Evolutionary Algorithms: Introduction

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http://cw.felk.cvut.cz/doku.php/courses/a4m33bia/start

Contents

- **Introduction to Evolutionary Algorithms (EAs)**
  - pioneers of EAs,
  - Simple Genetic Algorithm (SGA),
  - areas for EA's applications,
  - SGA example: Evolving strategy for an artificial ant problem.
  - Schema theory – a schema, its properties, exponential growth equation and its consequences.
- **Real-coded EAs**
  - Evolution strategies,
  - crossover operators for real-coded representation,
  - differential evolution.
- **EAs for dynamic optimization problems.**
- **Multi-objective EAs (MOEAs).**
  - concept of dominance and Pareto-optimality,
  - NSGA, NSGA-II, SPEA, SPEA2.

- **Genetic Programming (GP) and Grammatical Evolution (GE).**
  - tree representation, closure condition, 'strong typing',
  - application of GP to artificial ant problem,
  - other examples.

Evolutionary Programming

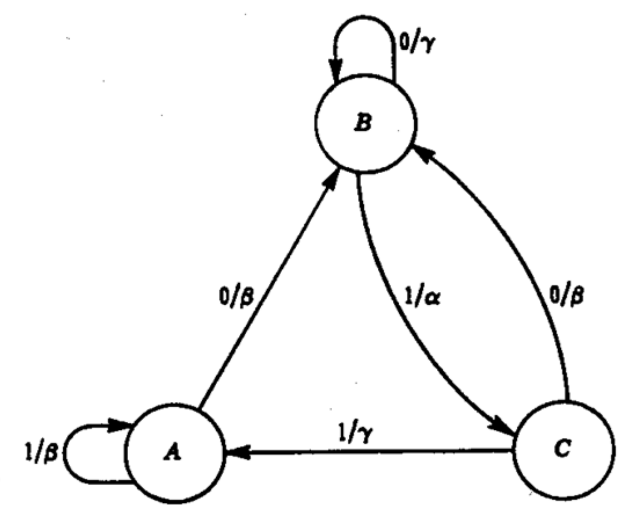
:: **Lawrence J. Fogel in 1960: Evolutionary programming (1960)**

:: **The goal** is to evolve an "intelligent behavior" that would exhibit the composite ability to (1) predict one's environment, coupled with (2) a translation of the predictions into a suitable response in light of the given goal.

- the environment was described as a sequence of symbols taken from a finite alphabet,
- finite state machines (FSMs) were used for representing the required behavior.

:: **Five modes of mutation**

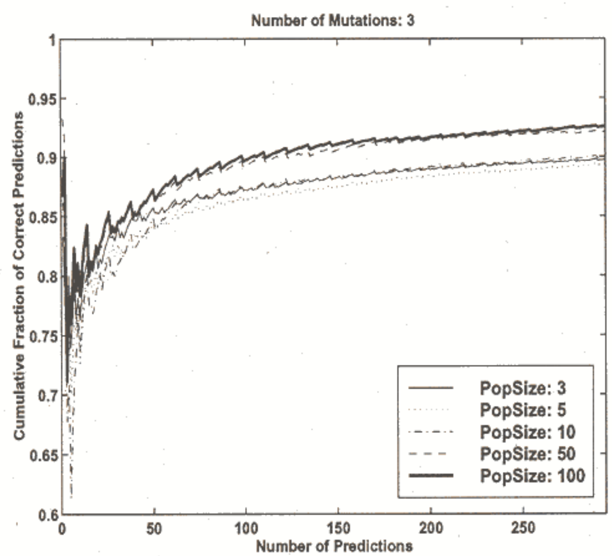
- add a state,
- delete a state,
- change the initial state,
- change an output symbol,
- change a next-state transition.



Evolutionary Programming: Prediction experiments

:: **The goal** was to evolve a population of FSMs that would predict the next symbol of the periodic sequence consisting of $(101110011101)*$ in one period.

- The alphabet for the FSMs was $0, 1$.
- Evolution was started with a training set consisting of the first 20 symbols of the cyclic sequence, which served as the initial observation.
- A population of random FSMs was generated and evolved for five generations.
- The evaluation function for evolving the population was the mean absolute error over the symbols in the training set.
- Then, the next symbol was added to the training set (as an additional observation) and the population was evolved for another five generations.
- This was repeated 300 times, resulting in a total of 1500 generations.



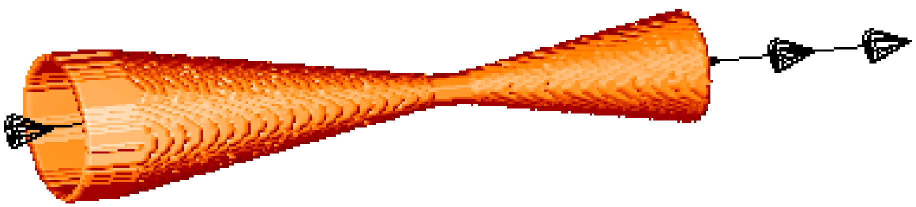
©David B. Fogel and Kumar Chellapilla: Revisiting Evolutionary Programming, Aerospace/Defense Sensing and controls, Orlando, Apr. 1998.

Evolution Strategies: Optimization of a Two-Phase Nozzle

:: **Ingo Rechenberg and Hans-Paul Schwefel: Evolution Strategy (early 1960s)**

:: **The task** was to determine the internal shape of a two-phase jet nozzle with maximum thrust under constant starting conditions.

- The nozzles were built of conical pieces such that no discontinuity within the internal shape was possible.
- Every nozzle shape could be represented by its overall length and the inside diameters at the borders between the segments (every 10mm).
- For technical reasons the incoming diameter of the first segment had to be 32mm and the smallest diameter was fixed to 6mm resulting in an convergent-divergent structure of the nozzles.



http://ls11-www.cs.uni-dortmund.de/people/kursawe/Demos/Duese/dueseGIFE.html

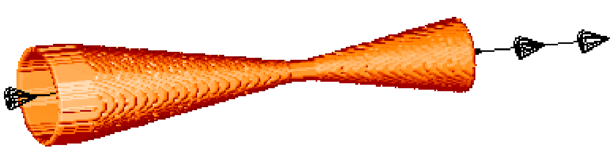
Evolution Strategies: Optimization of a Two-Phase Nozzle

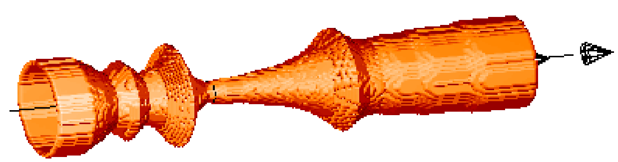
:: (1+1) Evolution Strategy.

- The genotype of each nozzle had the following form: $Z1, Z2, D(-nc), ..., D(-1), D(1), ..., D(nd)$, where Z1 and Z2 denote the number of segments in the convergent (divergent, respectively) part of the nozzle, i.e. the number of segments in front of or behind the smallest diameter.

  $D(-nc), ..., D(-1)$ designate the diameters in the convergent part and D(1), ..., D(nd) those in the divergent part of the nozzle.
- Mutation was carried out in the following form:
  - Z1 and Z2 were mutated with the help of a probability table that puts a stronger emphasis on no mutation at all.
  - According to the new lengths Z1' or Z2', segments were added or deleted at positions chosen at random.

    In the case of gene duplication the additional element(s) had the same diameter as the element to be duplicated.
  - Finally, all diameters Di, i = -nc, ..., -1, 1, ..., nd are varied in 2mm steps.





Evolutionary Algorithms: Characteristics

:: **EA are stochastic optimization algorithms**

- **Stochastic** – but not random search,
- **Use an analogy of natural evolution**
  - genetic inheritance (J.G. Mendel) – the basic principles of transference of hereditary factors from parent to offspring – genes, which present hereditary factors, are lined up on chromosomes.
  - strife for survival (Ch. Darwin) – the fundamental principle of natural selection – is the process by which individual organisms with favorable traits are more likely to survive and reproduce.
- **Not fast in some sense** – population-based algorithm,
- **Robust** – efficient in finding good solutions in difficult searches.

EA: Vocabulary

:: Vocabulary borrowed from natural genetics

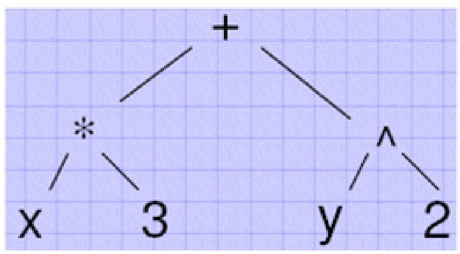
- **Individual** (chromosome + its quality measure "fitness value") – a solution to a problem.
- **Chromosome** – entire representation of the solution.
- **Fitness** – quality measure assigned to an individual, expresses how well it is adapted to the environment.
- **Gene** (also features, characters) – elementary units from which chromosomes are made.
  - each gene is located at certain place of the chromosome called locus (pl. loci),
  - a particular value for a locus is an allele.

    example: the "thickness" gene (which might be at locus 8) might be set to allele 2, meaning its second-thinnest value.
- **Genotype** – what's on the chromosome.
- **Phenotype** – what it means in the problem context (e.g., binary sequence may map to integers or reals, or order of execution, etc.).

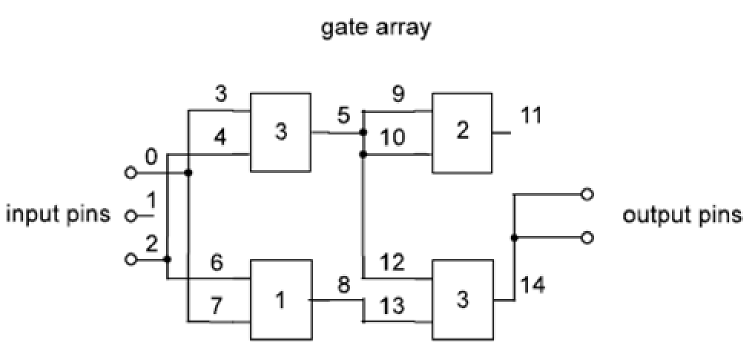
Representation

:: Problem can be represented as

- **binary string** – 1 0 1 1 0 1 1 0 0 1 0 1 1 0 1
- **real-valued string** – 3,24 1,78 -2,61
- **string of chars** – D→E→A→C→B
- or as a **tree**



- or as a **graph**, and others.



Evaluation Function

:: Objective (Fitness) function

- the only information about the sought solution the algorithm dispose of,
- must be defined for every possible chromosome.

:: Fitness function may be

- multimodal,
- discrete,
- multidimensional,
- nonlinear,
- noisy,
- multiobjective.

:: Fitness does not have to be define analytically

- simulation results,
- classification success rate.

:: Fitness function should not be too costly!!!

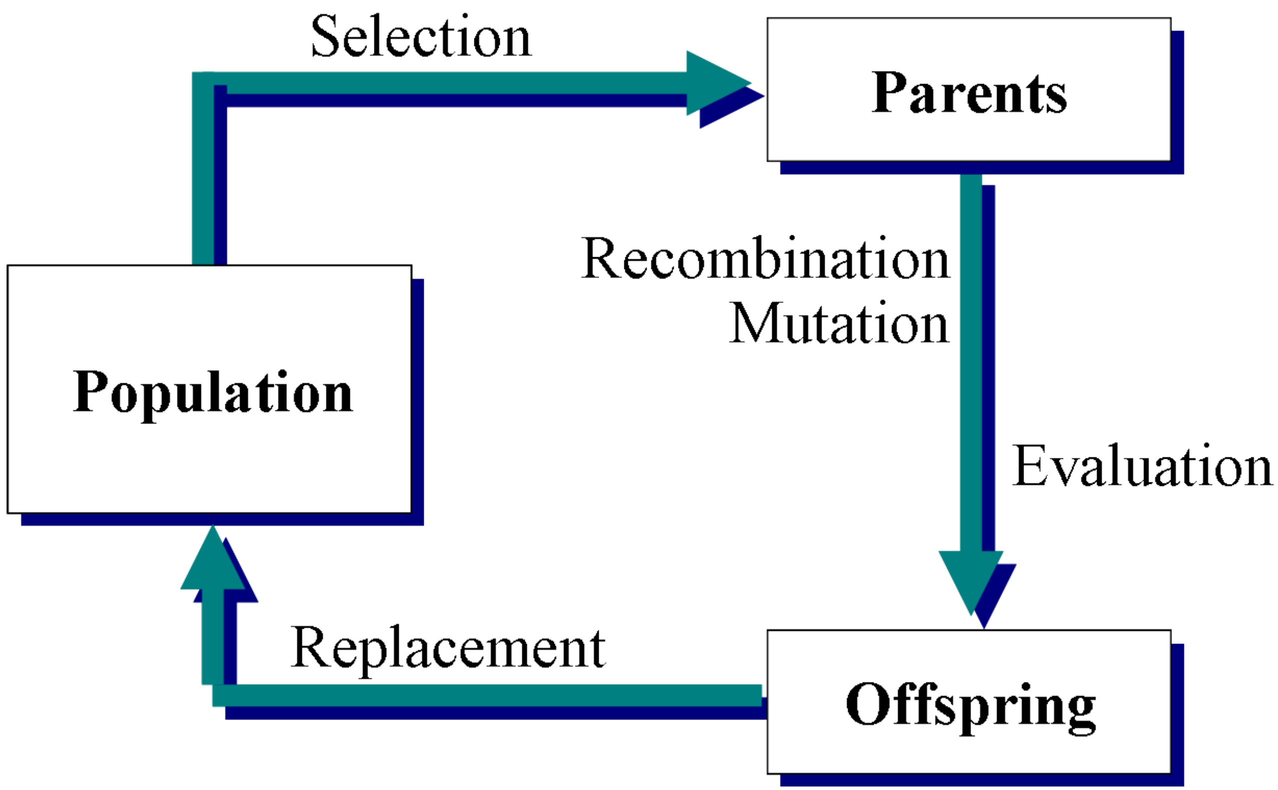
Example: Coding & Evaluation

:: **Function optimization**

- maximization of $f(x, y) = x^2 + y^2$,
- parameters x and y take on values from interval $< 0, 31 >$,
- and are code on 5 bits each.

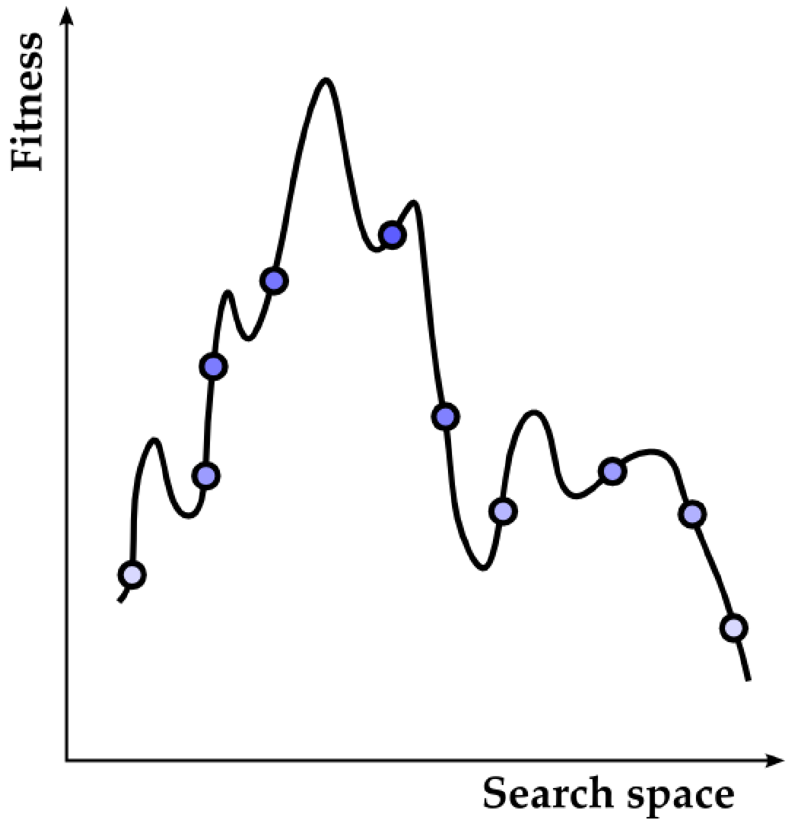
genotype	phenotype	fitness
0 0 0 0 0, 0 1 0 1 0	**0, 10**	**100**
0 0 0 0 1, 1 1 0 0 1	**1, 25**	**625 + 1 = 626**
0 1 0 1 1, 0 0 0 1 1	**11, 3**	**121 + 9 = 130**
1 1 0 1 1, 1 0 0 1 0	**27, 18**	**729 + 324 = 1053**

Evolutionary Cycle

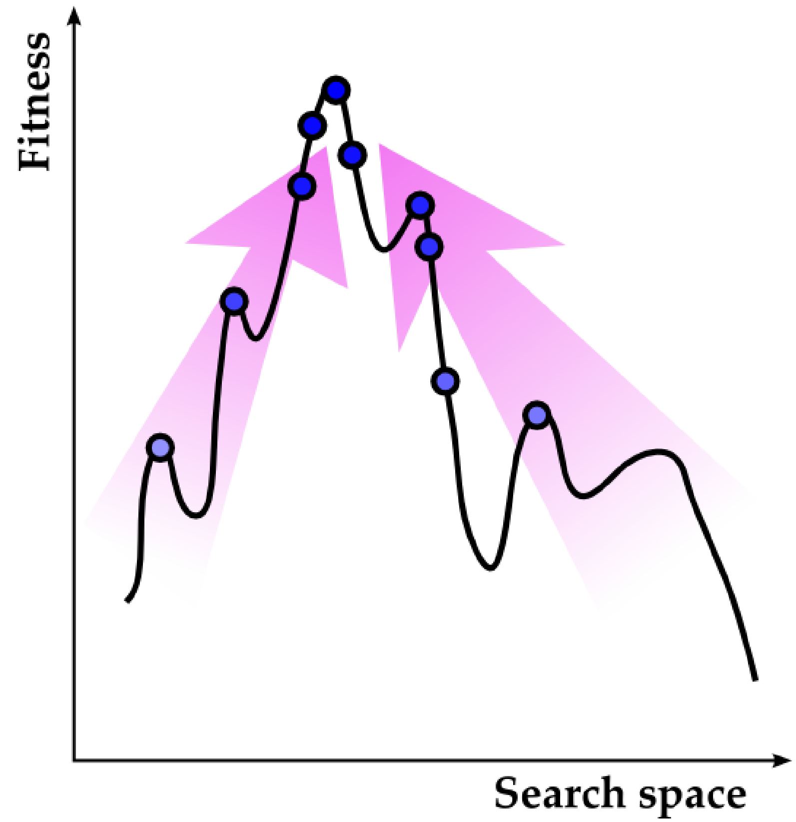


Idealized Illustration of Evolution

- Uniform sampled population.



- Population converged to promising regions.



Initialization

:: Random

- randomly generated solutions,
- no prior information about the shape of the sought solution,
- relies just on "lucky" sampling of the whole search space by a finite set of samples.

:: Informed (pre-processing)

- (meta)heuristic routines used for seeding the initial population,
- biased random generator sampling regions of the search space that are likely to contain the sought solutions,
  + may help to find better solutions,
  + may speed up the search process,
  – may cause irreversible focusing of the search process on regions with local optima.

Reproduction

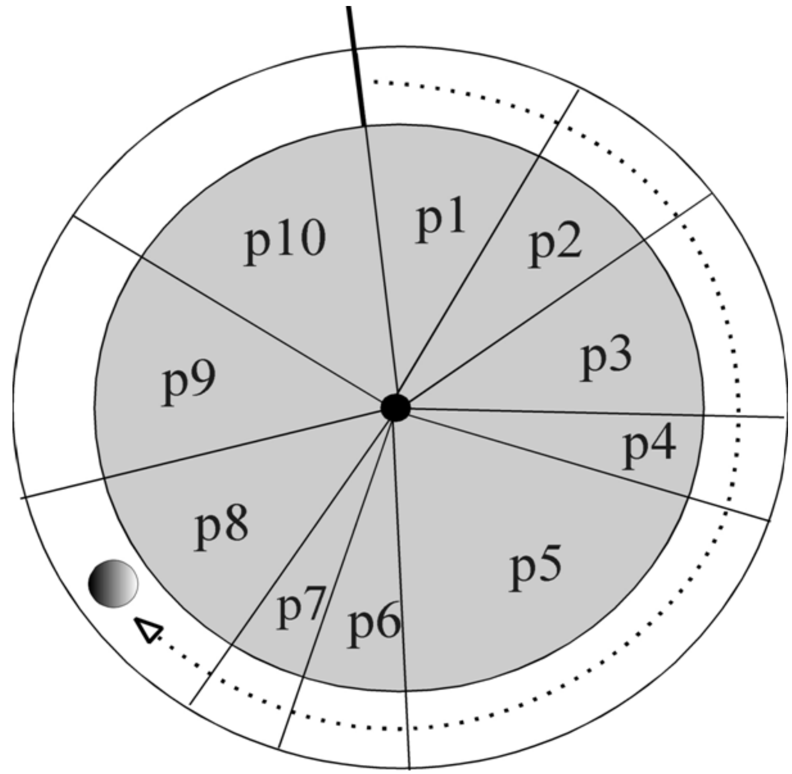
:: Models nature's survival-of-fittest principle

- prefers better individuals to the worse ones,
- still, every individual should have a chance to reproduce.

:: **Roulette wheel**

- probability of choosing some solution is directly proportional to its fitness value

$$P_i = \frac{f_i}{\sum_{j=1}^{PopSize} f_j}$$



:: Other methods

- Stochastic Universal Sampling,
- Tournament selection,
- Reminder Stochastic Sampling.

Genetic Operators: Crossover

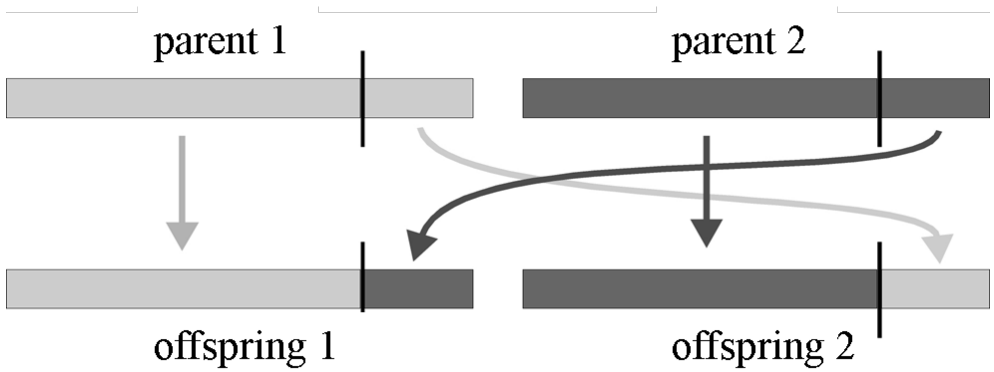
:: Idea

- given two well-fit solutions to the given problem, it is possible to get a new solution by properly mixing the two that is even better than both its parents.

:: Role of crossover

- sampling (exploration) of the search space

Example: 1-point crossover

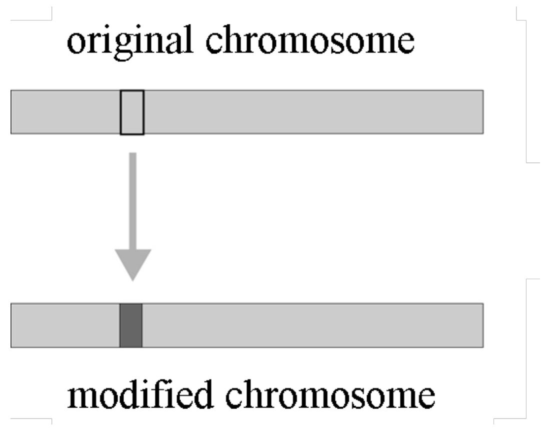


Genetic Operators: Mutation

:: Role of mutation

- preservation of a population diversity,
- minimization of a possibility of loosing some important piece of genetic information.

Example: Single bit-flipping mutation



Replacement Strategy

:: **Replacement strategy** defines

- how big portion of the current generation will be replaced in each generation, and
- which solutions in the current population will be replaced by the newly generated ones.

:: Two extreme cases

- **Generational** – the whole old population is completely rebuild in each generation (analogy of short-lived species).
- **Steady-state** – just a few individuals are replaced in each generation (analogy of longer-lived species).

Application Areas of Evolutionary Algorithms

:: EAs are popular for their

- simplicity,
- effectiveness,
- robustness.

:: **Holland:** *"It's best used in areas where you don't really have a good idea what the solution might be. And it often surprises you with what you come up with."*

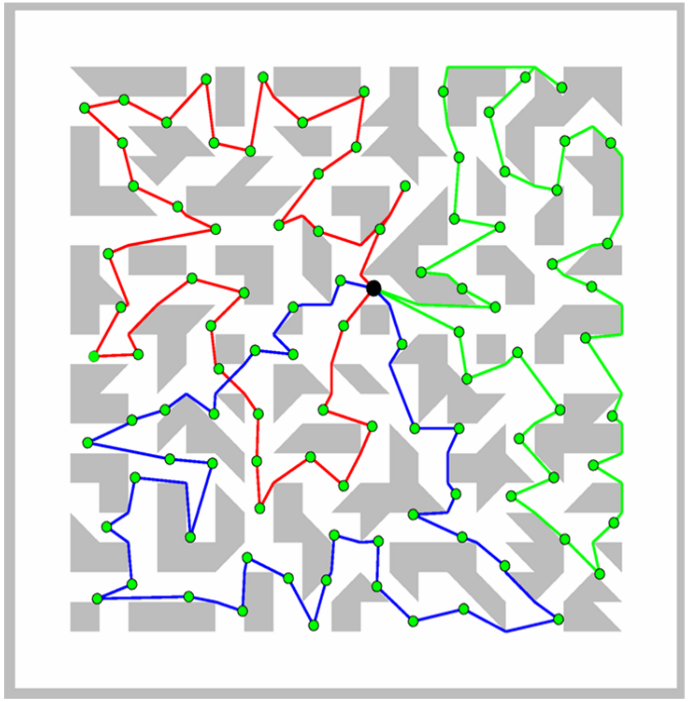
:: Applications

- control,
- engineering design,
- image processing,
- planning & scheduling,
- VLSI circuit design,
- network optimization & routing problems,
- optimal resource allocation,
- marketing,
- credit scoring & risk assessment,
- and many others.

Multiple Traveling Salesmen Problem

:: **Rescue operations planning**

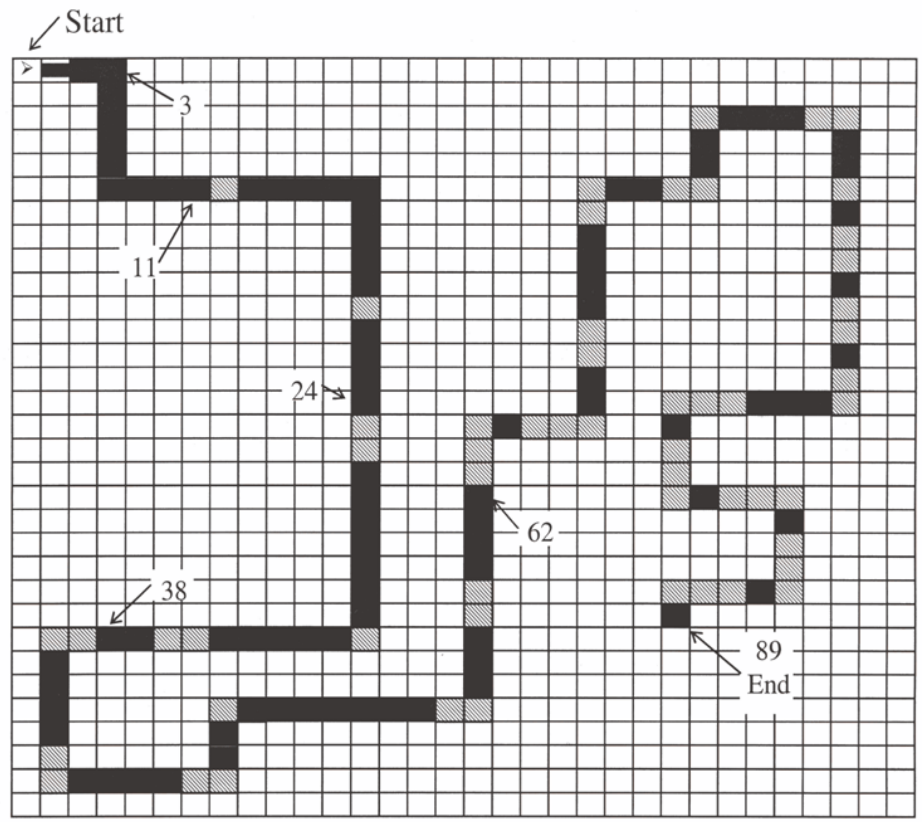
- Given N cities and K agents, find an optimal tour for each agent so that every city is visited exactly once.
- A typical criterion to be optimized is the overall time spent by the squad (i.e., the slowest team member) during the task execution.



Artificial Ant Problem

:: **Santa Fe trail**

- 32×32 **grid** with 89 food pieces.
- **Obstacles**
  - $1\times, 2\times$ strait,
  - $1\times, 2\times, 3\times$ right/left.



:: **Ant capabilities**

- **detects** the food right in front of him in direction he faces.
- **actions** observable from outside
  - MOVE – makes a step and eats a food piece if there is some,
  - LEFT – turns left,
  - RIGHT – turns right,
  - NO-OP – no operation.

:: **Goal** is to find a strategy that would navigate an ant through the grid so that it finds all the food pieces in the given time (600 time steps).

Artificial Ant Problem: GA Approach

:: **Collins a Jefferson 1991, standard GA using binary representation**

:: **Representation**

- strategy represented by finite state machine,
- table of transitions coded as binary chromosomes of fixed length.

Example: 4-state FSM, 32-bit long chromosomes

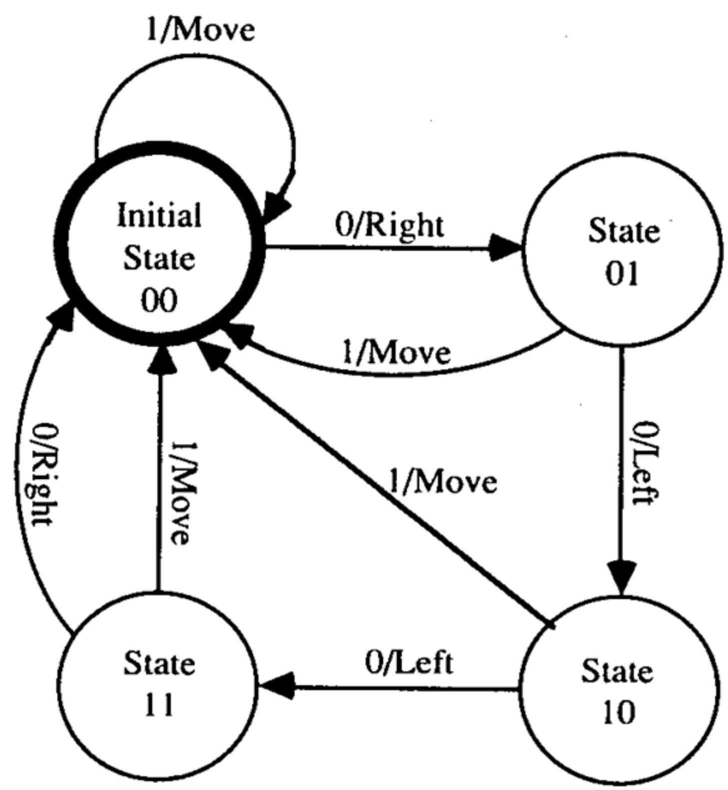
	Current state	Input	New state	Operation
1	00	0	01	10 = Right
2	00	1	00	11 = Move
3	01	0	10	01 = Left
4	01	1	00	11 = Move
5	10	0	11	01 = Left
6	10	1	00	11 = Move
7	11	0	00	10 = Right
8	11	1	00	11 = Move

00	0110	0011	1001	0011	1101	0011	0010	0011

Artificial Ant Problem: Example cont.

:: **Ant behavior**

- What happens if the ant hits an obstacle?
- What is strange with transition from state 01 to the initial state 00?
- When does the ant succeed?
- Is the number of states sufficient to solve the problem?
- Do all of the possible 32-bit chromosomes represent a feasible solution?



Artificial Ant Problem: GA result

:: **Representation**

- 32 states,
- $453 = 64 \times 7 + 5$ bits !!!

:: **Population size: 65.536 !!!**

:: **Number of generations: 200**

:: **Total number of samples tried: 13×10^6 !!!**

Why and How Evolutionary Algorithms Work?

:: **Schema theory** – tries to analyze effect of selection, crossover and mutation in order to answer the above questions. In its original form it assumes:

- binary representation,
- proportionate roulette wheel selection,
- 1-point crossover and bit-flip mutation.

Schema theory

:: **Schema** – a template, which defines set of solutions from the search space with certain specific similarities.

- consists of 0s, 1s (fixed values) and wildcard symbols * (any value),
- covers 2^r strings, where r is a number of $*$ used in the schema.

  Example: schema $S =${11*0*} covers strings 11000, 11001, 11100, and 11101

:: **Schema properties**

- **Defining length** $\delta(S)$ (compactness) – distance between first and last non-* in a schema (= number of positions where 1-point crossover can disrupt it).
- **Order** $o(S)$ (specificity) – a number of non-*'s (= number of positions where simple bit swapping mutation can disrupt it).
  - Chromosomes are order l schemata, where l is length of chromosome (in bits or loci).
  - Chromosomes are instances (or members) of lower-order schemata.
  - How many schemata is matched by a string of length l?
- **Fitness** $f(S)$ (quality) – average fitness computed over all covered strings.

  Example: $S =${**1*01*0**}: $\delta(S) = 5$, $o(S) = 4$

Schema Properties: Example

:: **8-bit Count Ones problem** – maximize a number of ones in 8-bit string.

string	fitness		string	fitness
00000000	0		11011111	7
00000001	1	...	10111111	7
00000010	1		01111111	7
00000100	1		11111111	8

Assume schema S_a ={1*1**10*} vs. S_b ={*0*0****}:

- **defining length**: $\delta(S_a) = 7 - 1 = 6$, $\delta(S_b) = 4 - 2 = 2$
- **order**: $o(S_a) = 4$, $o(S_b) = 2$
- **fitness of** S_a: S_a covers 2^4 strings in total

1 string of fitness 3
4 string of fitness 4
6 string of fitness 5
4 string of fitness 6
1 string of fitness 7

$$f(S_a) = (1 \cdot 3 + 4 \cdot 4 + 6 \cdot 5 + 4 \cdot 6 + 1 \cdot 7)/16$$
$$f(S_a) = 80/16 = 5$$

fitness of S_b: $S_b = (1 \cdot 0 + 6 \cdot 1 + 15 \cdot 2 + 20 \cdot 3 + 15 \cdot 4 + 6 \cdot 5 + 1 \cdot 6)/2^6 = 192/64 = 3$

Question: What would be a fitness of S ={*0*1****} compared to S_b?

Schema Theorem Derivation: Effect of Reproduction

Let $m(S, t)$ be number of instances (strings) of schema S in population of size n at time t.

Question: How do schemata propagate? What is a lower bound on change in sampling rate of a single schema from generation t to $t + 1$?

Effect of fitness-proportionate roulette wheel selection

A string a_i is copied according to its fitness; it gets selected with probability

$$p_i = \frac{f_i}{\sum f_j}.$$

After picking n strings with replacement from the population at time t, we expect to have $m(S, t + 1)$ representatives of the schema S in the population at time $t + 1$ as given by the equation

$$m(S, t + 1) = m(S, t) \cdot n \cdot \frac{f(S)}{\sum f_j},$$

where $f(S)$ is the fitness of schema S at time t.

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$$m(S,t+1) = m(S,t) \cdot n \cdot \frac{f(S)}{\sum f_j},$$

where $f(S)$ is the fitness of schema S at time t.

The formula can be rewritten as

$$m(S,t+1) = m(S,t) \cdot \frac{f(S)}{f_{avg}},$$

where f_{avg} is the average fitness of the population.

Schema Theorem Derivation: Effect of Crossover and Mutation

Effect of 1-point Crossover

- Survival probability p_s – let's make a conservative assumption that crossover within the defining length of S is always disruptive to S, and ignore gains.
- Crossover probability p_c – fraction of population that undergoes crossover.

$$p_s \geq (1 - p_c \cdot \delta(S)/(L-1))$$

Example: Compare survival probability of $S = (11 * * * *)$ and $S = (1 * * * * 0)$.

Effect of Mutation

Each fixed bit of schema (o(S) of them) changes with probability p_m, so they all stay unchanged with probability

$$p_s = (1 - p_m)^{o(S)}$$

that can be approximated as

$$p_s = (1 - o(S) \cdot p_m)$$

assuming $p_m \ll 1$.

Schema Theorem Derivation (cont.)

:: Finally, we get a "classical" form of the **reproductive schema growth equation**:

$$m(S, t+1) \geq m(S, t) \cdot \frac{f(S)}{f_{avg}} \cdot [1 - p_c \cdot \frac{\delta(S)}{L-1} - o(S) \cdot p_m].$$

What does it tell us?

Schema Theorem Derivation (cont.)

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$$m(S, t+1) \geq m(S, t) \cdot \frac{f(S)}{f_{avg}} \cdot [1 - p_c \cdot \frac{\delta(S)}{L-1} - o(S) \cdot p_m].$$

What does it tell us?

:: **Schema theorem**: Short, low-order, above-average schemata receive exponentially increasing trials in subsequent generations of a genetic algorithm.

:: **Building Block Hypothesis**: A genetic algorithm seeks near-optimal performance through the juxtaposition of short, low-order, high-performance schemata, called the building blocks.

David Goldberg: "*Short, low-order, and highly fit schemata are sampled, recombined, and resampled to form strings of potentially higher fitness... we construct better and better strings from the best partial solutions of the past samplings.*"

:: Y. Davidor: "*The whole GA theory is based on the assumption that one can state something about the whole only by knowing its parts.*"

Corollary: The problem of coding for a GA is critical for its performance, and that such a coding should satisfy the idea of short building blocks.

EA Materials: Reading, Demos, Software

:: Reading

- D. E. Goldberg: Genetic Algorithms in Search, Optimization, and Machine Learning, Addison-Wesley, 1989.
- Z. Michalewicz: Genetic Algorithms + Data Structures = Evolution Programs, Springer, 1998.
- Z. Michalewicz: How to solve it? Modern heuristics. 2nd ed. Springer, 2004.

:: Demos

- M. Obitko: Introduction to genetic algorithms with java applets,
  http://cs.felk.cvut.cz/ xobitko/ga/

:: Software

- ECJ 16 – A Java-based Evolutionary Computation Research System
  http://cs.gmu.edu/ eclab/projects/ecj/
- PISA – A Platform and Programming Language Independent Interface for Search Algorithms
  http://www.tik.ee.ethz.ch/sop/pisa/?page=selvar.php