Modal Logics for Multi-Agent Systems

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Section 3. Logics of Action and Time

Logics of Action and Time
3.1 Dynamic Logic
3.2 Temporal Logic
3.3 Linear Time Logic
3.4 Computation Tree Logic

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But:

- MAS are dynamic!

3.1 Dynamic Logic

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As usual, $\langle\alpha\rangle\varphi \equiv \neg[\alpha]\neg\varphi$.

3rd idea: Programs/actions can be combined (sequentially, nondeterministically, iteratively), e.g.:

$$[\alpha; \beta]\varphi$$

would mean "after every execution of α and then β, formula φ holds".

Definition 3.1 (Labelled Transition System)

A labelled transition system is a pair

$$\langle St, \{\xrightarrow{\alpha} : \ \alpha \in \mathbf{L}\}\rangle$$

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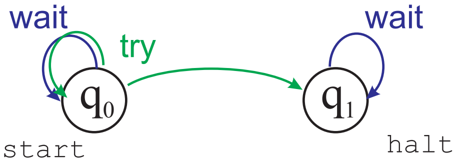
Definition 3.2 (Dynamic Logic: Models)

A model of propositional dynamic logic is given by a labelled transition systems and an evaluation of propositions.

Definition 3.3 (Semantics of DL)

$\mathcal{M}, s \models [\alpha]\varphi$ iff for every t such that $s \xrightarrow{\alpha} t$, we have $\mathcal{M}, t \models \varphi$.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try
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q0
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start
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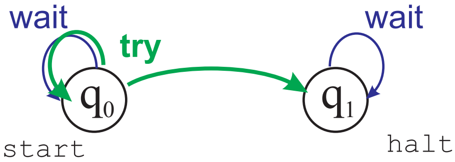
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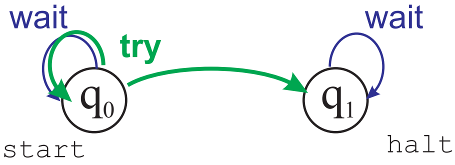

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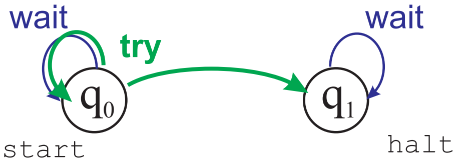

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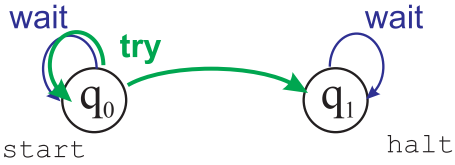

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3.2 Temporal Logic

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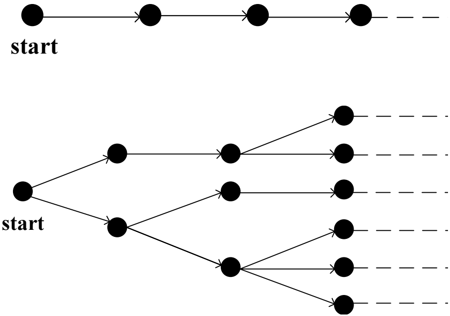

Typical temporal operators

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$\mathcal{X}\varphi$	φ is true in the next moment in time
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$$\mathcal{G}((\neg\text{passport} \vee \neg\text{ticket}) \rightarrow \mathcal{X}\neg\text{board\_flight})$$

$$\text{send}(\text{msg}, \text{rcvr}) \rightarrow \mathcal{F}\text{receive}(\text{msg}, \text{rcvr})$$

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- safety properties
- liveness properties
- fairness properties

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"something bad will not happen"

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Fairness:

- Useful when scheduling processes, responding to messages, etc.
- Good for specifying properties of the environment.

3.3 Linear Time Logic

Linear Time: LTL

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Notation:

- $\lambda[i]$: ith time moment
- $\lambda[i \dots j]$: all time moments between i and j
- $\lambda[i \dots \infty]$: all timepoints from i on

Important: computational vs. behavioral structures

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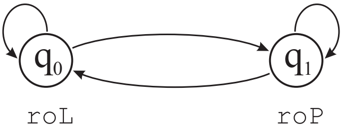
Computational



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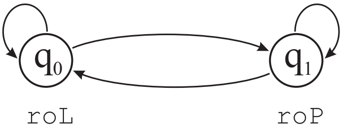

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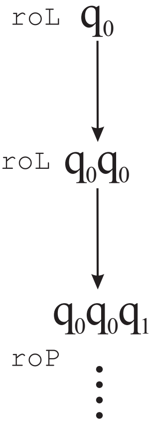

LTL models are defined as behavioral structures!

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Note that:

$$\mathcal{G}\varphi \equiv \neg\mathcal{F}\neg\varphi$$

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$$\mathcal{F}\varphi \equiv \top\mathcal{U}\varphi$$

3.4 Computation Tree Logic

Branching Time: CTL

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- CTL: Computation Tree Logic.
- Reasoning about possible computations of a system
- Time is branching: we want all alternative paths included!
- Models: states (time points, situations), transitions (changes)
- Paths: courses of action, computations.

- Path quantifiers: **A** (for all paths), **E** (there is a path);
- Temporal operators: $\mathcal{X}$ (nexttime), $\mathcal{F}$ (sometime), $\mathcal{G}$ (always) and $\mathcal{U}$ (until);

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- Reasoning in "vanilla" CTL can be automatized.

Definition 3.6 (CTL models: transition systems)

A transition system is a pair

$$\langle St, \longrightarrow \rangle$$

where:

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Note that, formally, transition relation is just a modal accessibility relation.

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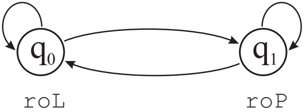

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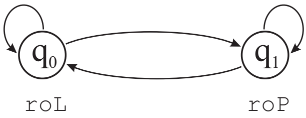

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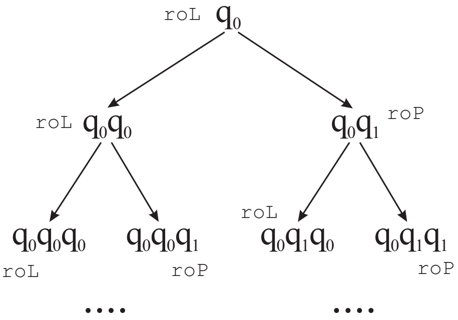

CTL models are defined as computational structures!

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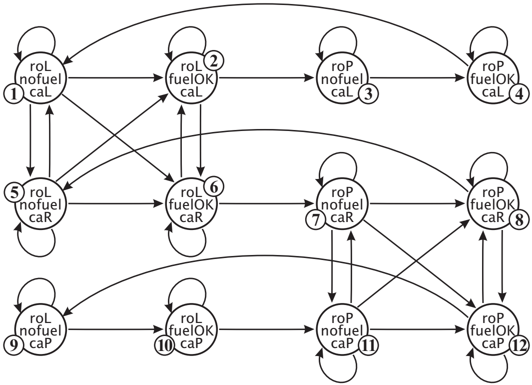
Usually, we assume that the transition relation is serial (time flows forever).
Then, all paths are infinite.

Example: Rocket and Cargo

- A rocket and a cargo,
- The rocket can be moved between London (proposition roL) and Paris (proposition roP),
- The cargo can be in London (caL), Paris (caP), or inside the rocket (caR),
- The rocket can be moved only if it has its fuel tank full (fuelOK),
- When it moves, it consumes fuel, and nofuel holds after each flight.

Example: Rocket and Cargo



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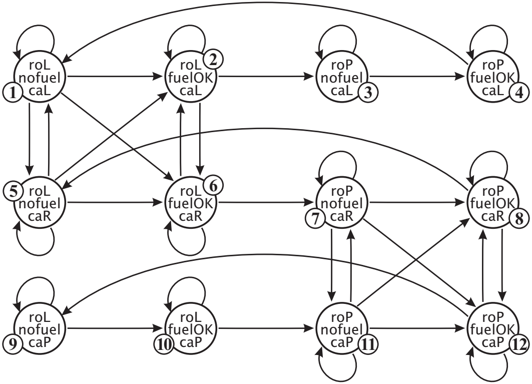

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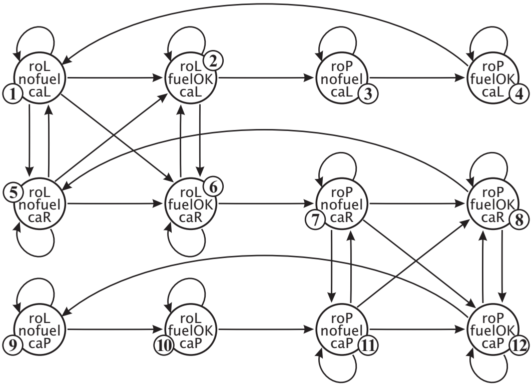

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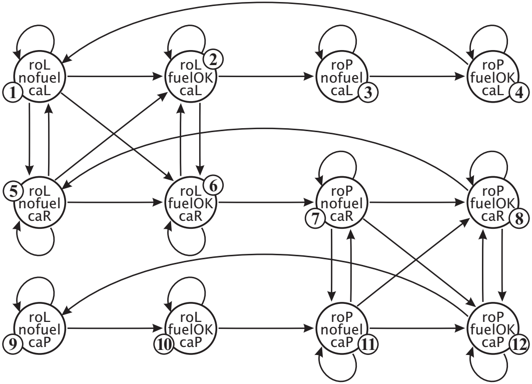

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$$\mathsf{roL} \rightarrow \mathbf{A}\mathcal{X}(\mathsf{roP} \rightarrow \mathsf{nofuel})$$

Definition 3.8 (Semantics of CTL*: state formulae)

$M, q \models \mathbf{E}\varphi$	iff there is a path λ, starting from q, such that $M, \lambda \models \varphi$;
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Definition 3.9 (Semantics of CTL*: path formulae)

Exactly like for LTL!

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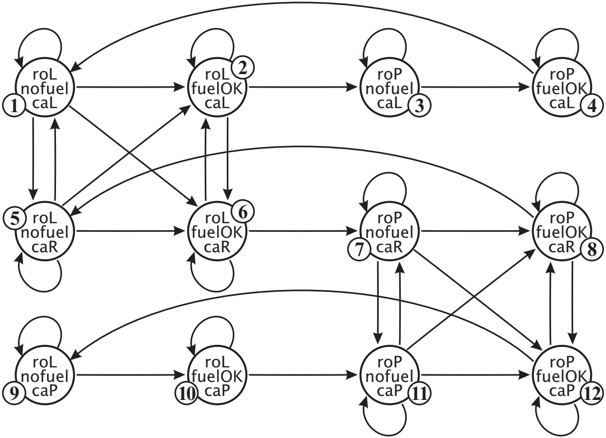

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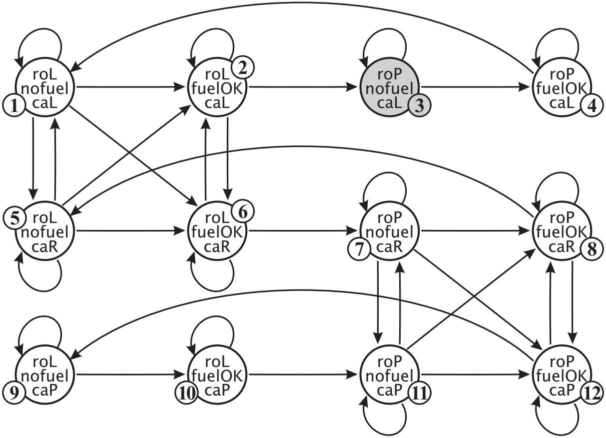

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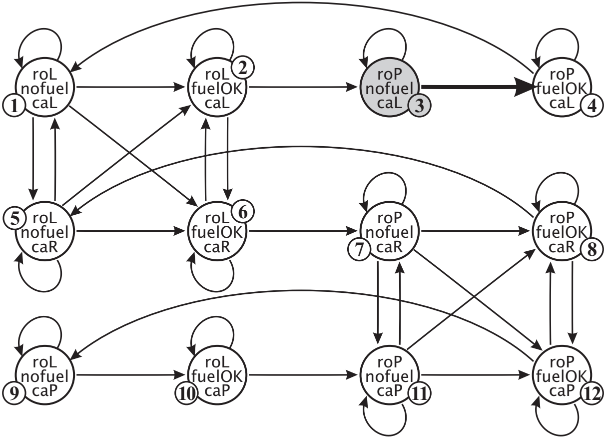

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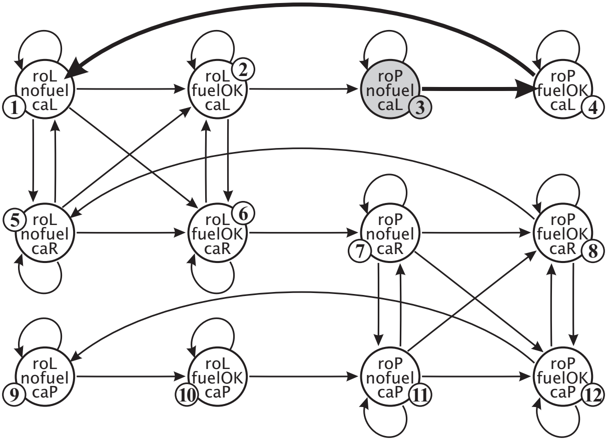

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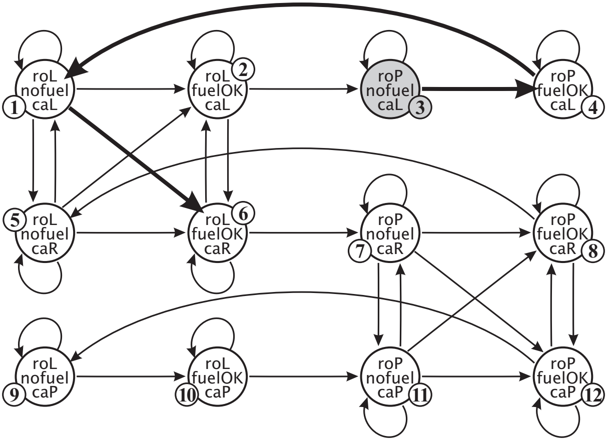

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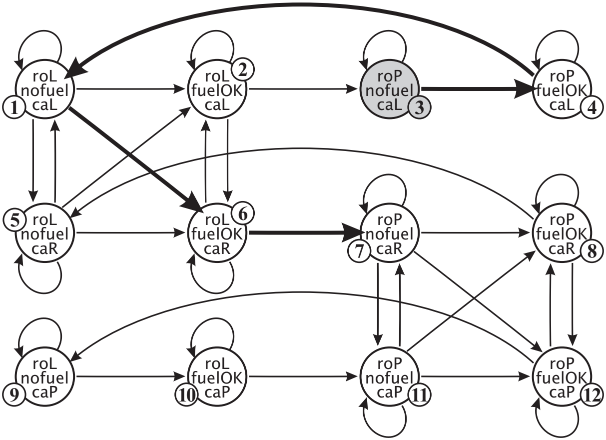

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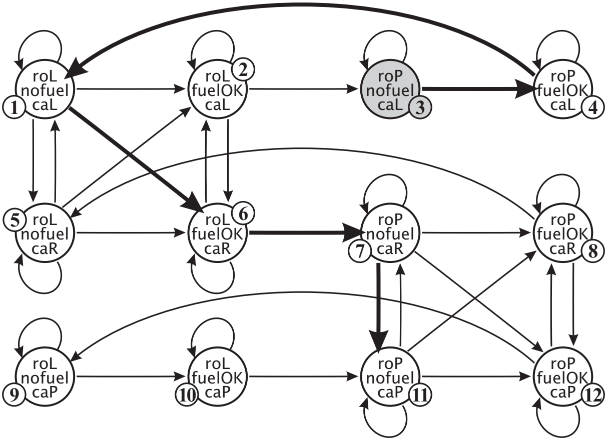

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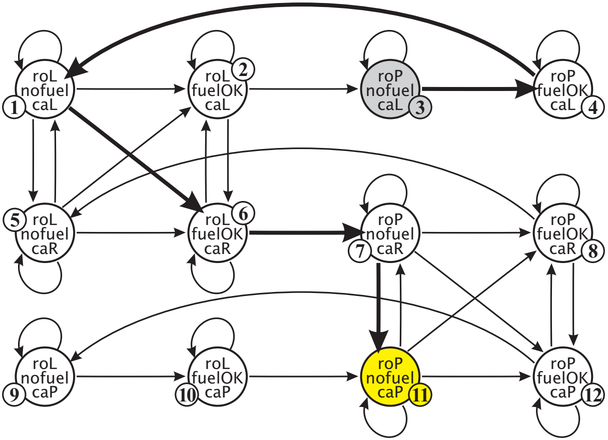

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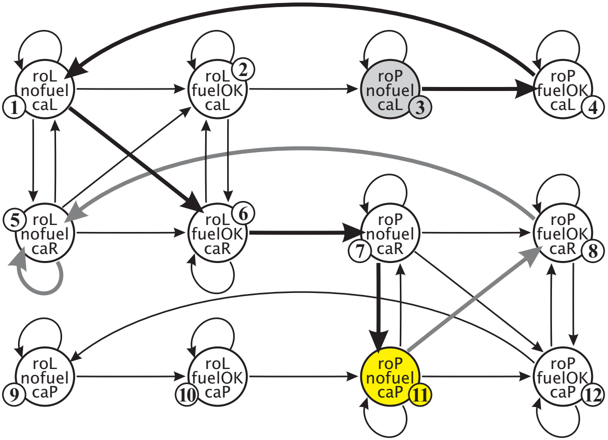

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Exercise:

How to express that there is no possibility of a deadlock?

Practical Importance of Temporal and Dynamic Logics:

Automatic verification in principle possible (model checking).

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Note:

When we combine time (actions) with knowledge (beliefs, desires, intentions, obligations...), we finally obtain a fairly realistic model of MAS.