source file (`hw1.w1` in your repository), implement the semantics described below.

Grammar rules:

$$\begin{aligned}
 \text{Expr} ::= & \text{Num} \mid \\
 & \mathbf{x} \mid \\
 & \text{Expr} \oplus \text{Expr} \mid \\
 & \text{Expr} \odot \text{Expr} \mid \\
 & \text{Expr}'
 \end{aligned} \tag{1}$$

Convention: $n, m \in \text{Num}$, $e, f, g \in \text{Expr}$ and $\cdot, +$ are standard operators on numbers.

$$\overline{n' \Rightarrow 0} \tag{2}$$

$$\overline{\mathbf{x}' \Rightarrow 1} \tag{3}$$

$$\overline{(e \oplus f)' \Rightarrow e' \oplus f'} \tag{4}$$

$$\overline{(e \odot f)' \Rightarrow e \odot f' \oplus e' \odot f} \tag{5}$$

$$\frac{e \Rightarrow g}{e \oplus f \Rightarrow g \oplus f} \tag{6}$$

$$\frac{f \Rightarrow g}{e \oplus f \Rightarrow e \oplus g} \tag{7}$$

$$\frac{e \Rightarrow g}{e \odot f \Rightarrow g \odot f} \tag{8}$$

$$\frac{f \Rightarrow g}{e \odot f \Rightarrow e \odot g} \tag{9}$$

Neutral element rules:

$$\overline{0 \oplus e \Rightarrow e} \tag{10}$$

$$\overline{e \oplus 0 \Rightarrow e} \tag{11}$$

$$\overline{1 \odot e \Rightarrow e} \tag{12}$$

$$\overline{e \odot 1 \Rightarrow e} \tag{13}$$

Absorbing element rules:

$$\overline{0 \odot e \Rightarrow 0} \tag{14}$$

$$\overline{e \odot 0 \Rightarrow 0} \tag{15}$$

Evaluation rules:

$$\overline{n \oplus m \Rightarrow n + m} \tag{16}$$

$$\overline{n \odot m \Rightarrow n \cdot m} \tag{17}$$

Naming convention

Use the following names and symbols for expressions (take care about the case sensitivity in Wolfram *Mathematica*):

- Num – `_Integer`
- X – `x`
- $Expr \oplus Expr$ – `plus`
- $Expr \odot Expr$ – `times`
- $Expr'$ – `derivative`

Function for expression rewrite (one application of SOS rules) is called `oneStepRewrite`.

Examples

- x' – `derivative[x]`
- $(42 + x)'$ – `derivative[plus[42,x]]`
- $(42 \cdot x)'$ – `derivative[times[42,x]]`
- `oneStepRewrite[derivative[x]] = {1}`
- `oneStepRewrite[derivative[plus[42,x]]] = {plus[derivative[42], derivative[x]}`

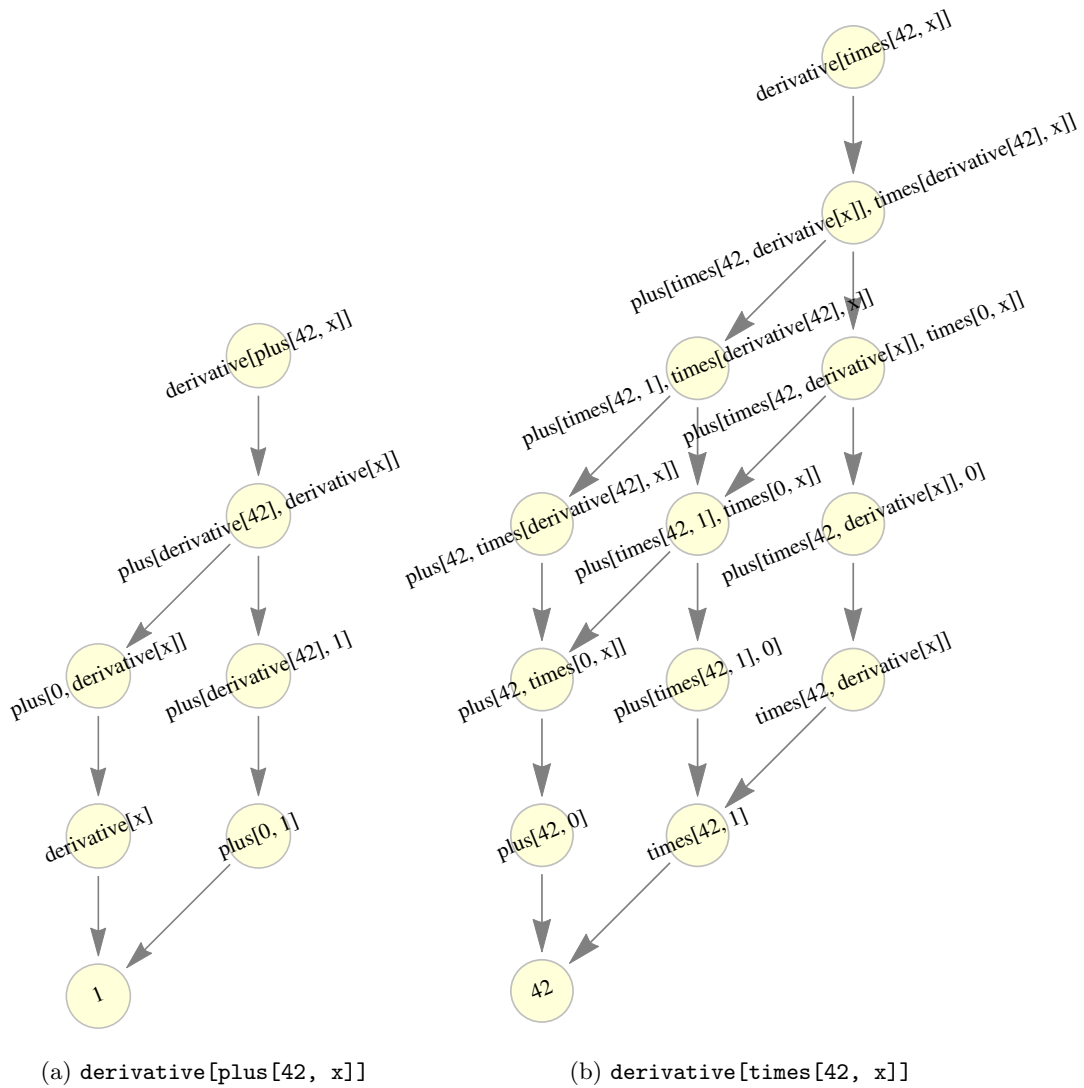


Figure 1: Example expressions evaluation, each arrow represents one call of `oneStepRewrite` function.