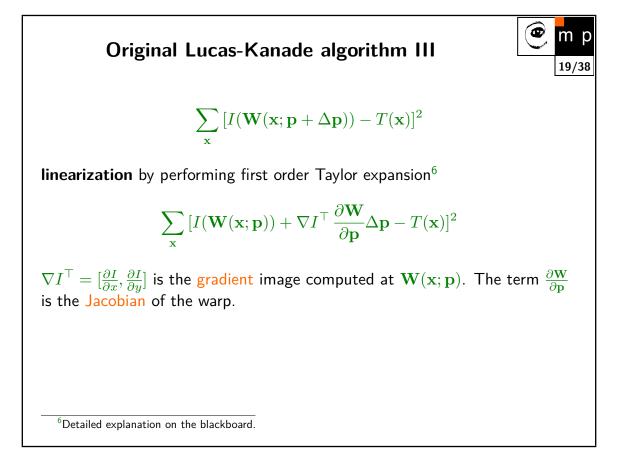


ncc - remind the notes from statistics ...

Image intensities, though organized in a matrix form, can be re-arranged into vectors. Best visualized with plots. Remember *variance, correlation*?

Sketch about coordinate systems



Taylor series and gradient of a compound function

Few notes that may help in understanding of the derivation. General first order Taylor series expansion of a scalar-valued function f of more than one variable (**x**, **a** are vectors):

$$T(\mathbf{x}) = f(\mathbf{a}) + (\mathbf{x} - \mathbf{a})^{\top} D f(\mathbf{a}), \qquad (1)$$

where $Df(\mathbf{a})$ is the gradient of f evaluated at $\mathbf{x} = \mathbf{a}$.

Gradient of a function f is a vector of partial derivatives

$$\nabla f = \left(\frac{\partial f}{\partial x_1}, \frac{\partial f}{\partial x_2}, \dots\right)^\top \tag{2}$$

Gradient of a compound function (chain rule). Suppose that $f : A \to R$ is a real-valued function defined on a subset A of R^n , and that f is differentiable at a point **a**. If function g is also differentiable and $g(\mathbf{c}) = \mathbf{a}$ then for the gradient of the compound function hold

$$D(f \circ g)(\mathbf{c}) = (Dg(\mathbf{c}))^{\top} \nabla f(\mathbf{a}), \qquad (3)$$

where $(Dg)^{\top}$ denotes the transpose of the Jacobian matrix.

Our problem is the linearization of the multidimensional warp that affects one pixel at a position $\mathbf{x} = (x, y)^{\top}$, $I(W(\mathbf{x}, \mathbf{p} + \Delta \mathbf{p}))$. Taylor expansion (1) becomes

$$T(\mathbf{p}^{k+1}) = I(\mathbf{p}^k) + \Delta \mathbf{p}^\top D I(\mathbf{p}^k)$$
(4)

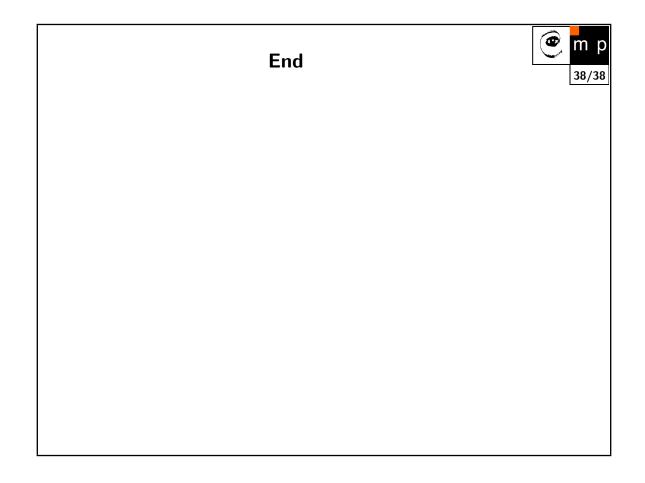
The inside function is the geometric warp hence, $g(\mathbf{p}) = W(\mathbf{x}, \mathbf{p})$. From the above it follows that the linearization is

$$I(W(\mathbf{x}, \mathbf{p} + \Delta \mathbf{p})) \approx I(W(\mathbf{x}, \mathbf{p})) + \Delta \mathbf{p}^{\top} \frac{\partial W}{\partial \mathbf{p}}^{\top} \nabla I , \qquad (5)$$

where

$$\nabla I = \left(\frac{\partial I}{\partial x}, \frac{\partial I}{\partial y}\right)^{\top} , \qquad (6)$$

is the image gradient and $\frac{\partial W}{\partial \mathbf{p}}$ is the Jacobian of the warp.



References