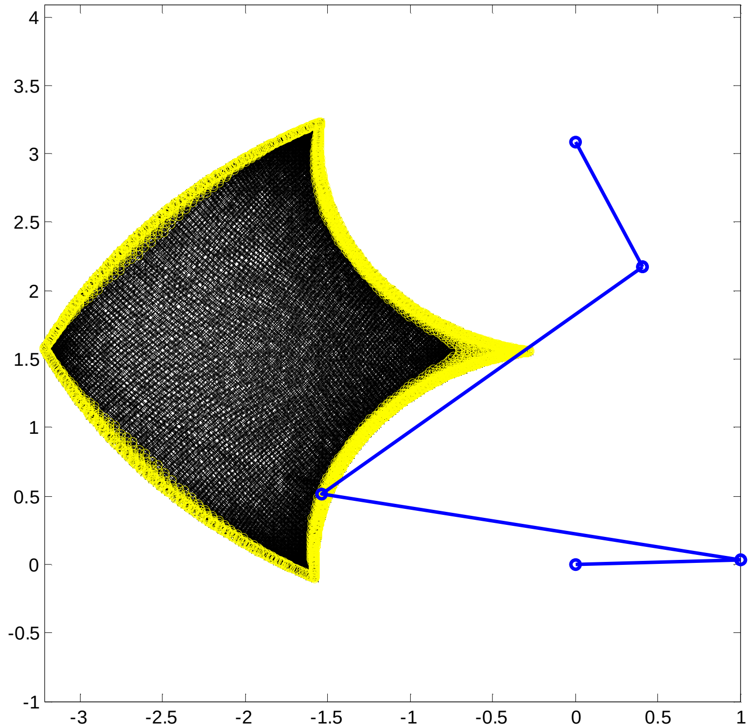
Lecture 1 – Dexterity



Example:
2 DOF

$$\begin{bmatrix} v_C \\ v_D \end{bmatrix} = \mathbf{J} \begin{bmatrix} \dot{x}_M \\ \dot{y}_M \end{bmatrix}$$

$$D = 1/cond(\mathbf{J})$$

$$\begin{bmatrix} v_C \\ v_D \end{bmatrix} = \begin{bmatrix} \dfrac{x_M - x_C}{-(x_M - x_C)\sin(\varphi_{12}) + (y_M - y_C)\cos(\varphi_{12})} & \dfrac{y_M - y_C}{-(x_M - x_C)\sin(\varphi_{12}) + (y_M - y_C)\cos(\varphi_{12})} \\ \dfrac{x_M - x_D}{-(x_M - x_D)\sin(\varphi_{15}) + (y_M - y_D)\cos(\varphi_{15})} & \dfrac{y_M - y_D}{-(x_M - x_D)\sin(\varphi_{15}) + (y_M - y_D)\cos(\varphi_{15})} \end{bmatrix} \begin{bmatrix} \dot{x}_M \\ \dot{y}_M \end{bmatrix}$$

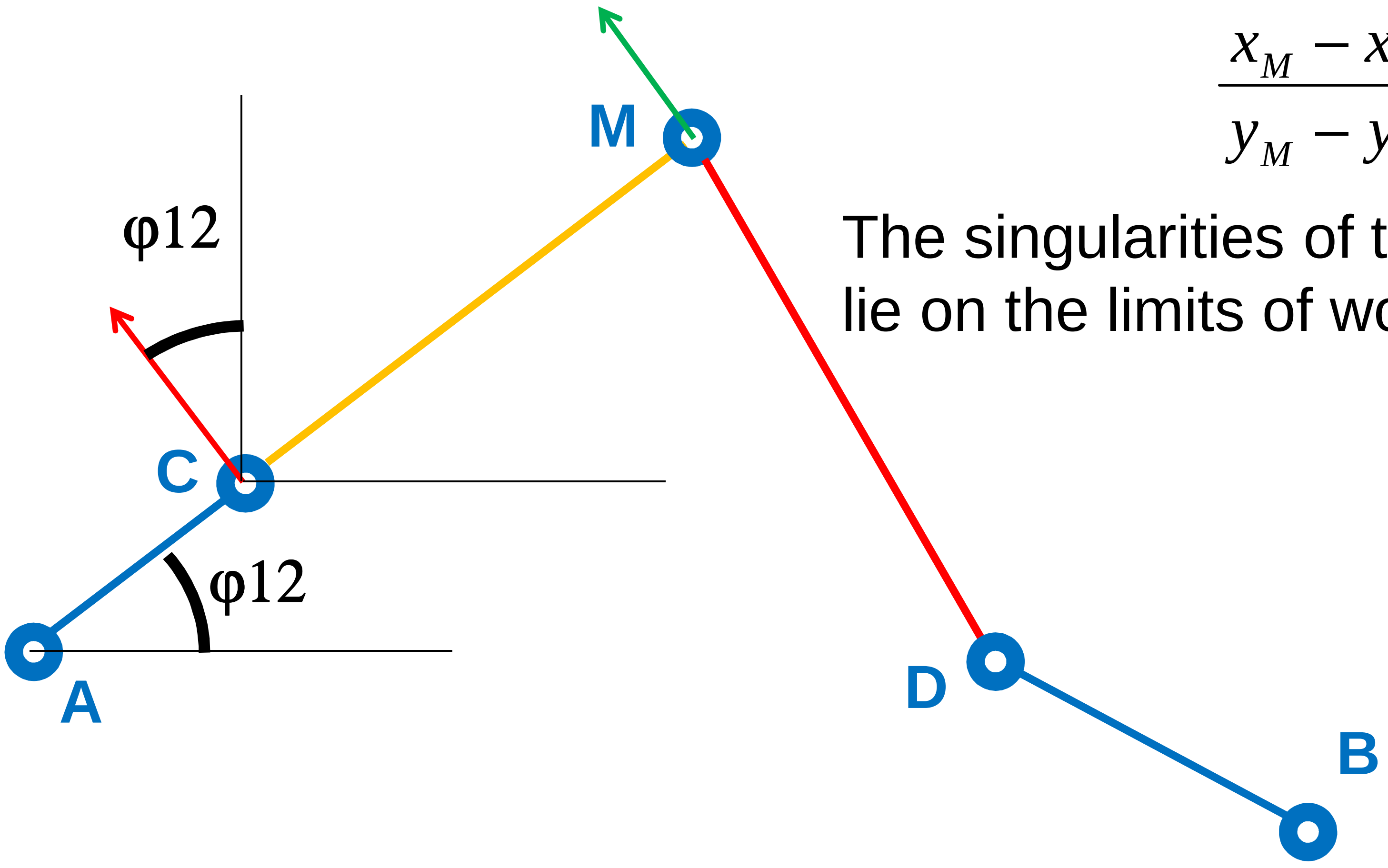
$$\begin{bmatrix} \left(-(x_M - x_C)\sin(\varphi_{12}) + (y_M - y_C)\cos(\varphi_{12})\right)v_C \\ \left(-(x_M - x_D)\sin(\varphi_{15}) + (y_M - y_D)\cos(\varphi_{15})\right)v_D \end{bmatrix} = \begin{bmatrix} x_M - x_C & y_M - y_C \\ x_M - x_D & y_M - y_D \end{bmatrix} \begin{bmatrix} \dot{x}_M \\ \dot{y}_M \end{bmatrix}$$

Dexterity – first type of singularity

$$\begin{bmatrix} \left(-(x_M - x_C)\sin(\varphi_{12}) + (y_M - y_C)\cos(\varphi_{12})\right)v_C \\ \left(-(x_M - x_D)\sin(\varphi_{15}) + (y_M - y_D)\cos(\varphi_{15})\right)v_D \end{bmatrix} = \begin{bmatrix} x_M - x_C & y_M - y_C \\ x_M - x_D & y_M - y_D \end{bmatrix} \begin{bmatrix} \dot{x}_M \\ \dot{y}_M \end{bmatrix}$$

$$-(x_M - x_C)\sin(\varphi_{12}) + (y_M - y_C)\cos(\varphi_{12}) = 0 \quad \Rightarrow \quad v_C \text{ is arbitrary}$$

$$\frac{x_M - x_C}{y_M - y_C} = -\frac{\dot{y}_M}{\dot{x}_M}$$



The singularities of the first type lie on the limits of workspace.

Dexterity – second type of singularity

$$\begin{bmatrix} \dfrac{y_M - y_D}{\Delta} & \dfrac{-y_M + y_C}{\Delta} \\ \dfrac{-x_M + x_D}{\Delta} & \dfrac{x_M - x_C}{\Delta} \end{bmatrix} \begin{bmatrix} \left(-(x_M - x_C)\sin(\varphi_{12}) + (y_M - y_C)\cos(\varphi_{12})\right)v_C \\ \left(-(x_M - x_D)\sin(\varphi_{15}) + (y_M - y_D)\cos(\varphi_{15})\right)v_D \end{bmatrix} = \begin{bmatrix} \dot{x}_M \\ \dot{y}_M \end{bmatrix}$$

where

$$\Delta = (x_M - x_C)(y_M - y_D) - (x_M - x_D)(y_M - y_C)$$

alternatively

$$\begin{bmatrix} y_M - y_D & -y_M + y_C \\ -x_M + x_D & x_M - x_C \end{bmatrix} \begin{bmatrix} \left(-(x_M - x_C)\sin(\varphi_{12}) + (y_M - y_C)\cos(\varphi_{12})\right)v_C \\ \left(-(x_M - x_D)\sin(\varphi_{15}) + (y_M - y_D)\cos(\varphi_{15})\right)v_D \end{bmatrix} = \Delta \begin{bmatrix} \dot{x}_M \\ \dot{y}_M \end{bmatrix}$$

$$\det \begin{bmatrix} y_M - y_D & -y_M + y_C \\ -x_M + x_D & x_M - x_C \end{bmatrix} = \Delta$$

Dexterity – second type of singularity

$$\Delta = 0 \quad \Rightarrow \dot{x}_M\,,\, \dot{y}_M \ are\ arbitrary$$

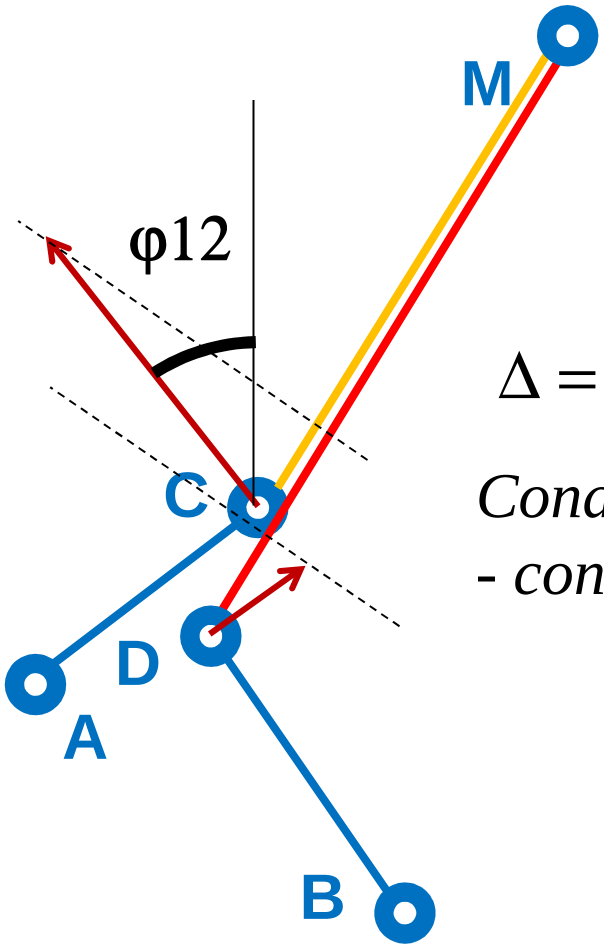
$$(y_M - y_D)\big(-(x_M - x_C)\sin(\varphi_{12}) + (y_M - y_C)\cos(\varphi_{12})\big)v_C +$$

$$+(-y_M + y_C)\big(-(x_M - x_D)\sin(\varphi_{15}) + (y_M - y_D)\cos(\varphi_{15})\big)v_D = 0$$

$$(-x_M + x_D)\big(-(x_M - x_C)\sin(\varphi_{12}) + (y_M - y_C)\cos(\varphi_{12})\big)v_C +$$

$$+(x_M - x_C)\big(-(x_M - x_D)\sin(\varphi_{15}) + (y_M - y_D)\cos(\varphi_{15})\big)v_D = 0$$

Dexterity – second type of singularity (geometrical interpretation)

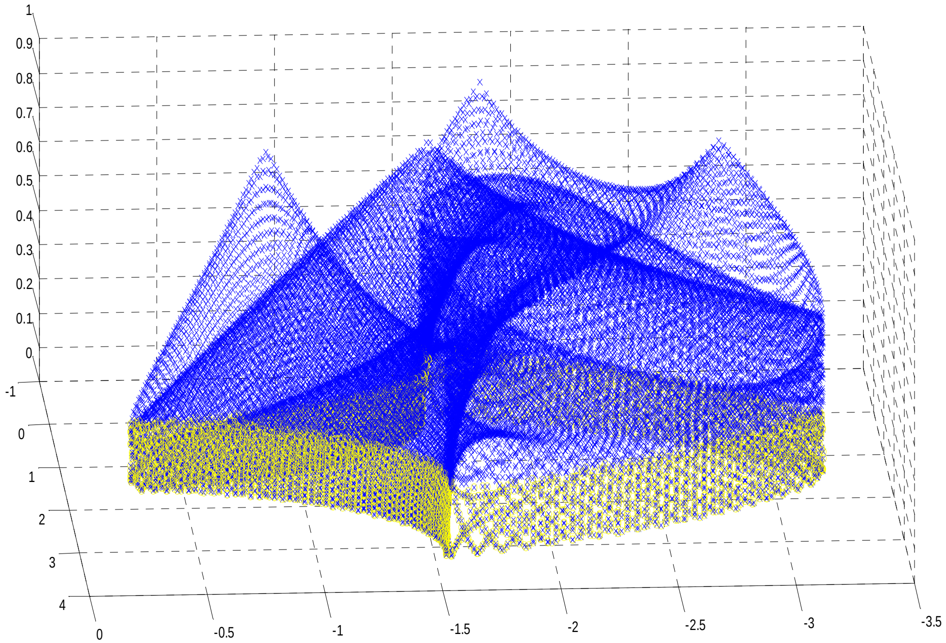


$$\Delta = 0 \quad \Rightarrow \dot{x}_M, \dot{y}_M \text{ are arbitrary}$$

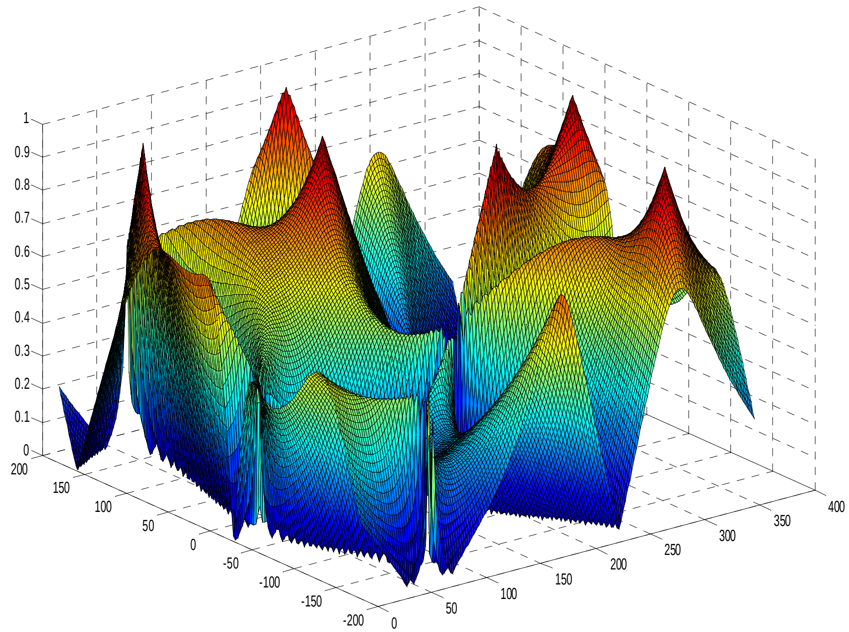
Condition of rigidity of CD

- constraint between v_C *and* v_D

Dexterity



Dexterity in x_M y_M space



Dexterity in φ_{12} φ_{15} space