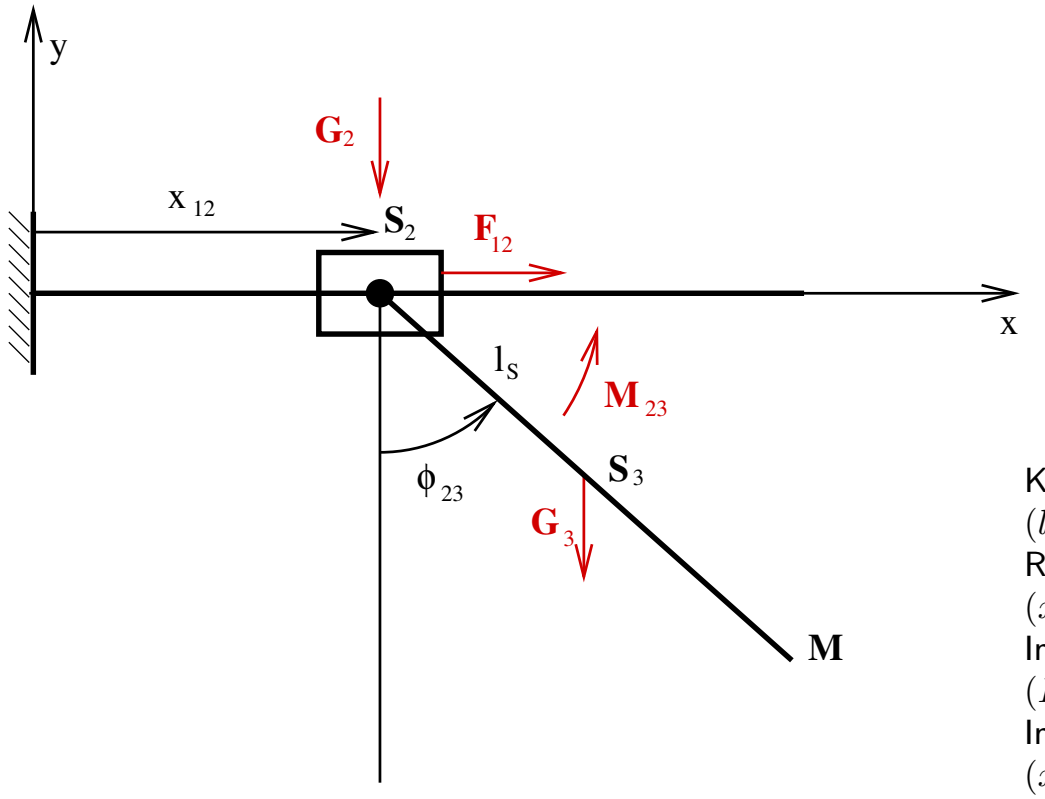
Torque Control Example



$$S_2 = \begin{pmatrix} x_{12} \\ 0 \end{pmatrix}$$
$$l_S = |S_2 S_3|$$
$$l_3 = |S_2 M|$$

Known:
$(l_s, l_3, m_2, m_3, I_{3S3}, b, \mu)^T$
Required motion:
$(x_{1M}, y_{1M})^T$
Input variables:
$(F_{12}, M_{23})^T$
Independent variables:
$(x_{12}, \phi_{23})^T$

Kinetic energy (it is not used in further derivations):

$$E_k = \frac{1}{2} m_2 \dot{x}_{12}^2 + \frac{1}{2} m_3 (\dot{x}_{1S3}^2 + \dot{y}_{1S3}^2) + \frac{1}{2} I_{3S3} \dot{\phi}_{13}^2 \quad (1)$$

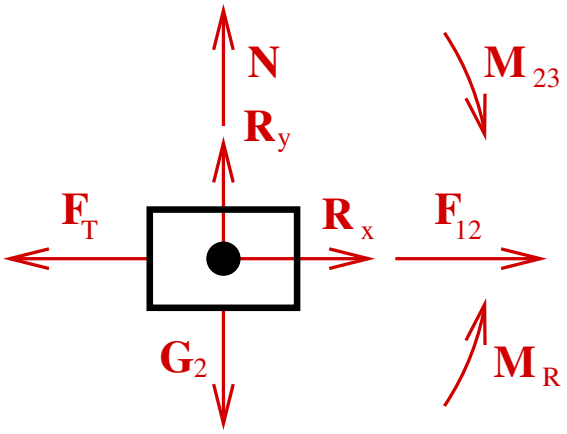
Direct kinematics

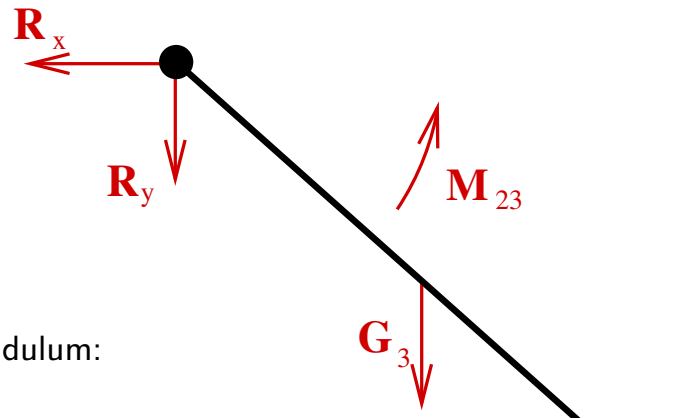
$$\begin{pmatrix} x_{12} \\ \phi_{23} \end{pmatrix} -> \begin{pmatrix} x_{1M} \\ y_{1M} \end{pmatrix} \quad (2)$$
$$x_{1S3} = x_{12} + l_S \sin \phi_{23} \quad (3)$$
$$y_{1S3} = -l_S \cos \phi_{23} \quad (4)$$
$$x_{1M} = x_{12} + l_3 \sin \phi_{23} \quad (5)$$
$$y_{1M} = -l_3 \cos \phi_{23} \quad (6)$$

Inverse kinematics, the configuration should be selected based on the current configuration and joint coordinates velocities $(\dot{x}_{12}, \dot{\phi}_{23})$

$$x_{12} = x_{1M} \pm \sqrt{l_3^2 - y_{1M}^2} \quad (7)$$
$$\phi_{23} = \arctan 2(x_{1M} - x_{12}, y_{1M}) \quad (8)$$

Uvolňování





Slider:

$$
\begin{aligned}
m_2\ddot{x}_{12} &= F_{12} + R_x - F_T \\
0 &= N + R_y - m_2 g \\
0 &= M_{23} - M_r \\
F_T &= b\dot{x}_{12} + \mu \mathrm{sgn}(\dot{x}_{12})|N|
\end{aligned}
$$

Pendulum:

$$
\begin{aligned}
m_3\ddot{x}_{1S3} &= -R_x \\
m_3\ddot{y}_{1S3} &= -R_y - m_3 g \\
I_{3S3}\ddot{\phi}_{23} &= R_y l_S \sin\phi_{23} + R_x l_S \cos\phi_{23} + M_{23}
\end{aligned}
$$

Coordinates of S_3 substituted:

$$m_3(\ddot{x}_{12} - l_S\dot{\phi}_{23}^2 \sin\phi_{23} + l_S\ddot{phi}_{23}\cos\phi_{23}) + R_x = 0 \quad (9)$$

$$m_3(l_S\dot{\phi}_{23}^2 \cos\phi_{23} + l_S\ddot{phi}_{23}\sin\phi_{23}) + R_y = -m_3 g \quad (10)$$

Dynamics equations:

$$
\begin{aligned}
(m_2+m_3)\ddot{x}_{12} + b\dot{x}_{12} + \mu \mathrm{sgn}(\dot{x}_{12})|m_2 g + m_3 g + m_3 l_S \dot{\phi}_{23}^2 \cos\phi_{23} + m_3 l_S \ddot{\phi}_{23} \sin\phi_{23}| &= \\
= F_{12} - m_3 l_S \ddot{\phi}_{23} \cos\phi_{23} + m_3 l_S \dot{\phi}_{23}^2 \sin\phi_{23} & \\
I_{3S3}\ddot{\phi}_{23} &= \\
= M_{23} - m_3 g l_S \sin\phi_{23} - m_3 l_S^2 \ddot{\phi}_{23} - m_3 l_S \ddot{x}_{12} \cos\phi_{23} &
\end{aligned}
$$

Nonlinear transformation of input, the plus/minus sign is applied according $\mathrm{sgn}(\dot{x}_{12})$ and $\mathrm{sgn}(\vec{N})$

$$
\begin{aligned}
\mathbf{M} &= \begin{pmatrix} m_2+m_3 & m_3 l_S \cos\phi_{23} \pm m_3 l_S \sin\phi_{23} \\ m_3 l_S \cos\phi_{23} & I_{3S3} + m_3 l_S^2 \end{pmatrix} \\
\vec{N} &= \begin{pmatrix} b\dot{x}_{12} \pm \mu(m_2 g + m_3 g + m_3 l_S \dot{\phi}_{23}^2 \cos\phi_{23}) - m_3 l_S \dot{\phi}_{23}^2 \sin\phi_{23} \\ m_3 g l_S \sin\phi_{23} \end{pmatrix}
\end{aligned}
$$

Structure of the Torque Control Controller