Name: *Date: January 19, 2022*

Written Exam — Logical Reasoning and Programming

Convention: Variables are uppercased while other symbols are lowercased.

1. Write a Prolog predicate

```
my_max(A, B, Max)
```

which will take two numbers A and B and "return" the larger one from them. For instance, the query ?-my_max(1,2,2) should return true and ?-my_max(1,2,1) should return false. *(2 points)*

2. Write a Prolog predicate

```
my_sum(List, Result)
```

which will take the list `List` and sum its elements (you can assume that the list contains only numbers). For instance, the query ?-my_sum([1,2,3],6) should return true. *(5 points)*

3. In the 3rd column, write the substitution under which the Prolog terms in the first two columns unify, or leave the cell empty if they do not unify.

2	$1 + X$	$X =$		
a	$f(X, Y)$	$X =$	$Y =$	
$[a, X]$	$[a, [c, d]]$	$X =$		
$f(X, f(Y, Z))$	$f(a, f(b, f(c, W)))$	$X =$	$Y =$	$Z =$

(4 points)

4. Consider the following Prolog program

```
edge(1,2).
edge(2,3).
edge(1,3).

connected(X,Y) :- edge(X,Y).
connected(X,Y) :- edge(X,Z), connected(Z,Y).
```

Show the SLD tree of this program for the query

```
?-connected(1,3).
```

(3 points)

5. Explain the difference between proof trees and SLD trees. *(2 points)*

6. Find the answer sets (= stable models) of the following normal logic program.

```
a :- not b, c.
b :- not a, c.
c.
```

(4 points)

7. Select exactly one theory from the following list that is usually considered the core theory of an SMT solver

   (a) arrays,
   (b) bit-vectors,
   (c) difference logic,
   (d) linear arithmetic,
   (e) non-linear arithmetic,
   (f) uninterpreted functions.

(1 point)

8. Write down all the pairs of clauses (implicitly universally quantified) from the following list (5 clauses) such that the first clause subsumes the second one

   (a) $\{p(X, Y), q(g(Y))\}$,
   (b) $\{p(X, X)\}$,
   (c) $\{p(X, a), p(b, Y)\}$,
   (d) $\{\neg q(X)\}$,
   (e) $\{p(b, a), q(g(a))\}$.

(3 points)

9. Let Γ and Δ be non-empty satisfiable sets of formulas in first-order logic. Decide which of the following claims are *always* true. Mark them either T (always true), or F (not always true).

   (a) $\{\varphi\colon \Gamma \not\models \varphi\}$ is unsatisfiable,

   (b) $\{\,\neg\varphi\colon \varphi \in \Gamma\,\}$ is unsatisfiable,

   (c) if $\varphi \in \Gamma$ and $\psi \in \Delta$, then $\varphi \vee \psi$ is satisfiable,

   (d) it is impossible to derive the empty clause from $\Gamma \cup \Delta$ by resolution,

   (e) there exist clauses $\varphi \in \Gamma$ and $\psi \in \Delta$ such that $\Gamma \models \psi$ or $\Delta \models \varphi$.

(5 points)

10. It is possible to show that the set

$$\{s \rightarrow (q \vee \neg p),\ r \rightarrow s,\ (\neg x \wedge v) \rightarrow u,\ r \rightarrow \neg t,\ r,\ q \rightarrow \neg u,\ u \rightarrow p,\ x \rightarrow w,\ \neg t \rightarrow (v \wedge \neg w)\}$$

is unsatisfiable solely by using unit propagations and simplifications. Find such a sequence of unit propagations.

(3 points)

11. Briefly explain the saturation procedure (also called the given-clause algorithm) used in resolution-based provers for first-order logic. Do not forget to list the possible outcomes and their meaning.

(4 points)

12. Use the congruence closure algorithm to decide whether it is possible to satisfy simultaneously

$$\{g(a) = f(b),\ f(f(b)) \neq h(c),\ f(g(a)) = h(d),\ c = d\}.$$

(3 points)

13. In conflict-driven clause learning (CDCL), we not only learn new clauses. From a practical point of view, it is also essential to do

    (a) restarts and

    (b) delete some learned clauses.

    Briefly explain why we do these counter-intuitive steps. *(3 points)*

14. Show by resolution that from

$$\begin{aligned}&\forall X(\neg r(f(X), X),\\&\forall X \forall Y(r(X,Y) \rightarrow r(Y,X)),\\&\forall X \forall Y \exists Z(r(X,Z) \vee r(Z,Y))\end{aligned}$$

all the following formulas are provable

(a) $\exists X \forall Y(\neg r(Y, f(Y)))$,

(b) $\forall X \exists Y(r(X,Y))$,

(c) $\forall X \exists Y \exists Z(r(Y,X) \wedge \neg r(Z,Y))$.

(8 points)