2. BASIC CALIBRATION ALGORITHM

The investigated kinematical structures include the kinematical loops, at least the virtual ones through the end-effector positioned on calibration artifact. The kinematical loops are described by the kinematical constraints in given position

$$\mathbf{f}(\mathbf{d},\mathbf{s},\mathbf{v})=\mathbf{0}, \tag{1}$$

where $\mathbf{d}$ are the dimensions of the mechanism, $\mathbf{s}$ are the input (measured) coordinates in the joints and the guides and $\mathbf{v}$ are the output coordinates, i.e. the position of the end-effector. The basic calibration algorithm e.g. [9] uses Newton's method modified for overconstrained system of nonlinear algebraic equations (more equations than unknowns) that follow from the constraints (1) formulated for many instances of measurements. If *j=1, …, n* positions of the kinematical structure are considered (measured) then the constraint equations (1) are coupled into the constraint equations for the calibration

$$\mathbf{F}(\mathbf{d},\mathbf{S},\mathbf{V})=\mathbf{0}, \tag{2}$$

where for the position *j* the constraint $\mathbf{f}_j=\mathbf{f}(\mathbf{d},\mathbf{s}_j,\mathbf{v}_j)=\mathbf{0}$ from the equation (1) holds and $\mathbf{F}=[\mathbf{f}_1,\mathbf{f}_2,\ldots,\mathbf{f}_n]^T$, $\mathbf{S}=[\mathbf{s}_1,\mathbf{s}_2,\ldots,\mathbf{s}_n]^T$, $\mathbf{V}=[\mathbf{v}_1,\mathbf{v}_2,\ldots,\mathbf{v}_n]^T$. In traditional (non-redundant) calibration approach the output coordinates $\mathbf{V}$ are measured by external devices. In the case of redundant (self) calibration approach the used constraints (2) do not include necessarily measurement of $\mathbf{V}$ by external devices/artifacts. The equation (2) covers both variants.

The calibration is based on the fact that the dimensions $\mathbf{d}$ are the same (constant) for all positions. Nevertheless the real values of the manufactured dimensions $\mathbf{d}$ differ from their design values $\bar{\mathbf{d}}$. Thus the only unknown variables in the equation (2) are the manufactured dimensions $\mathbf{d}$. The Newton method of the calibration is derived from the Taylor series of (2)

$$\mathbf{F}(\bar{\mathbf{d}},\mathbf{S},\mathbf{V})+\mathbf{J}_{\mathbf{d}}\delta\mathbf{d}+\ldots=\mathbf{0} \tag{3}$$

with Jacobi matrix $\mathbf{J}_{\mathbf{d}}$ of partial derivatives of the kinematical constraints (2) with respect to the calibrated dimensions $\mathbf{d}$. Hence

$$\mathbf{J}_{\mathbf{d}}\delta\mathbf{d}=-\mathbf{F}(\bar{\mathbf{d}},\mathbf{S},\mathbf{V})=\delta\mathbf{r} \tag{4}$$

and the ***i-th*** iteration step of Newton's method [9] is

$$\delta\mathbf{d}_i=(\mathbf{J}_{\mathbf{d}i}{}^{\mathbf{T}}\mathbf{J}_{\mathbf{d}i})^{-1}\mathbf{J}_{\mathbf{d}i}{}^{\mathbf{T}}\delta\mathbf{r}_i, \tag{5}$$

where $\mathbf{J}_{\mathbf{d}i}$ is the Jacobi matrix and $\delta\mathbf{r}_i=-\mathbf{F}(\mathbf{d}_i,\mathbf{S},\mathbf{V})$ is the vector of deviations computed from measured quantities and calibrated quantities $\mathbf{d}_i$ from the previous step. The new values of the dimensions are then computed

$$\mathbf{d}_{i+1}=\mathbf{d}_i+\delta\mathbf{d}_i \tag{6}$$

and the iterations continue until the deviations are decreasing. The basic calibration procedure provides us with the unique solution for the given data. This solution is typically unique for very broad region of initial guesses of parameters of iterative solution by Newton's method.

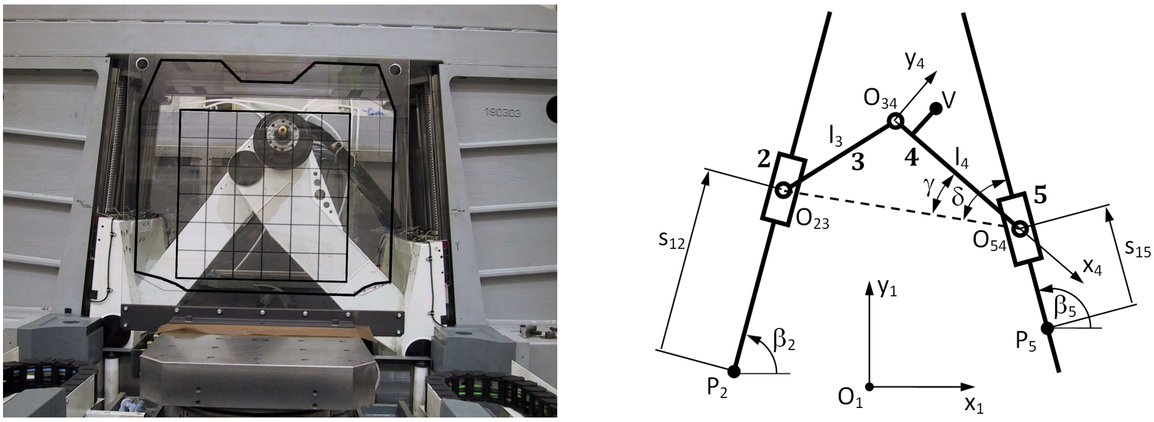
3. CALIBRATION OF MACHINE TOOL TRIJOINT 900H

Horizontal machine centre TRIJOINT 900H [10, 21] is a machine tool of hybrid concept developed in cooperation of KOVOSVIT MAS Inc. Sezimovo Ustí and Department of Mechanics, Faculty of Mechanical Engineering CTU in Prague. The machine consists of two parts, the cutting tool part and workpiece part. The cutting tool part realizes the planar motion of cutting tool and represents a planar mechanism with 2 DOFs (in Fig. 1 a) there is the real machine, in Fig. 1 b) right the kinematical scheme). The workpiece part consists of moving and rotating table and mechanism of palette exchange. It realizes translational motion perpendicular to the plane of cutting tool motion.

On two linear guidances there are moving the carriages 2 and 5 to which the arms 3 and 4 are attached by rotational joints. The tool is fixed to the arm 4.

The basis of the non-redundant calibration problem formulation for TRIJOINT 900H is the kinematic transformation between the coordinates of drive (the positions of carriages $s_{12}=s_{12}(t)$, $s_{15}=s_{15}(t)$), the dimensions of the mechanism $\mathbf{d}$=[x_{1P2}, y_{1P2}, x_{1P5}, y_{1P5}, β_2, β_5, l_3, l_4, x_{4V}, y_{4V}] and the positions of the cutting tool on the machine platform ($x_V=x_V(t)$, $y_V=y_V(t)$) measured by calibration artifact (Fig. 2). Actually

$$[x_{1V}, y_{1V}] = \mathbf{f}_{KT}(s_{12}, s_{15}, \mathbf{d}) \tag{7}$$



a) Machine with scheme of workspace *b) Kinematical scheme with calibration parameters*

Figure 1. Machine tool Trijoint 900H

is the direct kinematical solution of the mechanism. In the case of TRIJOINT 900H it is simply solvable in closed analytical form, where

$$\begin{aligned} &x_{1O_{23}}(t) = x_{1P_2} + s_{12}(t)\cos(\beta_2), y_{1O_{23}}(t) = y_{1P_2} + s_{12}(t)\sin(\beta_2) \\ &x_{1O_{54}}(t) = x_{1P_5} + s_{15}(t)\cos(\beta_5), y_{1O_{54}}(t) = y_{1P_5} + s_{15}(t)\sin(\beta_5) \end{aligned}, \tag{8}$$

$$\begin{aligned} &l_3^2 = l_4^2 + (x_{1O_{23}}(t) - x_{1O_{54}}(t))^2 + (y_{1O_{23}}(t) - y_{1O_{54}}(t))^2 - \\ &- 2l_4\sqrt{(x_{1O_{23}}(t) - x_{1O_{54}}(t))^2 + (y_{1O_{23}}(t) - y_{1O_{54}}(t))^2}\cos(\gamma(t)) \\ &\text{therefore} \\ &\gamma(t) = \pm\arccos\left(\frac{l_4^2 + (x_{1O_{23}}(t) - x_{1O_{54}}(t))^2 + (y_{1O_{23}}(t) - y_{1O_{54}}(t))^2 - l_3^2}{2l_4\sqrt{(x_{1O_{23}}(t) - x_{1O_{54}}(t))^2 + (y_{1O_{23}}(t) - y_{1O_{54}}(t))^2}}\right), \\ &\sqrt{(x_{1O_{23}}(t) - x_{1O_{54}}(t))^2 + (y_{1O_{23}}(t) - y_{1O_{54}}(t))^2}\cos(\delta(t)) = \\ &= \begin{bmatrix}(x_{1O_{23}}(t) - x_{1O_{54}}(t)) & (y_{1O_{23}}(t) - y_{1O_{54}}(t))\end{bmatrix}\begin{bmatrix}\cos(\beta_5) \\ \sin(\beta_5)\end{bmatrix} \\ &\text{and consequently} \\ &\delta(t) = \arccos\left(\frac{\cos(\beta_5)(x_{1O_{23}}(t) - x_{1O_{54}}(t)) + \sin(\beta_5)(y_{1O_{23}}(t) - y_{1O_{54}}(t))}{\sqrt{(x_{1O_{23}}(t) - x_{1O_{54}}(t))^2 + (y_{1O_{23}}(t) - y_{1O_{54}}(t))^2}}\right). \end{aligned} \tag{9}$$

Finally the actual position of the spindle centre V is evaluated concerning the appropriate configuration in $\gamma(t)$ formula (9)

$$\begin{aligned} x_{1V}(t) &= x_{1O_{54}}(t) + (l_4 - x_{4V})\cos(\beta_5 + \delta(t) - \gamma(t)) + y_{4V}\sin(\beta_5 + \delta(t) - \gamma(t)) \\ y_{1V}(t) &= y_{1O_{54}}(t) + (l_4 - x_{4V})\sin(\beta_5 + \delta(t) - \gamma(t)) - y_{4V}\cos(\beta_5 + \delta(t) - \gamma(t)). \end{aligned} \tag{10}$$

The equations (10) for considered calibration position is used for the formulation of equation (1) in the form

$$\begin{aligned} x_{1V,measured} - x_{1V,computed} &= 0 \\ y_{1V,measured} - y_{1V,computed} &= 0. \end{aligned} \tag{11}$$

Thanks to the analytical form of (1) and consequently (2) the Jacobi matrix $\mathbf{J_d}$ (3) of partial derivatives of constraint equations with respect to the calibration parameters can be simply analytically computed. Accordingly the algorithm of the iterative solution (3)-(6) for the unknown dimensions **d** can be applied on the basis of measurements of positions of cutting tool spindle centre V by an external artefact (calibration plate with calibration pins) (Fig. 2) and simultaneous measurements of drive coordinates s_{12}, s_{15}.

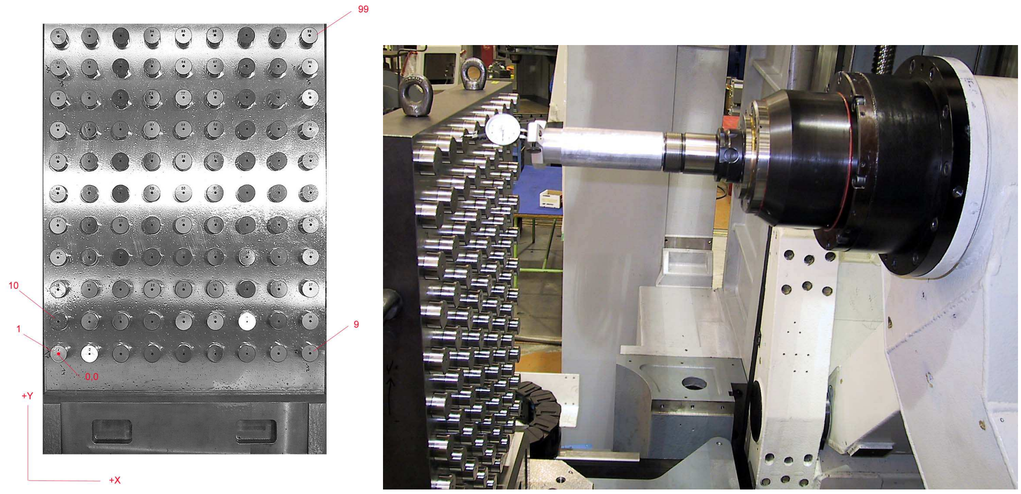
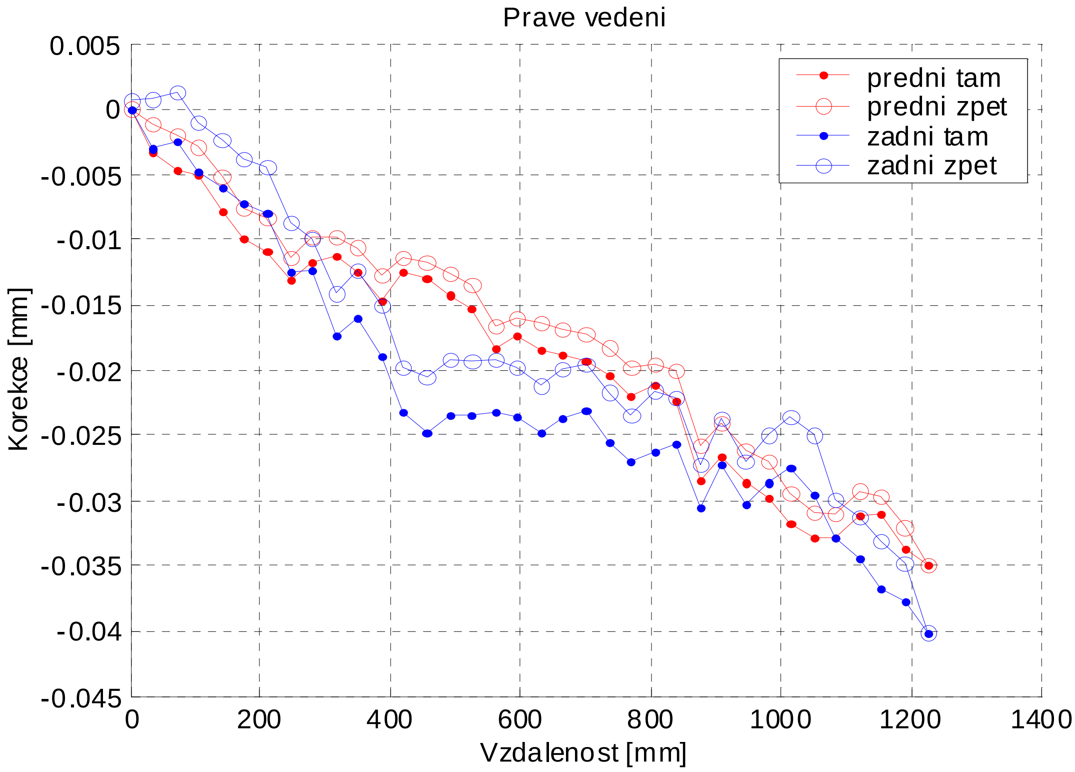
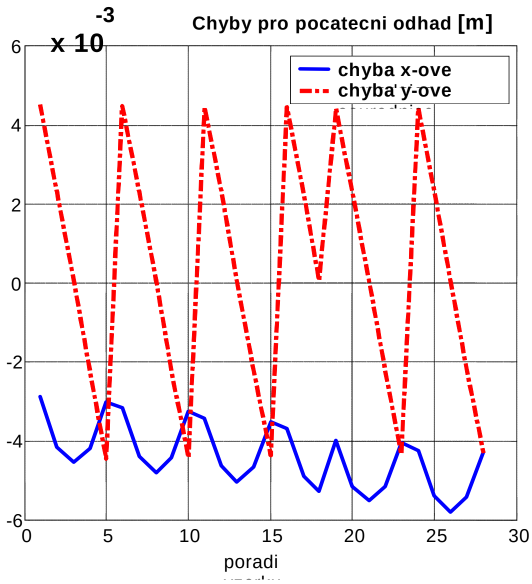


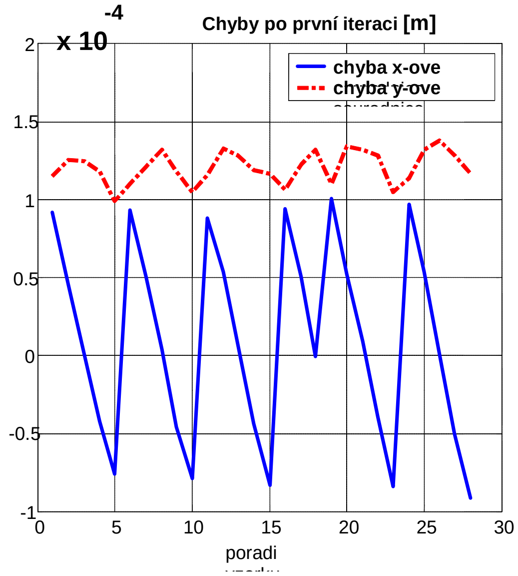
Plate with 99 calibration pins



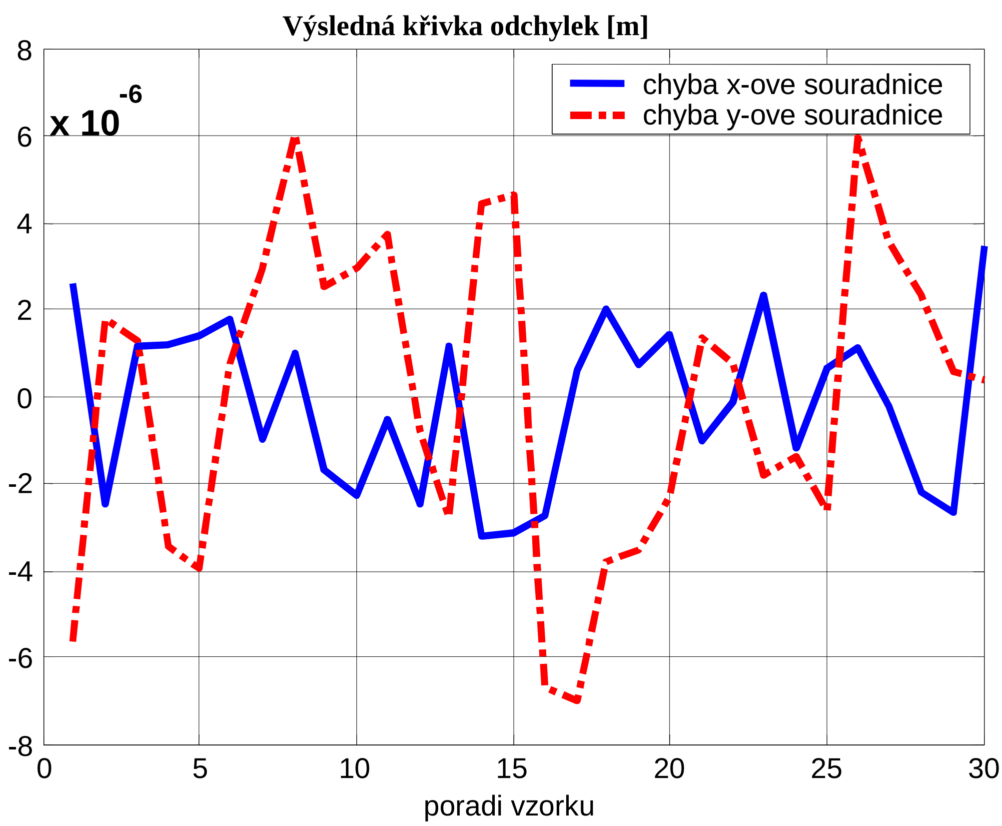
Calibration of s_{12} and s_{15} measurements by laser interferometer



Errors for initial guess of dimensions



Errors after 1-st iteration



Errors after 3-rd iteration

Figure 2. Calibration procedure using plate with 99 calibration pins, iterations results

The calibration of Trijoint 900H has been successfully realized with the final spindle positioning error in the range 5-10 μm within the whole machine workspace with the area roughly 1 m^2 (Fig. 1 a)). However it has been found out, that the parameters determined from the different realizations of calibration measurements vary considerably [25, 27]. Therefore the basic procedure from the section 2 has been modified. The section 4 explains the modification generally, whereas its application to Trijoint is shown in section 5.

4. MODIFIED CALIBRATION ALGORITHM

Very often the convergency of the basic calibration procedure doesn't guarantee better machine performance [16]. The fundamental reason of this phenomenon is an interaction of the inferior conditionality of linear systems solved during the iterations of Newton's method, measurement errors, and errors of model simplifications regarding real machine. Consequently it is very useful to acquire deeper insight into relations between parameter space and space of calibration results. The crucial step towards efficient mapping of the parameter space is singular value decomposition (SVD) of system matrices of iterations (5) of Newton method

$$\mathbf{J}_i^{\mathrm{T}}\mathbf{J}_i = \mathbf{U}_i\mathbf{S}_i\mathbf{V}_i^{\mathrm{T}} \tag{12}$$

The matrices $\mathbf{U}_i$ and $\mathbf{V}_i$ are orthonormal ($\mathbf{U}_i^{-1} = \mathbf{U}_i^{\mathrm{T}}, \mathbf{V}_i^{-1} = \mathbf{V}_i^{\mathrm{T}}$) and $\mathbf{S}_i$ is diagonal matrix of singular values sequenced in the descending order. Considering SVD, equation (5) can be rewritten into form

$$\mathbf{U}_i\mathbf{S}_i\mathbf{V}_i^{\mathrm{T}}\delta\mathbf{d}_i = \mathbf{J}_i^{\mathrm{T}}\delta\mathbf{r}_i\,. \tag{13}$$

The singular value decomposition introduces vector of auxiliary variables $\mathbf{y}_i = \mathbf{V}_i^{\mathrm{T}}\delta\mathbf{d}_i$, which are generally evaluated from equation

$$\mathbf{S}_i\mathbf{y}_i = \mathbf{U}_i^{\mathrm{T}}\mathbf{J}_i^{\mathrm{T}}\delta\mathbf{r}_i\,. \tag{14}$$

If the rank of system matrix is reduced by *r* (matrix is singular, last *r* singular values are zeros), the last *r*-tuple of elements of auxiliary vector $\mathbf{y}_i$ serves as a free parameters of solution. Unique solution is replaced by *r*-parametric solution. However also for the non-singular cases (like the Trijoint calibration) the lowest singular values identify the subspace of parameters mostly influenced by the measurement errors. The mapping of the possible calibration solutions within this subspace has been performed as follows.

1. Only few iterations of the Newton method are considered. Experience indicates that two or three iterations are typically enough for reaching solution from the reasonable (design) starting point within the parameter space.
2. The last (corresponding to lowest singular values) elements of the auxiliary vectors $\mathbf{y}_i$ (*i*=1,2) are considered as a free optimisation parameters, whereas the rest of elements is computed standardly from the equations (14).
3. The appropriate objective functions representing the calibration error using different norms are put together (e.g. $\sum_{j=1}^{n}(|dx_{1V,j}|+|dy_{1V,j}|)/n$, or $\max_{j=1}^{n}(|dx_{1V,j}|,|dy_{1V,j}|)$, where *n* is the number of calibration positions and $dx_{1V,j}, dy_{1V,j}$ are final computational errors for the *j*-th position).
4. The multiobjective genetic optimization is used for the finding of the Pareto set of the objective functions because of its natural mapping of solution space within the favourable region.

The optimization can be realized by the minimization of the composed single objective function using weighted sum of the partial objective functions (error norms) [30] or using the complete multiobjective optimization [31]. The number of distinctively low singular values and consequently the number of free optimisation parameters is typically very low (up to 10 optimisation parameters). Also the necessary interval of parameter seeking is narrow. Therefore there are no problems with the optimization convergence. The described calibration modified by the optimization is typically the modest computation, which takes few minutes on the common PC.

5. MODIFIED CALIBRATION ALGORITHM FOR TRIJOINT 900H

The singular values of TRIJOINT calibration problem were typically in the range from $2*10^2$ to $2*10^{-4}$. The calibration problem is far from pure singularity, however the part of solution connected to the lowest singular values has been mapped using the algorithm from section 4 in order to further improve obtained machine accuracy. The optimization using weighted sum of the partial objective functions (error norms) [30] has been used. The number of the optimization parameters has been 6, for 2 lowest singular values and 3 iterations of the Newton method. The improvement of the objective functions stagnates after approximately 600 – 700 evaluations of the objective functions. The total number of the objective functions evaluations during optimization was 2000. Example of the results of the calibration optimization is given by Fig. 3. The two alternative error norms are depicted. The parametric variants for the experimental testing have been selected from the results on the frontier of the best results region (Pareto set).

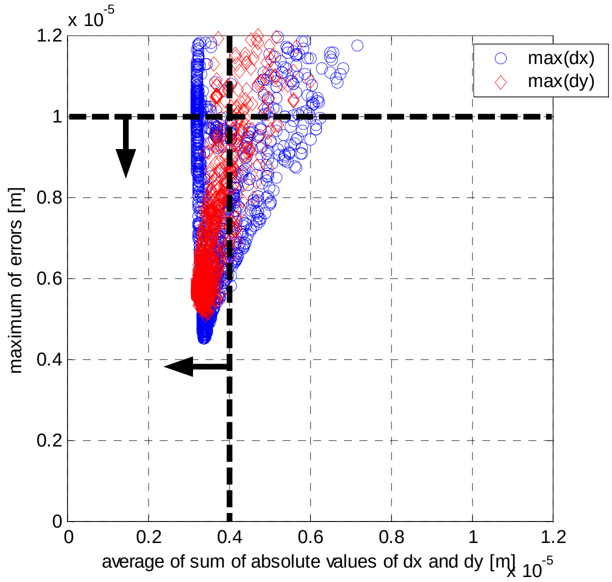


Figure 3. Example of results of calibration optimization of alternative error norms

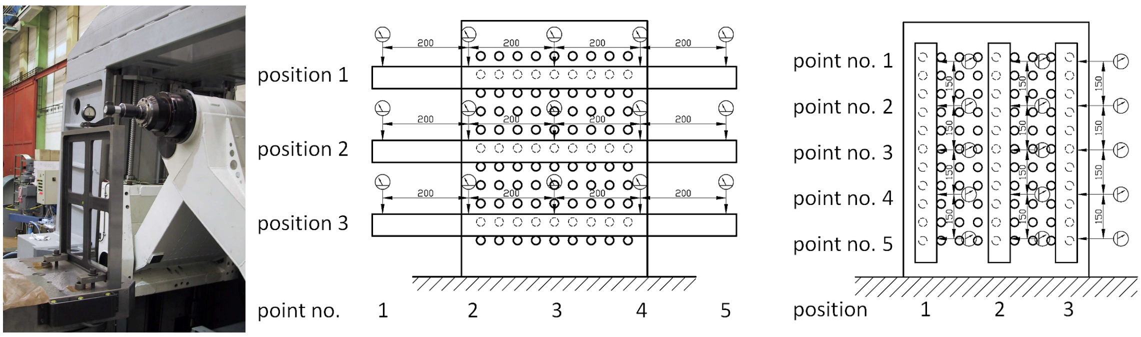


Figure 4. Trijoint 900H - measurement of straightness for horizontal and vertical direction

Finally several parametric variants from the Pareto set have been experimentally tested by the straightness measurements (Fig. 4). The best one (Fig. 5) has been implemented to the machine control algorithms. The important generalization of the experience from the Trijoint calibration is that the condition number of the calibration task should be optimized during the very early stage of machine design.

6. CALIBRABILITY AS ADDITIONAL DESIGN CRITERION

As concluded in the previous section, it is very useful to acquire a good conditionality of the calibration task already during the design process. It can be influenced by several design properties namely the machine structure, values of its geometrical parameters and the number and positioning of the sensors. Based on that the concept of calibrability is introduced and the design measure of calibrability C is defined as a pendant of other traditional design criterions, namely

$$C = cond(\mathbf{J}_{\mathbf{d}_i}{}^{\mathbf{T}}\mathbf{J}_{\mathbf{d}_i}). \tag{15}$$

The smaller value of the calibrability C the more accurate determination of the unknown actual values of the manufactured parameters $\mathbf{d}$ and the more accurate determination of the output coordinates $\mathbf{v}$ from the input coordinates $\mathbf{s}$, i.e. the smaller resulting positioning errors for the same accuracy of the accuracy of the particular sensors. Further crucial after-design aspect is the choice of the set of calibration positions of the machine.

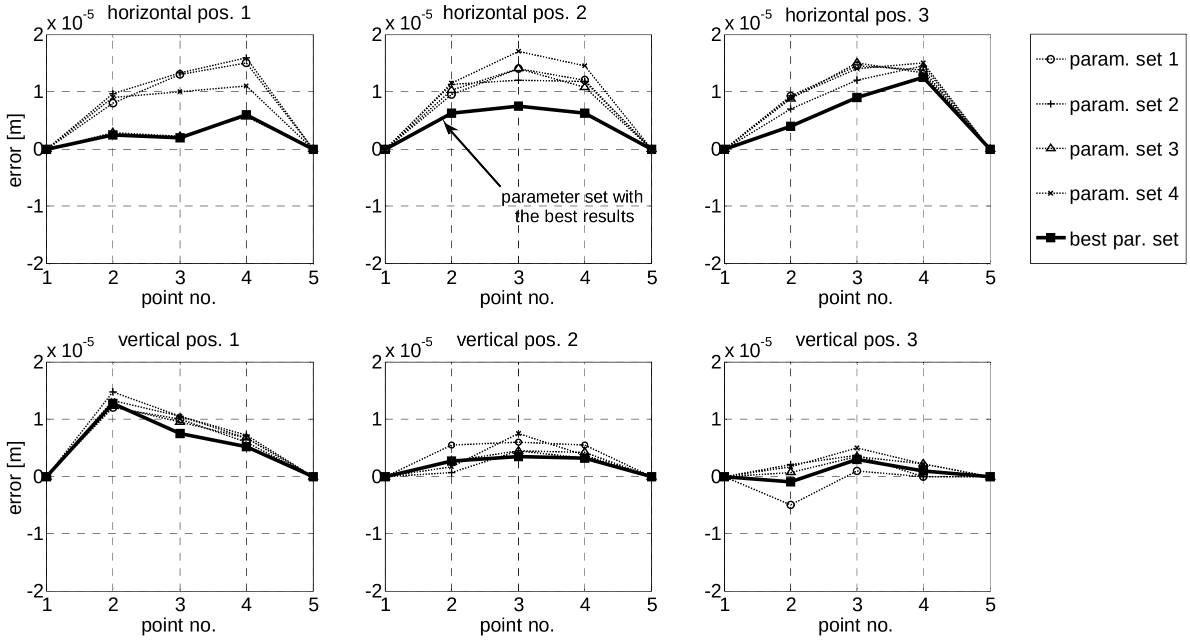


Figure 5. Results of experimental testing of straightness of parametric variants from the Pareto set

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