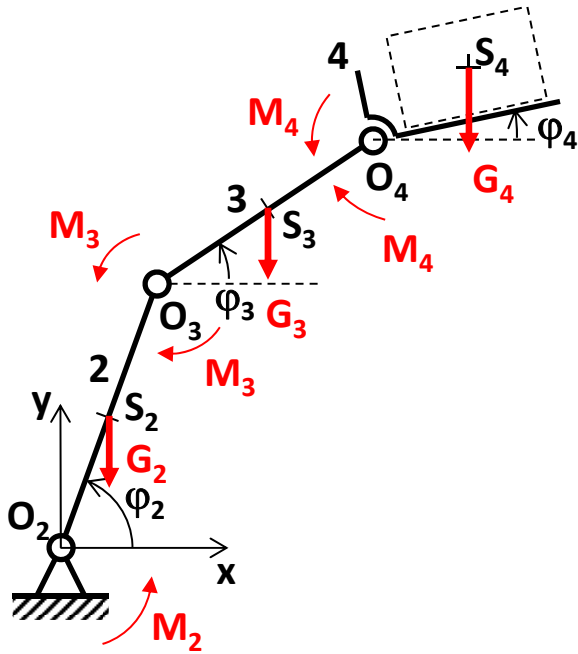


## Example 2:

Solve the problem of manipulator inverse dynamics using Lagrange's equations of mixed type.



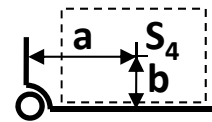
Known parameters:

$$m_2, m_3, m_4, l_{2S_2}, l_{3S_3}, l_{4S_4},$$

$$|O_2S_2| = |S_2O_3| = l_2,$$

$$|O_3S_3| = |S_3O_4| = l_3, a, b,$$

$$\varphi_2(t), \varphi_3(t), \varphi_4(t),$$

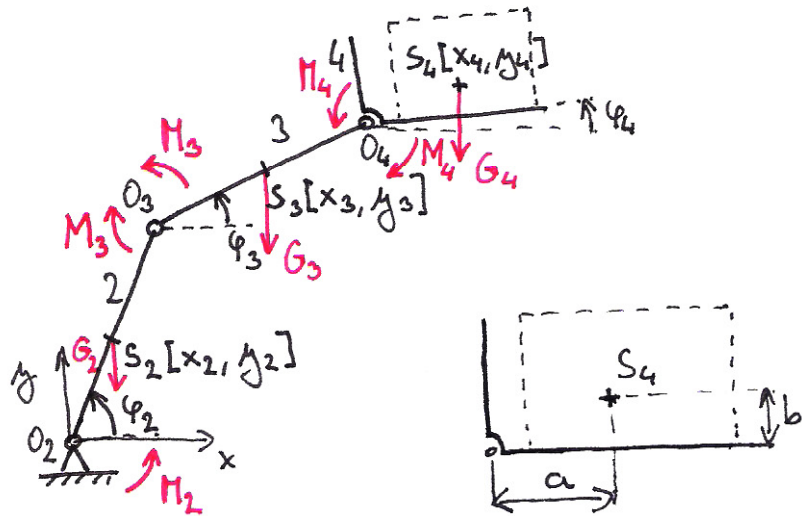


$$M_2(t) = ?$$

$$M_3(t) = ?$$

$$M_4(t) = ?$$

$$\text{LEMT} : \frac{d}{dt} \left( \frac{\partial E_k}{\partial \dot{s}_j} \right) - \frac{\partial E_k}{\partial s_j} = Q_j + \sum_{k=1}^4 \lambda_k \frac{\partial f_k}{\partial s_j}$$



Fyzikální souřadnice:

$$S = [x_2, y_2, \varphi_2, x_3, y_3, \varphi_3, x_4, y_4, \varphi_4]$$

Stupně volnosti:

$$i = 3(4-1) - 2 \cdot 3 = 9 - 6 = 3^\circ \text{ volnosti}$$

Kinetická energie:

$$\begin{aligned} E_k(s) = & \frac{1}{2} m_2 (\dot{x}_2^2 + \dot{y}_2^2) + \frac{1}{2} I_2 s_2 \cdot \dot{\varphi}_2^2 + \\ & + \frac{1}{2} m_3 (\dot{x}_3^2 + \dot{y}_3^2) + \frac{1}{2} I_3 s_3 \cdot \dot{\varphi}_3^2 + \\ & + \frac{1}{2} m_4 (\dot{x}_4^2 + \dot{y}_4^2) + \frac{1}{2} I_4 s_4 \cdot \dot{\varphi}_4^2 \end{aligned}$$

Zobecněné síly:

$$\begin{aligned} \sum_i Q_i \delta s_i = & -G_2 \delta y_2 - G_3 \delta y_3 - G_4 \delta y_4 + M_2 \delta \varphi_2 + M_3 \delta \varphi_3 + M_4 \delta \varphi_4 - M_3 \delta \varphi_2 - M_4 \delta \varphi_3 \\ = & -G_2 \delta y_2 - G_3 \delta y_3 - G_4 \delta y_4 + M_2 \delta \varphi_2 + M_3 (\delta \varphi_3 - \delta \varphi_2) + M_4 (\delta \varphi_4 - \delta \varphi_3) \end{aligned}$$

Vazbové podmínky:

$$O_2: \quad x_2 - l_2 \cdot \cos \varphi_2 = 0 : f_1 \quad O_3: \quad x_2 + l_2 \cdot \cos \varphi_2 - x_3 + l_3 \cdot \cos \varphi_3 = 0 : f_3$$

$$y_2 - l_2 \cdot \sin \varphi_2 = 0 : f_2 \quad y_2 + l_2 \cdot \sin \varphi_2 - y_3 + l_3 \cdot \sin \varphi_3 = 0 : f_4$$

$$O_4: \quad x_3 + l_3 \cdot \cos \varphi_3 - x_4 + a \cdot \cos \varphi_4 - b \cdot \sin \varphi_4 = 0 : f_5$$

$$y_3 + l_3 \cdot \sin \varphi_3 - y_4 + a \cdot \sin \varphi_4 + b \cdot \cos \varphi_4 = 0 : f_6$$