Normalized cross-correlation

You may know it as correlation coefficient (from statistics)

$$\rho_{X,Y} = \frac{\mathrm{cov}(X,Y)}{\sigma_X \sigma_Y} = \frac{E[(X-\mu_X)(Y-\mu_Y)]}{\sigma_X \sigma_Y}$$

where σ means standard deviation.

Having template $T(k,l)$ and image $I(x,y)$,

$$r(x,y) = \frac{\sum_k \sum_l \left(T(k,l) - \overline{T}\right)\left(I(x+k,y+l) - \overline{I(x,y)}\right)}{\sqrt{\sum_k \sum_l \left(T(k,l) - \overline{T}\right)^2}\sqrt{\sum_k \sum_l \left(I(x+k,y+l) - \overline{I(x,y)}\right)^2}}$$

ncc - remind the notes from statistics ...

Image intensities, though organized in a matrix form, can be re-arranged into vectors. Best visualized with plots. Remember *variance*, *correlation*?

Sketch about coordinate systems

Original Lucas-Kanade algorithm III

$$\sum_{\mathbf{x}} [I(\mathbf{W}(\mathbf{x};\mathbf{p}+\Delta\mathbf{p})) - T(\mathbf{x})]^2$$

linearization by performing first order Taylor expansion[6]

$$\sum_{\mathbf{x}} [I(\mathbf{W}(\mathbf{x};\mathbf{p})) + \nabla I^\top \frac{\partial \mathbf{W}}{\partial \mathbf{p}} \Delta\mathbf{p} - T(\mathbf{x})]^2$$

$\nabla I^\top = [\frac{\partial I}{\partial x}, \frac{\partial I}{\partial y}]$ is the gradient image computed at $\mathbf{W}(\mathbf{x};\mathbf{p})$. The term $\frac{\partial \mathbf{W}}{\partial \mathbf{p}}$ is the Jacobian of the warp.

[6] Detailed explanation on the blackboard.

Taylor series and gradient of a compound funtion

Few notes that may help in understanding of the derivation. General first order Taylor series expansion of a scalar-valued function f of more than one variable ($\mathbf{x}, \mathbf{a}$ are vectors):

$$T(\mathbf{x}) = f(\mathbf{a}) + (\mathbf{x} - \mathbf{a})^\top Df(\mathbf{a})\,, \tag{1}$$

where $Df(\mathbf{a})$ is the *gradient* of f evaluated at $\mathbf{x} = \mathbf{a}$.

Gradient of a function f is a vector of partial derivatives

$$\nabla f = \left(\frac{\partial f}{\partial x_1}, \frac{\partial f}{\partial x_2}, \dots \right)^\top \tag{2}$$

Gradient of a compound funtion (chain rule). Suppose that $f : A \to R$ is a real-valued function defined on a subset A of R^n, and that f is differentiable at a point $\mathbf{a}$. If function g is also differentiable and $g(\mathbf{c}) = \mathbf{a}$ then for the gradient of the compound function hold

$$D(f \circ g)(\mathbf{c}) = (Dg(\mathbf{c}))^\top \nabla f(\mathbf{a})\,, \tag{3}$$

where $(Dg)^\top$ denotes the transpose of the *Jacobian matrix*.

Our problem is the linearization of the multidimensional warp that affects one pixel at a position $\mathbf{x} = (x, y)^\top$, $I(W(\mathbf{x}, \mathbf{p} + \Delta\mathbf{p}))$. Taylor expansion (1) becomes

$$T(\mathbf{p}^{k+1}) = I(\mathbf{p}^k) + \Delta\mathbf{p}^\top DI(\mathbf{p}^k) \tag{4}$$

The inside function is the geometric warp hence, $g(\mathbf{p}) = W(\mathbf{x}, \mathbf{p})$. From the above it follows that the linearization is

$$I(W(\mathbf{x}, \mathbf{p} + \Delta\mathbf{p})) \approx I(W(\mathbf{x}, \mathbf{p})) + \Delta\mathbf{p}^\top \frac{\partial W}{\partial \mathbf{p}}^\top \nabla I\,, \tag{5}$$

where

$$\nabla I = \left(\frac{\partial I}{\partial x}, \frac{\partial I}{\partial y} \right)^\top\,, \tag{6}$$

is the image gradient and $\frac{\partial W}{\partial \mathbf{p}}$ is the Jacobian of the warp.

End

References