Modal Logics for Multi-Agent Systems

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Time:

31 July–4 August 2006, 17:00–18:30

Organization:

Lectures 1, 3, 5, 8 & 10: Wojtek Jamroga,
Lectures 2, 4, 6, 7, & 9: Valentin Goranko.

More course material, including updated notes, exercises, slides, and references will be placed on the course website:

http://www2.in.tu-clausthal.de/∼wjamroga/courses/MAS2006ESSLLI/

Section 1. Agents and Logics

1.1 Agents

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- So, what is an agent precisely?
- No commonly accepted definition

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- A new paradigm for computation

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Our claim:

MAS is a philosophical metaphor that induces a specific way of seeing the world.

Intuition:

We are agents!

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The metaphor:

- Makes us use specific vocabulary

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Intuition:

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The metaphor:

- Makes us use specific vocabulary
- Makes us use specific conceptual structures
- So:
- A new paradigm for thinking and talking about the world

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- Reactive: reacts to changes in the environment
- Pro-active: takes the initiative
- Goal-directed: acts to achieve a goal
- Social: interacts with others (cooperation, communication, coordination, competition)

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- Embodied: has sensors and effectors to read from and make changes to the environment
- Intelligent: ...whatever it means
- Rational: always does the right thing

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In planning, it is called a conditional plan.

1.2 Logic for Agents

Recall...

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- Makes us use specific conceptual structures
- Provides methodology for design and programming

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This view of MAS comes close to the role of logic in both philosophy and computer science!

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Can be used for practical things (also in MAS):

- automatic verification
- executable specifications
- planning as model checking

Logic and MAS can be a good match.

1.3 Modal Logic

Modal logic is an extension of classical logic by new connectives $\Box$ and $\Diamond$: necessity and possibility.

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Independently of the precise definition, the following holds:

$$\Diamond\varphi \leftrightarrow \neg\Box\neg\varphi$$

Definition 1.1 (Modal Logic with n modalities)

The language of modal logic with n modal operators $\Box_1, \ldots, \Box_n$ is the smallest set containing:

- atomic propositions $p, q, r, \ldots$;
- for formulae φ, it also contains $\neg\varphi, \Box_1\varphi, \ldots, \Box_n\varphi$;
- for formulae φ, ψ, it also contains $\varphi \wedge \psi$.

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Note that the modal operators can be nested:

$$(\Box_1\Box_2\Diamond_1 p) \;\vee\; \Box_3\neg p$$

More precisely, necessity/possibility is interpreted as follows:

- p is necessary $\Leftrightarrow$ p is true in all possible scenarios
- p is possible $\Leftrightarrow$ p is true in at least one possible scenario

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$\rightsquigarrow$ possible worlds semantics

Definition 1.2 (Kripke Structure)

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Definition 1.3 (Kripke model)

A possible worlds model $\mathcal{M} = \langle \mathcal{S}, \pi \rangle$ consists of a Kripke structure $\mathcal{S}$, and a valuation of propositions $\pi : \mathcal{W} \rightarrow \mathcal{P}(\{p, q, r, \ldots\})$.

Remarks:

- $\mathcal{R}$ indicates which worlds are relevant for each other; $w_1 \mathcal{R} w_2$ can be read as "world w_2 is relevant for (reachable from) world w_1"

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- $\mathcal{R}$ can be any binary relation from $\mathcal{W} \times \mathcal{W}$; we do not require any specific properties (yet).
- It is natural to see the worlds from $\mathcal{W}$ as classical propositional models, i.e. valuations of propositions $\pi(w) \subseteq \{p, q, r, \ldots\}$.

Definition 1.4 (Semantics of modal logic)

The truth of formulae is relative to a Kripke model $\mathcal{M} = \langle \mathcal{W}, \mathcal{R}, \pi \rangle$, and a world $w \in \mathcal{W}$. It can be defined through the following clauses:

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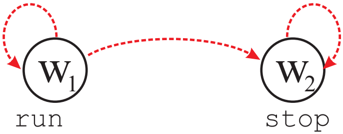
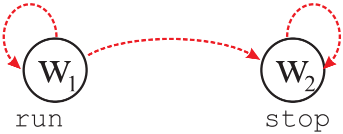
- $\mathcal{M}, w \models \Box\varphi$ iff, for every $w' \in \mathcal{W}$ such that $w\mathcal{R}w'$, we have $\mathcal{M}, w' \models \varphi$.



w_1
run
w_2
stop



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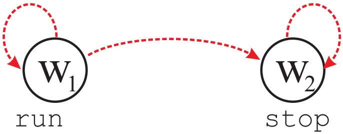

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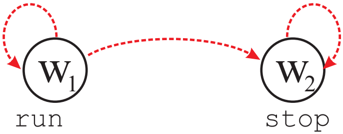

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- . . . but the result looks horribly ugly,
- . . . and in most cases it is much harder to automatize anything

1.4 Axioms for Modal Logics

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Definition 1.5 (System K)

System **K** is an extension of the propositional calculus by the axiom

$$\mathbf{K} \quad (\Box\varphi \wedge \Box(\varphi \rightarrow \psi)) \rightarrow \Box\psi$$

and the inference rule

$$\textbf{(Necessitation)} \quad \frac{\varphi}{\Box\varphi}.$$

Theorem 1.6 (Soundness/completeness of system K)

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Note: with n modalities, the calculus is called $\mathbf{K}_n$, and the theorem extends in a straightforward way.

Definition 1.7 (Extending K with axioms D, T, 4, 5)

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4	$\Box\varphi \rightarrow \Box\Box\varphi$
5	$\neg\Box\varphi \rightarrow \Box\neg\Box\varphi$

Best known extensions of system **K**:

- **S5** = **KDT45**: the standard logic of knowledge
- **KD45**: the standard logic of beliefs

Theorem 1.8 (Sound/complete subsystems of KDT45)

Let $\mathbf{X}$ *be any subset of* $\{\mathbf{D}, \mathbf{T}, \mathbf{4}, \mathbf{5}\}$ *and let* $\mathcal{X}$ *be any subset of* {*serial*, *reflexive*, *transitive*, *euclidean*} *corresponding to* $\mathbf{X}$.
Then $\mathbf{K} \cup \mathbf{X}$ *is sound and complete with respect to Kripke models the accessibility relation of which satisfies* $\mathcal{X}$.

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Corollary 1.9

System ***S5*** *is sound and complete with respect to Kripke models with equivalence accessibility relations.*

Exercise

Show that

1. Axiom **D** follows from **KT45**.
2. **KD45** is not equivalent to **K45**: axiom **D** does not follow from **K45**.

1.5 Methodology

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- φ can be satisfiable ($\mathcal{M}, q \models \varphi$ for some $\mathcal{M}, q$)
- φ can be a theorem (it can be derived from the axioms via inference rules)

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- validity: "given φ, is φ true in all models and their states?"
- theorem proving: "given φ, is it possible to prove (derive) φ?"

1.6 Logic for Agents ctd.

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- ability $\rightsquigarrow$ strategic logic,
- and combinations of the above

Modal logic seems very well suited for reasoning about various dimensions of multi-agent systems!

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Modal logic and MAS *are* a good match!