Combinatorial Optimization
Lab No. 4
Integer Linear Programming

Industrial Informatics Research Center

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Abstract

In this lab, we will further practice problem formulation with Integer Linear Programming. We use it to solve Game of Fivers puzzle and for modeling Power Plants Problem.

1 Game of Fivers

This is a puzzle that is played on a board $n \times n$ with n^2 stones. Each stone has two sides — black and white. In the begging, each stone is placed on one square facing with white side up. In each turn, the player turns one stone to the other side and with it also its *4-neighborhood*. The goal is to turn all stones to the black face using the minimal number of moves. For humans, boards with sizes $n > 7$ are already challenging ones.

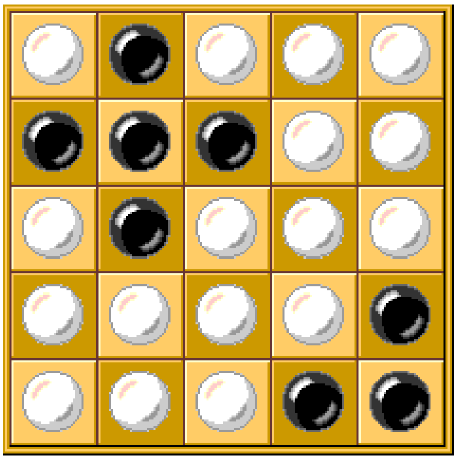


Figure 1: A state of a game in the Game of Fivers.

To formulate this game as an ILP, one needs suitable encoding of the problem — variables, objective and constrains. To do this, one needs to realize following properties of the problem:

1. the order of moves does not matter – the important is what stones will be turned
2. every stone is turned at most once – otherwise there is a way reaching the same state with less moves
3. the last move of the game is characterized by that for each stone, in its 4-neighborhood including the stone itself, an odd number of stones was turned – otherwise we do not end up with all stones facing black

The ILP solves this puzzle *implicitly*, i.e. not giving us a sequence of actions leading to the final state, but it rather encodes necessary conditions for the final state stated by Property 3. Therefore, if ILP solver finds a feasible solution, then it denotes a solution to the puzzle.

A lab exercise: Derive an ILP formulation for Game of Fivers. Then, implement it for general n using Gurobi solver. How large boards can you solve?

2 Power Plants

An electricity company Energy4U faces a following optimisation problem. Based on the contracts from the electricity market, the company knows what is the demand for the electrical energy in each hour of the following day, i.e. you are given parameters $d_t \in \mathbb{R}_{\geq 0}$ representing the demand during hour t and $t+1$, where $t \in \{0, 1, \ldots, 23\}$. The company wants to satisfy this demand by generating sufficient amount of electrical energy using its power plants.

The company has $n^{\text{base}} \in \mathbb{Z}_{\geq 0}$ *base load power plants* and $n^{\text{peak}} \in \mathbb{Z}_{\geq 0}$ *peaking power plants* that generate energy. These two types of power plants differ in how fast they can be turned on, cost for generating energy etc.

Generating energy using base load power plants is very cheap (usually nuclear plants), however, they have slow startup and shutdown, each power plant either runs for the whole day or is turned off. The amount of energy generated in each hour by one base load power plant is $e^{\text{base}} \in \mathbb{R}_{\geq 0}$ and the cost of running one base load power plant for one hour is $c^{\text{base}} \in \mathbb{Z}_{\geq 0}$.

On the other hand, peaking power plants (usually gas turbines) have fast startup and shutdown. Therefore, in each hour of a day they can be either turned on or off. However, the cost of generating energy is usually higher than by base load power plants. The amount of energy generated in each hour by one peaking power plant is $e^{\text{peak}} \in \mathbb{R}_{\geq 0}$ and the cost of running one peaking power plant for one hour is $c^{\text{peak}} \in \mathbb{Z}_{\geq 0}$.

Moreover, the company has energy storage where the excessive energy can be stored (i.e. excessive energy is the remaining energy after subtracting the demand). This storage can be used to cover the electricity demand in the future altought with a loss of γ. For example, if the amount of stored energy in hour 3 is 60, then taking 20 from the storage will cover $\gamma \cdot 20 = 16$ of the energy demand in hour 4 (we assume that putting energy into storage does not incur any loss). The amount of energy in the storage is initially 0 and it has a capacity of $s^{\max}$. It also has to be enforced that it is not possible to both store and take the energy to/from the storage during the same hour.

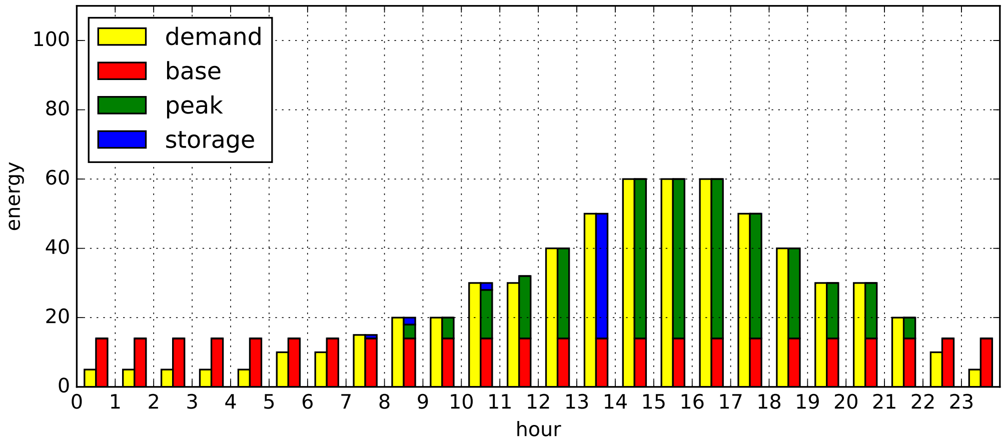
The goal is to find out how many base load and peaking power plants must be started so that the demand is satisfied using the minimum cost.

A lab exercise: Derive an ILP formulation for the Power Plants problem. Then, implement it using Gurobi solver.

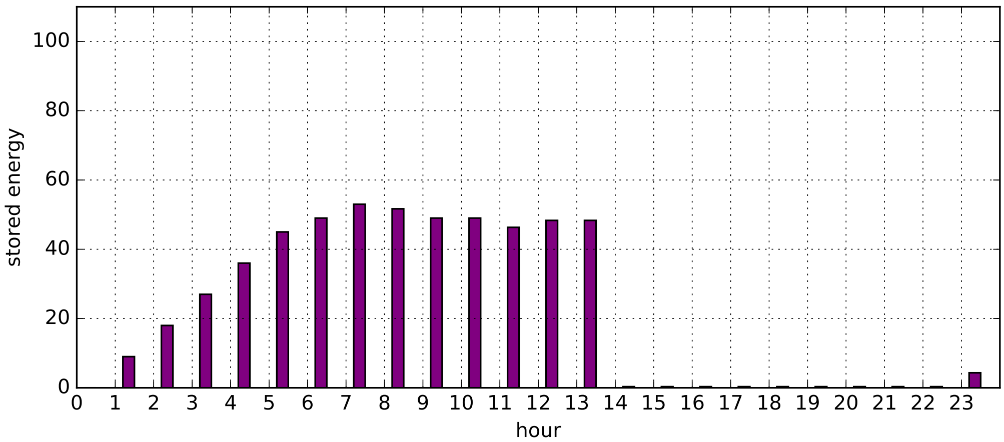
You can test your model on the following instance

```
d = [5, 5, 5, 5, 5, 10, 10, 15, 20, 20, 30, 30, 40, 50, 60, 60, 60, 50, 40, 30, 30,
     20, 10, 5]
n_base = 10
e_base = 7
c_base = 0.0833
n_peak = 40
e_peak = 2
c_peak = 12
s_max = 100
gamma = 0.75
```

The optimal objective value should be 1840. Fig. 2a shows how the energy demand is satisfied in the optimal solution and Fig. 2b shows how the state of the energy storage changes over time.



(a) Demand and generation of energy.



(b) State of the storage.

Figure 2: Visualization of the optimal solution for the example instance.