Differentiating density estimator II

$$\nabla f_{h,K}(\mathbf{x}) = \frac{1}{nh^d} \sum_{i=1}^{n} \nabla K \left(\frac{\mathbf{x} - \mathbf{x}_i}{h} \right)$$

using profiles, instead of kernels

$$K \left(\frac{\mathbf{x} - \mathbf{x}_i}{h} \right) = c_k k \left(\left\| \frac{\mathbf{x} - \mathbf{x}_i}{h} \right\|^2 \right)$$

Detailed derivation/explanation on the board and in the talk-note.pdf.

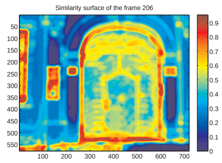
$$\begin{aligned}
\nabla f_{h,K}(\mathbf{x}) &= \frac{2c_k}{nh^{d+2}} \sum_{i=1}^{n} (\mathbf{x} - \mathbf{x}_i) k' \left(\left\| \frac{\mathbf{x} - \mathbf{x}_i}{h} \right\|^2 \right) \\
&= \frac{2c_k}{nh^{d+2}} \sum_{i=1}^{n} (\mathbf{x} - \mathbf{x}_i) g \left(\left\| \frac{\mathbf{x} - \mathbf{x}_i}{h} \right\|^2 \right) \\
&= \frac{2c_k}{nh^{d+2}} \left(\sum_{i=1}^{n} g_i \right) \left(\frac{\sum_{i=1}^{n} \mathbf{x}_i g_i}{\sum_{i=1}^{n} g_i} - \mathbf{x} \right) ,
\end{aligned}$$

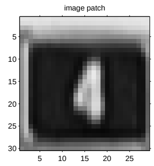
where

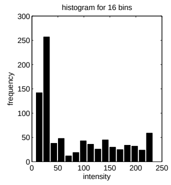
$$g_i = g \left(\left\| \frac{\mathbf{x} - \mathbf{x}_i}{h} \right\|^2 \right)$$

Mean-shift tracking - Bhattacharya coeficient

17/22







$s(\mathbf{y}) = \sum_{u=1}^{m} \sqrt{p_u(\mathbf{y}) q_u}$

model, coordinates $\mathbf{x}_i^*$ centered at $\mathbf{0}$:

$$q_u = C \sum_{i=1}^{n} k(\|\mathbf{x}_i^*\|^2)\delta(b(\mathbf{x}_i^*) - u)$$

target candidate centered at $\mathbf{y}$:

$$p_u(\mathbf{y}) = C_h \sum_{i=1}^{n_h} k\left(\left\|\frac{\mathbf{y} - \mathbf{x}_i}{h}\right\|^2\right)\delta(b(\mathbf{x}_i) - u)$$

Detailed derivation/explanation on the board and in the talk-note.pdf.

target candidate centered at $\mathbf{y}$:

$$p_u(\mathbf{y}) = C_h \sum_{i=1}^{n_h} k\left(\left\|\frac{\mathbf{y} - \mathbf{x}_i}{h}\right\|^2\right)\delta(b(\mathbf{x}_i) - u) \tag{1}$$

Bhattacharya coefficient:

$$s(\mathbf{y}) = \sum_{u=1}^{m} \sqrt{p_u(\mathbf{y}) q_u}$$

linearize around to $\mathbf{y}_0$

$$s(\mathbf{y}) \approx \sum_{u=1}^{m} \sqrt{p_u(\mathbf{y}_0) q_u} + \frac{1}{2}\sum_{u=1}^{m} \sqrt{\frac{q_u}{p_u(\mathbf{y}_0)}}\,(p_u(\mathbf{y}) - p_u(\mathbf{y}_0))$$

can be simplified to:

$$s(\mathbf{y}) \approx \frac{1}{2}\sum_{u=1}^{m} \sqrt{p_u(\mathbf{y}_0) q_u} + \frac{1}{2}\sum_{u=1}^{m} \sqrt{\frac{q_u}{p_u(\mathbf{y}_0)}}\,(p_u(\mathbf{y}))$$

in which we insert (1)

$$s(\mathbf{y}) \approx \frac{C_h}{2}\sum_{u=1}^{m} \sqrt{p_u(\mathbf{y}_0) q_u} + \frac{1}{2}\sum_{u=1}^{m} \sqrt{\frac{q_u}{p_u(\mathbf{y}_0)}} \sum_{i=1}^{n_h} k\left(\left\|\frac{\mathbf{y} - \mathbf{x}_i}{h}\right\|^2\right)\delta(b(\mathbf{x}_i) - u)$$

as we are looking for a local extrema of the Bhattacharrya coeffient we derive the above according to $\mathbf{y}$ which finally leads to the usual eqution for the shift in position $\mathbf{y}$ where the pixel weights are computed by

$$w_i = \sum_{u=1}^{m} \delta(b(\mathbf{x}_i) - u)\sqrt{\frac{q_u}{p_u(\mathbf{y}_0)}}$$

see [1] for the complete derivation. Note that evaluating histograms can be done very efficiently by using a look-up table [5, chapter 16.5]

References

Mean-shift originally from [3].

[1] Robert Collins. CSE/EE486 Computer Vision I. slides, web page. http://www.cse.psu.edu/~rcollins/CSE486/. Robert kindly gave general permission to reuse the material.

[2] Dorin Comaniciu and Peter Meer. Mean shift: A robust approach toward feature space analysis. IEEE Transactions on Pattern Analysis and Machine Analysis, 24(5):603–619, May 2002.

[3] Keinosuke Fukunaga and Larry D. Hostetler. The estimation of the gradient of a density function, with appilcations in pattern recognition. IEEE Transactions on Information Theory, 21(1):32–40, January 1975.

[4] Milan Šonka, Václav Hlaváč, and Roger Boyle. Image Processing, Analysis and Machine Vision. Thomson, 3rd edition, 2007.

[5] Tomáš Svoboda, Jan Kybic, and Václav Hlaváč. Image Processing, Analysis and Machine Vision. A MATLAB Companion. Thomson, 2007. Accompanying www site http://visionbook.felk.cvut.cz.

References

[1] Dorin Comaniciu, Visvanathan Ramesh, and Peter Meer. Real-time tracking of non-rigid objects using mean shift. In *IEEE Conference on Computer Vision and Pattern Recognition*, volume 2, pages 142–149. IEEE Computer Society Press, 2000.

[2] Tomáš Svoboda, Jan Kybic, and Václav Hlaváč. *Image Processing, Analysis and Machine Vision. A MATLAB Companion*. Thomson, 2007. Accompanying www site http://visionbook.felk.cvut.cz.