Introduction to Prolog (2)

Using some slides from Peter Flach's materials

Recap from the last lecture… Hebrand…

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- **Hebrand universe:**
  - {alice, prolog}
- **Hebrand base:**
  - {teaches(alice, prolog), teaches(prolog,alice), teaches(prolog,prolog), teaches(alice,alice), course(alice), course(prolog), person(alice), person(prolog)}

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- **For instance:**

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```

- **Hebrand universe:**
  - {alice, prolog}
- **Hebrand base:**
  - {teaches(alice, prolog), teaches(prolog,alice), teaches(prolog,prolog), teaches(alice,alice), course(alice), course(prolog), person(alice), person(prolog)}
- **Hebrand interpretation:**
  - {teaches(alice, prolog), course(prolog), person(alice)}
  - …which is also a Hebrand model of the program.

Recap from the last lecture… Functions

- **Function symbols** allow us to construct composite terms (besides that they also allow us to do **Skolemization**).

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```
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```

  - **Herbrand universe:** {0, s(0), s(s(0)), …}
  - **Hebrand base:** {num(0), num(s(0)), num(s(s(0))), …}
  - In this case, we do not have a finite Herbrand model.

A Digression: Skolemization

"Everybody knows somebody."

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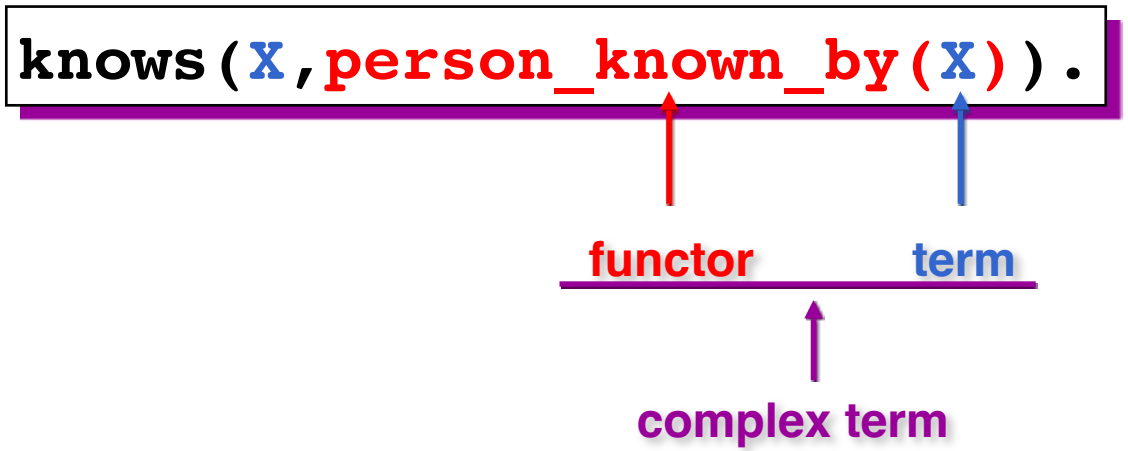
Skolemization to avoid an existential quantifier




```
knows(peter,person_known_by(peter)).
knows(anna,person_known_by(anna)).
knows(paul,person_known_by(paul)).
…
```

Recap from the last lecture… Logical consequence

- A clause ***C*** is a **logical consequence** of a program (set of clauses) ***P*** iff every model of ***P*** is a model of ***C***.

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Program **P**:

```
num(0).
num(s(X)) :- num(X).
```

Some logical consequences of **P**:

```
num(s(s(0))).
num(0)
num(s(s(X))) :- num(X).
```

An Example: Prague Metro

A Prolog DB (1)

```
connected(nemocnice_motol,petriny,green).
connected(petriny,nadrazi_veleslavin,green).
connected(nadrazi_veleslavin,borislavka,green).
connected(borislavka,dejvicka,green).
connected(dejvicka,hradcanska,green).
connected(hradcanska,malostranska,green).
connected(malostranska,staromestska,green).
connected(staromestska,mustek,green).
connected(mustek,muzeum,green).
connected(muzeum,namesti_miru,green).
connected(namesti_miru,jiriho_z_podebrad,green).
connected(jiriho_z_podebrad,flora,green).
connected(flora,zelivskeho,green).
connected(zelivskeho,strasnicka,green).
connected(strasnicka,skalka,green).
connected(skalka,depo_hostivar,green).
```

A Prolog DB (2)

```
connected(letnany,prosek,red).
connected(prosek,strizkov,red).
connected(strizkov,ladvi,red).
connected(ladvi,kobylisy,red).
connected(kobylisy,nadrazi_holesovice,red).
connected(nadrazi_holesovice,vltavska,red).
connected(vltavska,florenc,red).
connected(florenc,hlavni_nadrazi,red).
connected(hlavni_nadrazi,muzeum,red).
connected(muzeum,i_p_pavlova,red).
connected(i_p_pavlova,vysehrad,red).
connected(vysehrad,prazskeho_povstani,red).
connected(prazskeho_povstani,pankrac,red).
connected(pankrac,budejovicka,red).
connected(budejovicka,kacerov,red).
connected(kacerov,roztyly,red).
connected(roztyly,chodov,red).
connected(chodov,opatov,red).
connected(opatov,haje,red).
```

A Prolog DB (3)

```
connected(zlicin,stodulky,yellow).
connected(stodulky,luka,yellow).
connected(luka,luziny,yellow).
connected(luziny,hurka,yellow).
connected(hurka,nove_butovice,yellow).
connected(nove_butovice,jinonice,yellow).
connected(jinonice,radlicka,yellow).
connected(radlicka,smichov,yellow).
connected(smichov,andel,yellow).
connected(andel,karlovo_namesti,yellow).
connected(karlovo_namesti,narodni_trida,yellow).
connected(narodni_trida,mustek,yellow).
connected(mustek,namesti_republiky,yellow).
connected(namesti_republiky,florenc,yellow).
connected(florenc,krizikova,yellow).
connected(krizikova,invalidovna,yellow).
connected(invalidovna,palmovka,yellow).
connected(palmovka,ceskomoravska,yellow).
connected(ceskomoravska,vysocanska,yellow).
connected(vysocanska,kolbenova,yellow).
connected(kolbenova,hloubetin,yellow).
connected(hloubetin,rajska_zahrada,yellow).
connected(rajska_zahrada,cerny_most,yellow).
```

“Nearby”

Two stations are nearby if they are on the same line with at most one other station in between:

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```
nearby(zlicin,luka).
nearby(luka,zlicin).
nearby(zlicin,stodulky).
nearby(stodulky,zlicin).
nearby(luka,luziny).
nearby(luziny,luka).
nearby(luka,hurka).
```

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Two stations are nearby if they are on the same line with at most one other station in between:

```
nearby(zlicin,luka).
nearby(luka,zlicin).
nearby(zlicin,stodulky).
nearby(stodulky,zlicin).
nearby(luka,luziny).
nearby(luziny,luka).
nearby(luka,hurka).
...
```

or better

```
nearby(X,Y):-connectedS(X,Y,L).
nearby(X,Y):-connectedS(X,Z,L),connectedS(Z,Y,L).
connectedS(X,Y,W) :- connected(X,Y,W).
connectedS(X,Y,W) :- connected(Y,X,W).
```

“Not too far”

Compare

```
nearby(X,Y):-connectedS(X,Y,L).
nearby(X,Y):-connectedS(X,Z,L),connectedS(Z,Y,L).
```

with

```
not_too_far(X,Y):-connectedS(X,Y,L).
not_too_far(X,Y):-connectedS(X,Z,L1),connectedS(Z,Y,L2).
```

“Not too far”

Compare

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nearby(X,Y):-connectedS(X,Y,L).
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with

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not_too_far(X,Y):-connectedS(X,Y,L).
not_too_far(X,Y):-connectedS(X,Z,L1),connectedS(Z,Y,L2).
```

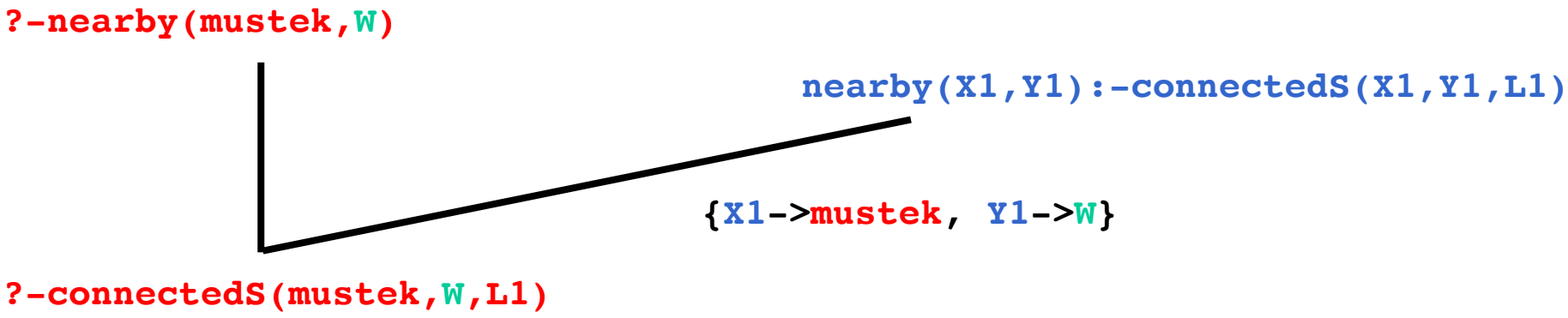
This can be rewritten with don’t cares:

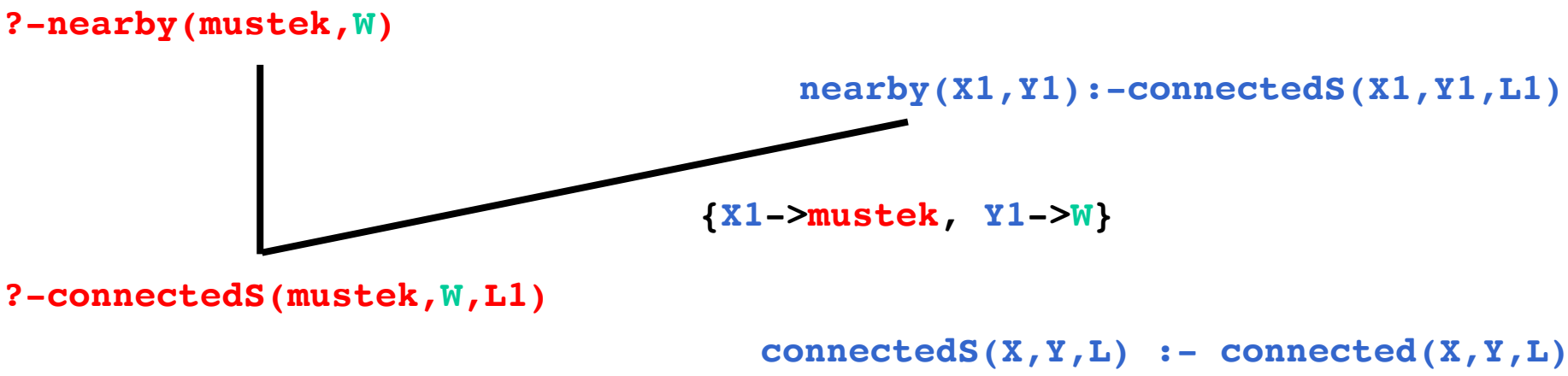
```
not_too_far(X,Y):-connectedS(X,Y,_).
not_too_far(X,Y):-connectedS(X,Z,_),connectedS(Z,Y,_).
```

```
?-nearby(mustek,W)
```

```
?-nearby(mustek,W)
```

```
nearby(X1,Y1):-connectedS(X1,Y1,L1)
```


?-nearby(mustek,W)
nearby(X1,Y1):-connectedS(X1,Y1,L1)
{X1->mustek, Y1->W}
?-connectedS(mustek,W,L1)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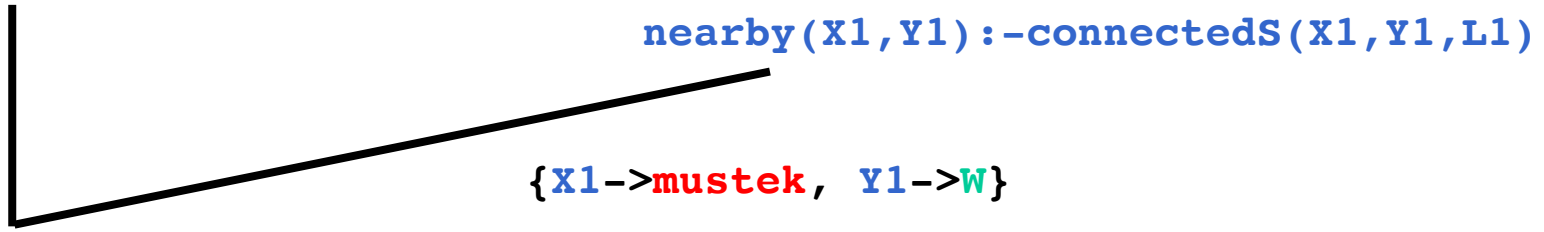
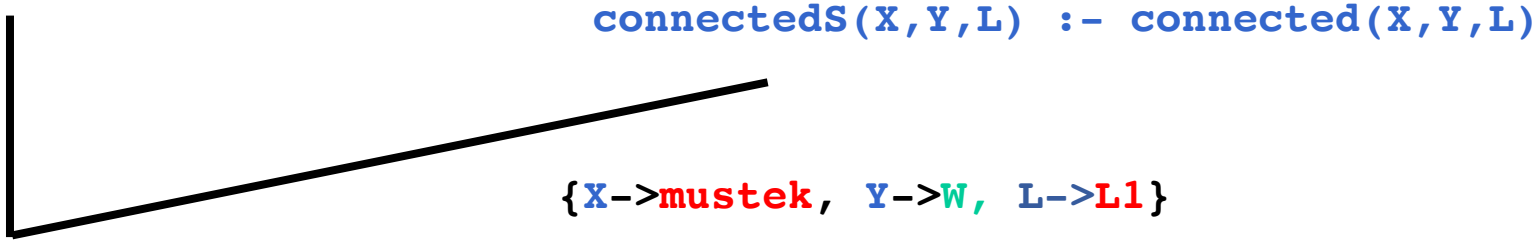
connectedS(X,Y,L) :- connected(X,Y,L)

?-nearby(mustek,W)

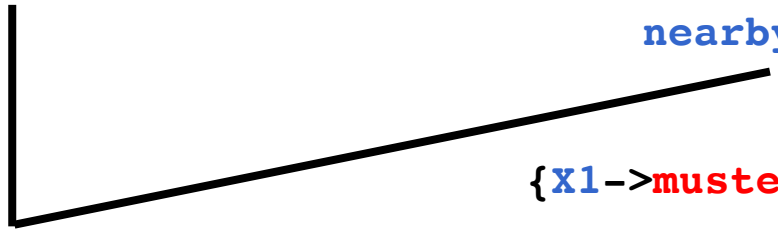


?-connectedS(mustek,W,L1)



?-connected(mustek,W,L1)

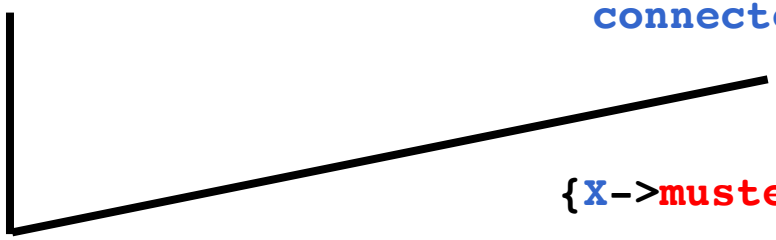
?-nearby(mustek,W)



nearby(X1,Y1):-connectedS(X1,Y1,L1)

{X1->mustek, Y1->W}

?-connectedS(mustek,W,L1)

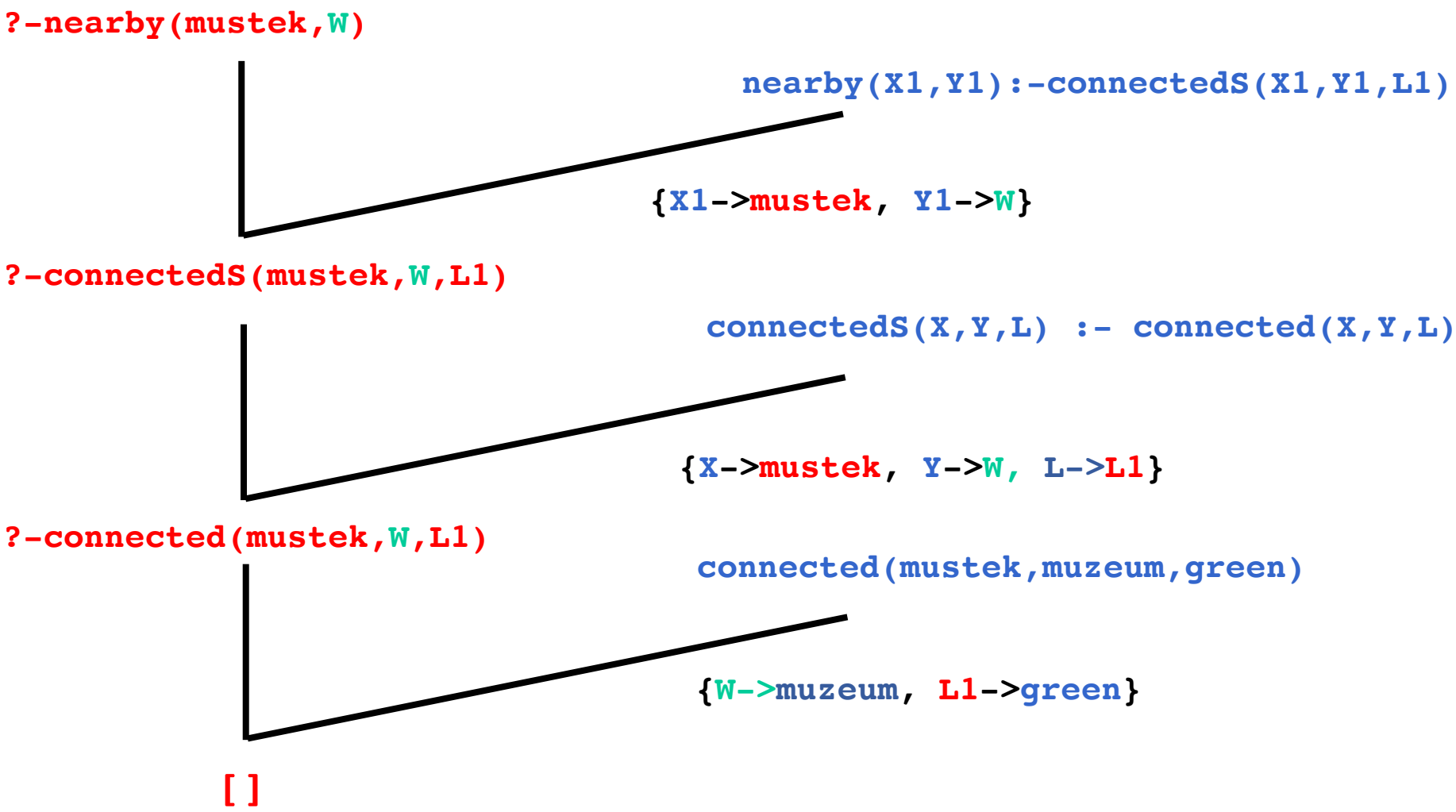


connectedS(X,Y,L) :- connected(X,Y,L)

{X->mustek, Y->W, L->L1}

?-connected(mustek,W,L1)

connected(mustek,muzeum,green)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?-connected(mustek,W,L1)
connected(mustek,muzeum,green)
{W->muzeum, L1->green}
[ ]

“Reachable”

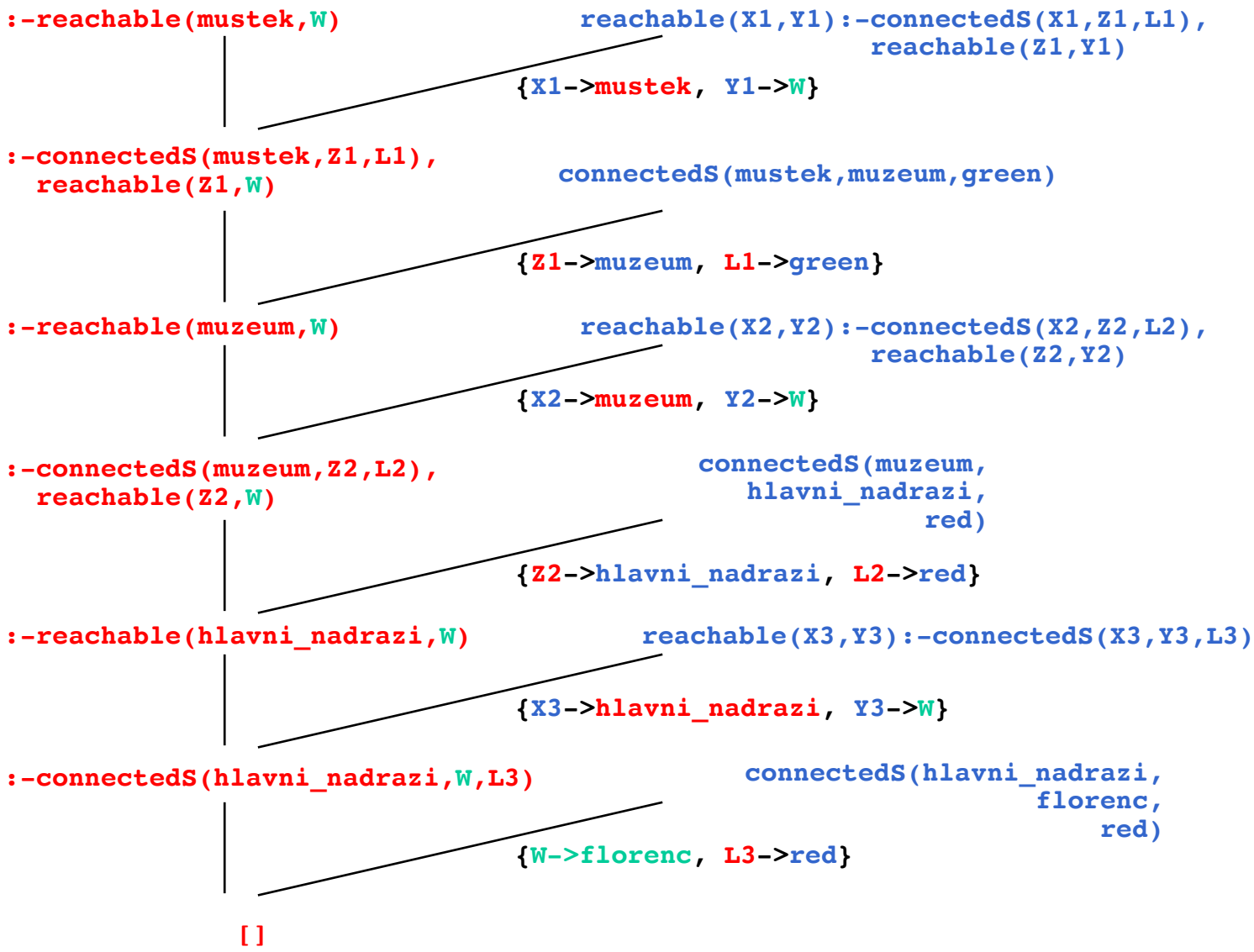
A station is reachable from another if they are on the same line, or with one, two, … changes:

```
reachable(X,Y):-connectedS(X,Y,L).
reachable(X,Y):-connectedS(X,Z,L1),connectedS(Z,Y,L2).
reachable(X,Y):-connectedS(X,Z1,L1),connectedS(Z1,Z2,L2),
                connectedS(Z2,Y,L3).
...
```

or better

```
reachable(X,Y):-connectedS(X,Y,L).
reachable(X,Y):-connectedS(X,Z,L),reachable(Z,Y).
```

Note: To make the example more compact, we assume that we have the predicate connectedS given as ground facts.



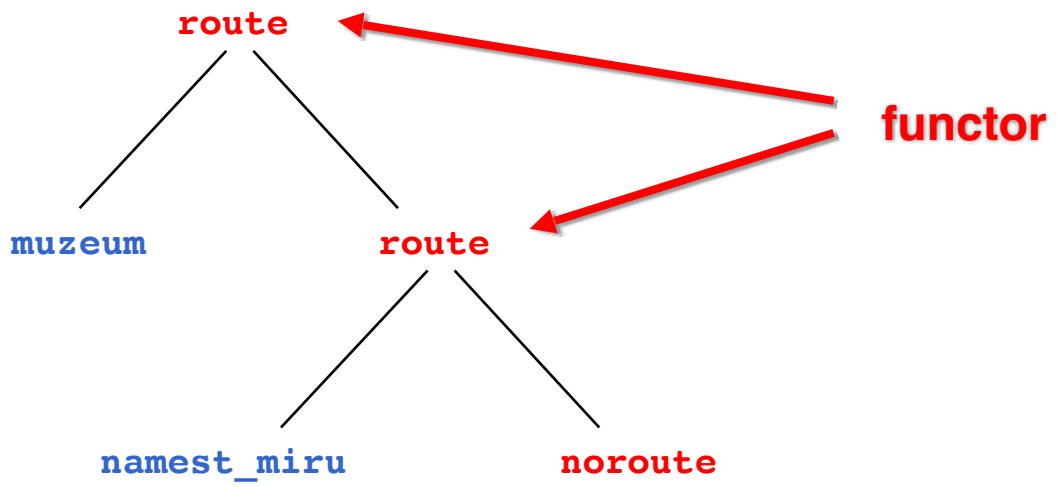
There is a catch!

- The answers that we get depend on the exact way Prolog works inside. We will talk about that next time.

“Recording the Path”

```
reachable(X,Y,noroute):-connected(X,Y,L).
reachable(X,Y,route(Z,R)):-connected(X,Z,L),
                           reachable(Z,Y,R).

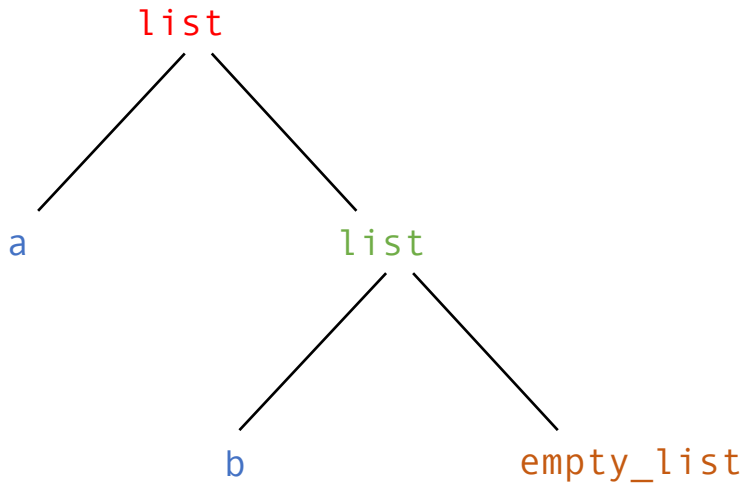
?-reachable(mustek,jiriho_z_podebrad,R).
R = route(muzeum,route(namesti_miru,noroute));
…
```



More examples (Still with our “own” lists)

- Here we will represent lists more or less like we represented routes (Prolog has built in support for lists but this is still to illustrate them as pure logic conepts), e.g.: `empty_list, list(X,empty_list), ….`

```
list(a,list(b,empty_list))
```



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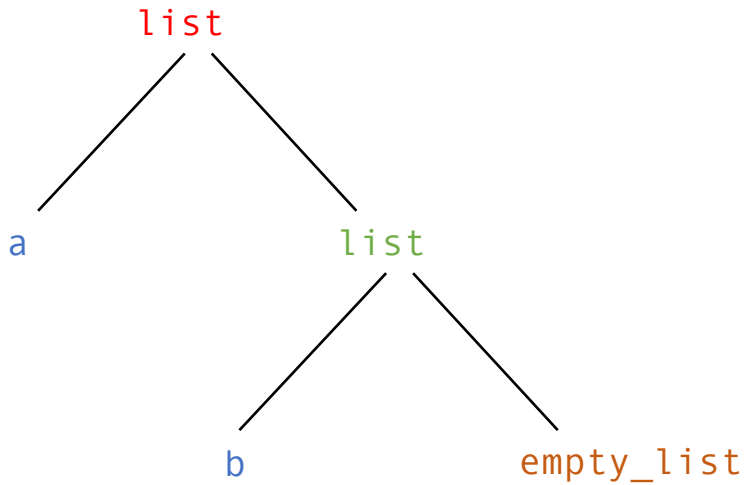
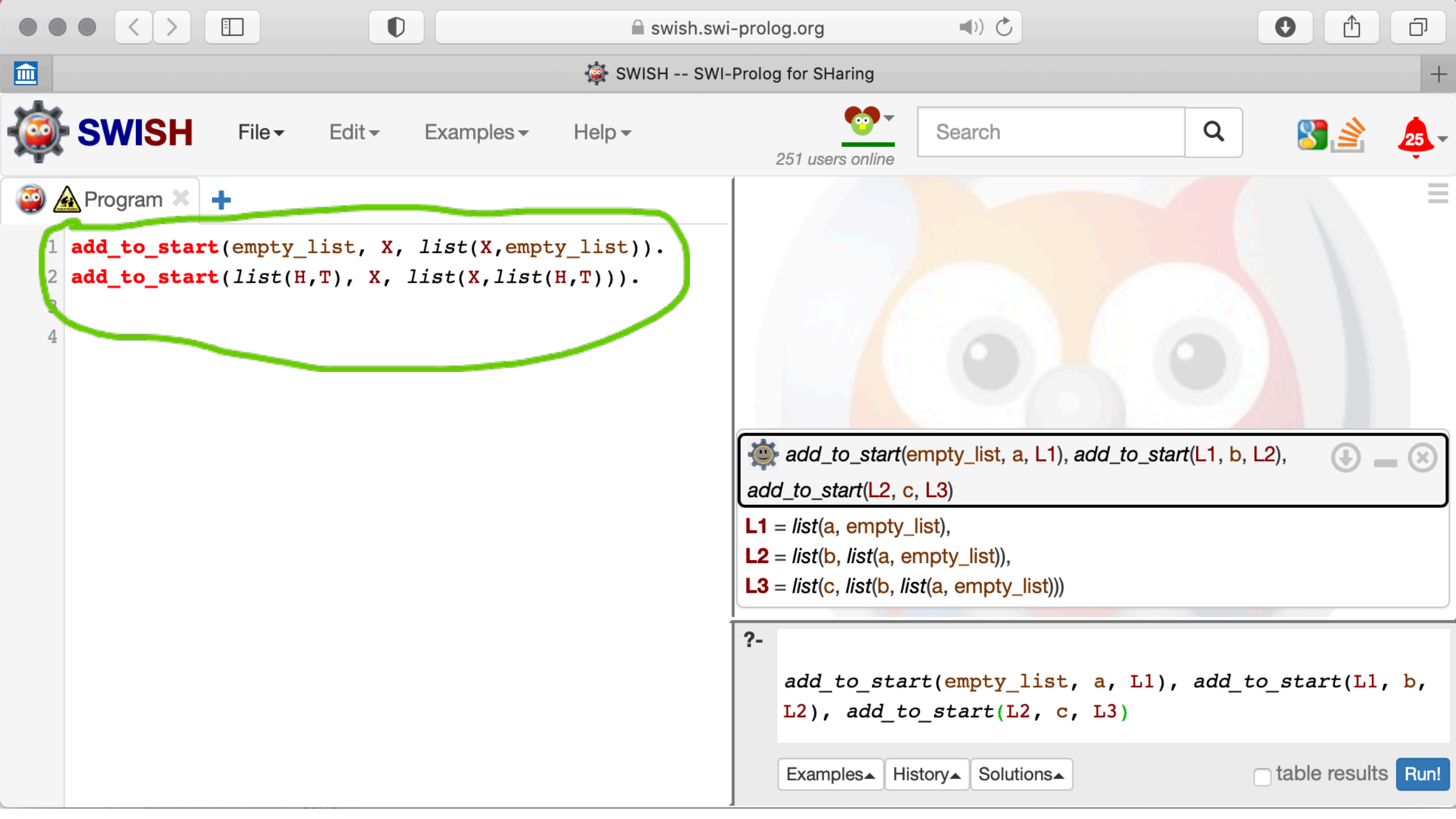
Example:

```
add_to_start(empty_list, X, list(X,empty_list)).
add_to_start(list(H,T), X, list(X,list(H,T))).
```

```
list(a,list(b,empty_list))
```

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Program
1 add_to_start(empty_list, X, list(X,empty_list)).
2 add_to_start(list(H,T), X, list(X,list(H,T))).
3
4
add_to_start(empty_list, a, L1), add_to_start(L1, b, L2),
add_to_start(L2, c, L3)
L1 = list(a, empty_list),
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?-
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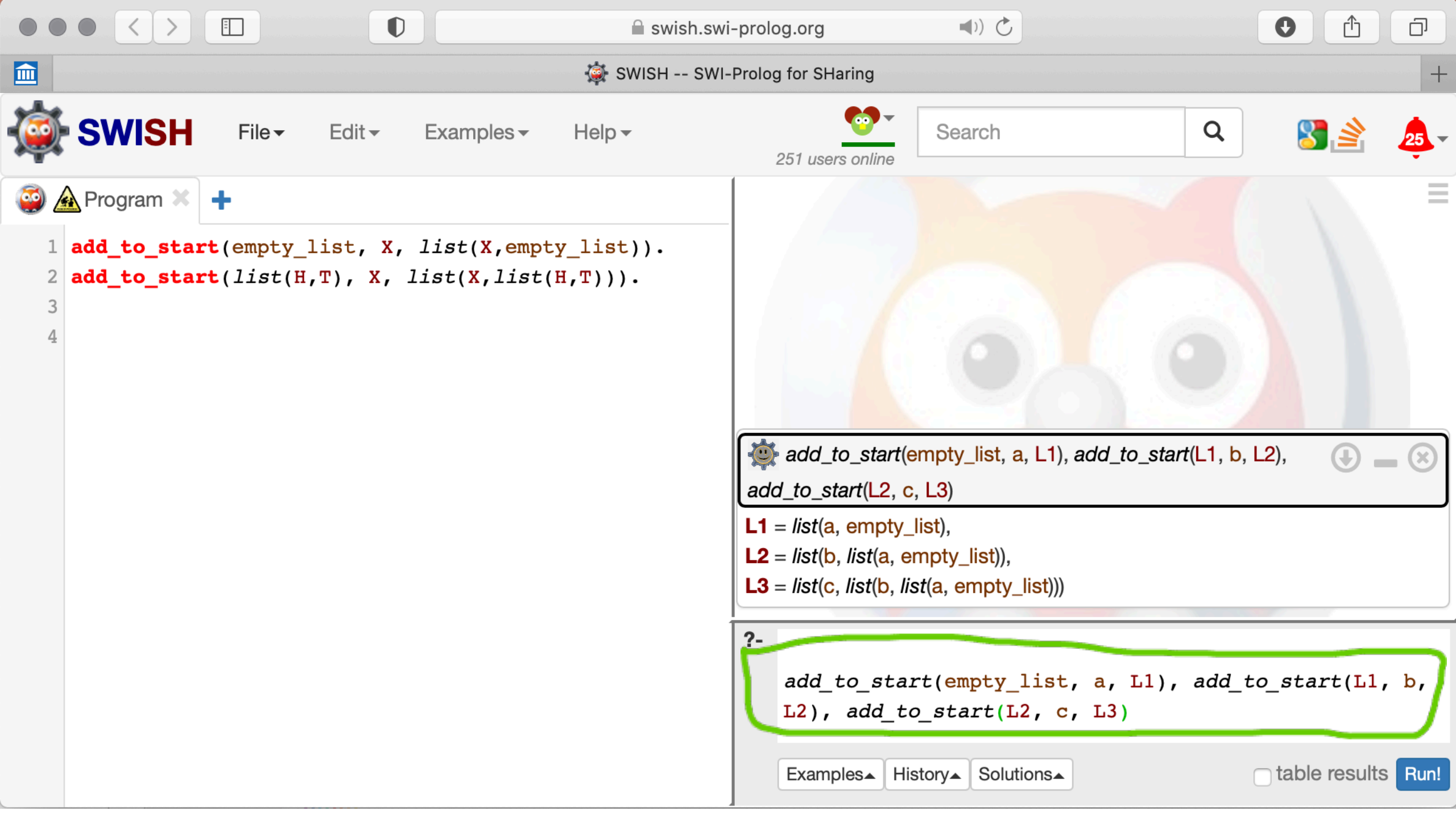
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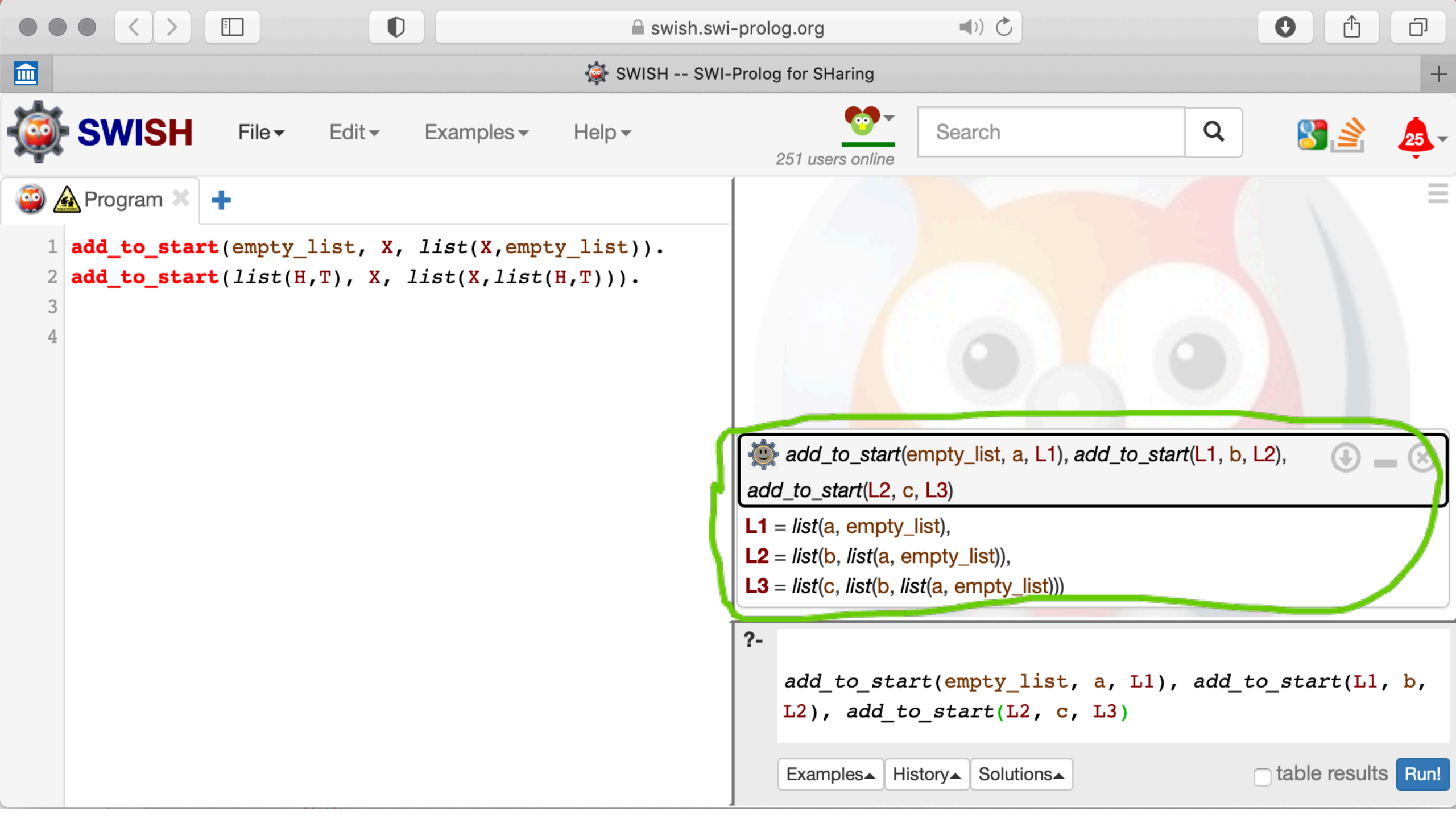
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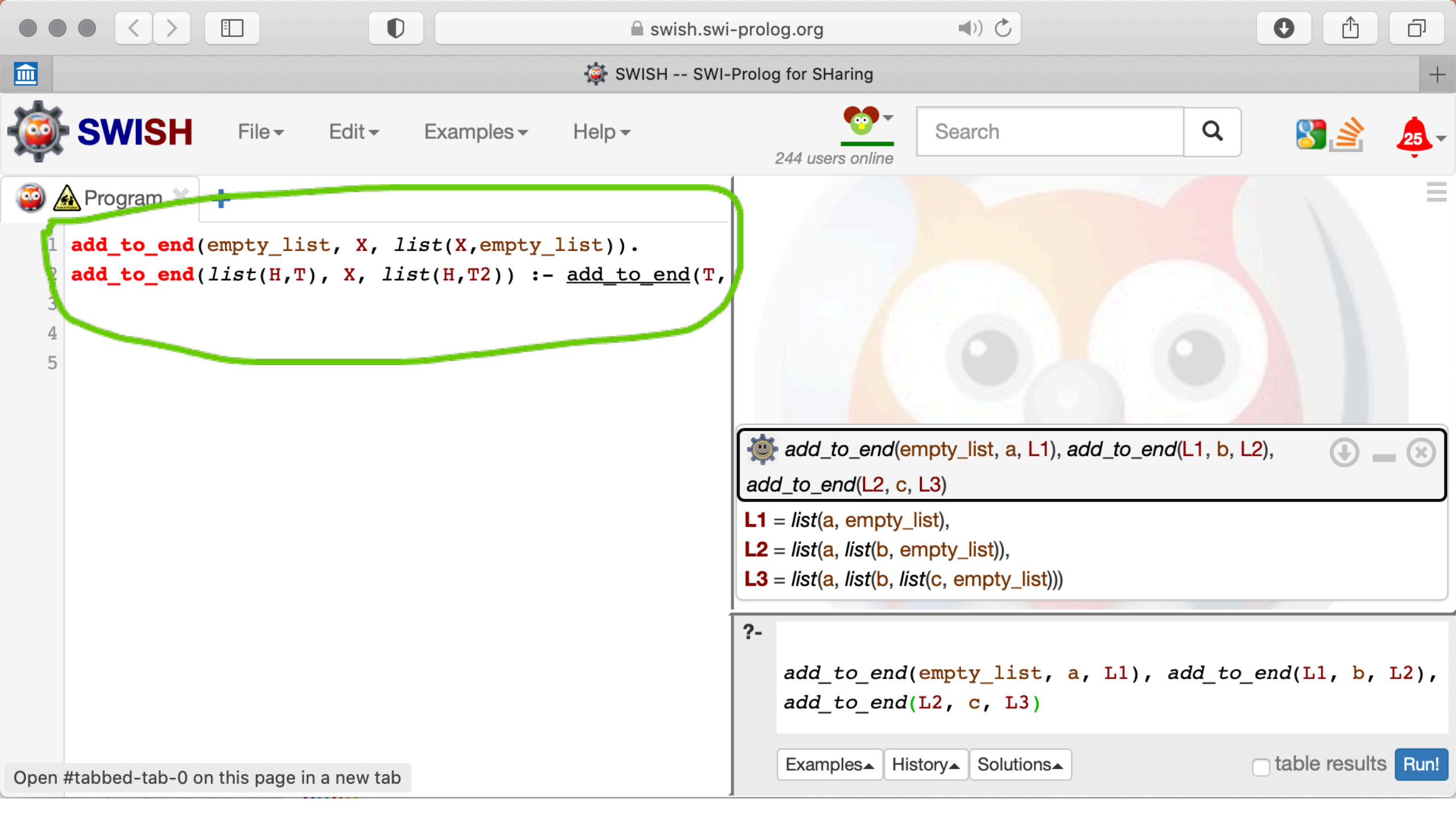
Yet another example

```
add_to_end(empty_list, X, list(X,empty_list)).
add_to_end(list(H,T), X, list(H,T2)) :- add_to_end(T,X,T2).
```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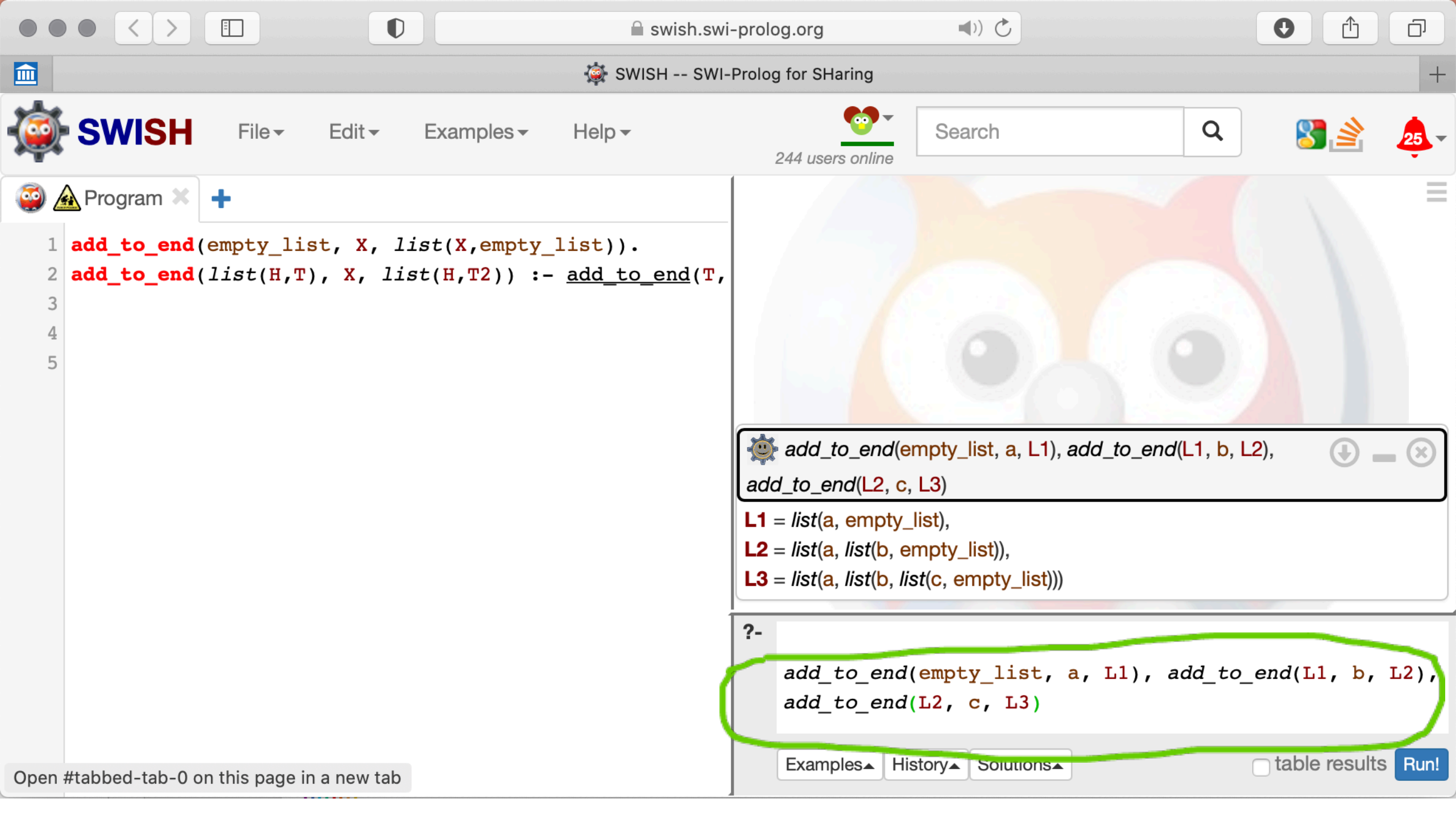
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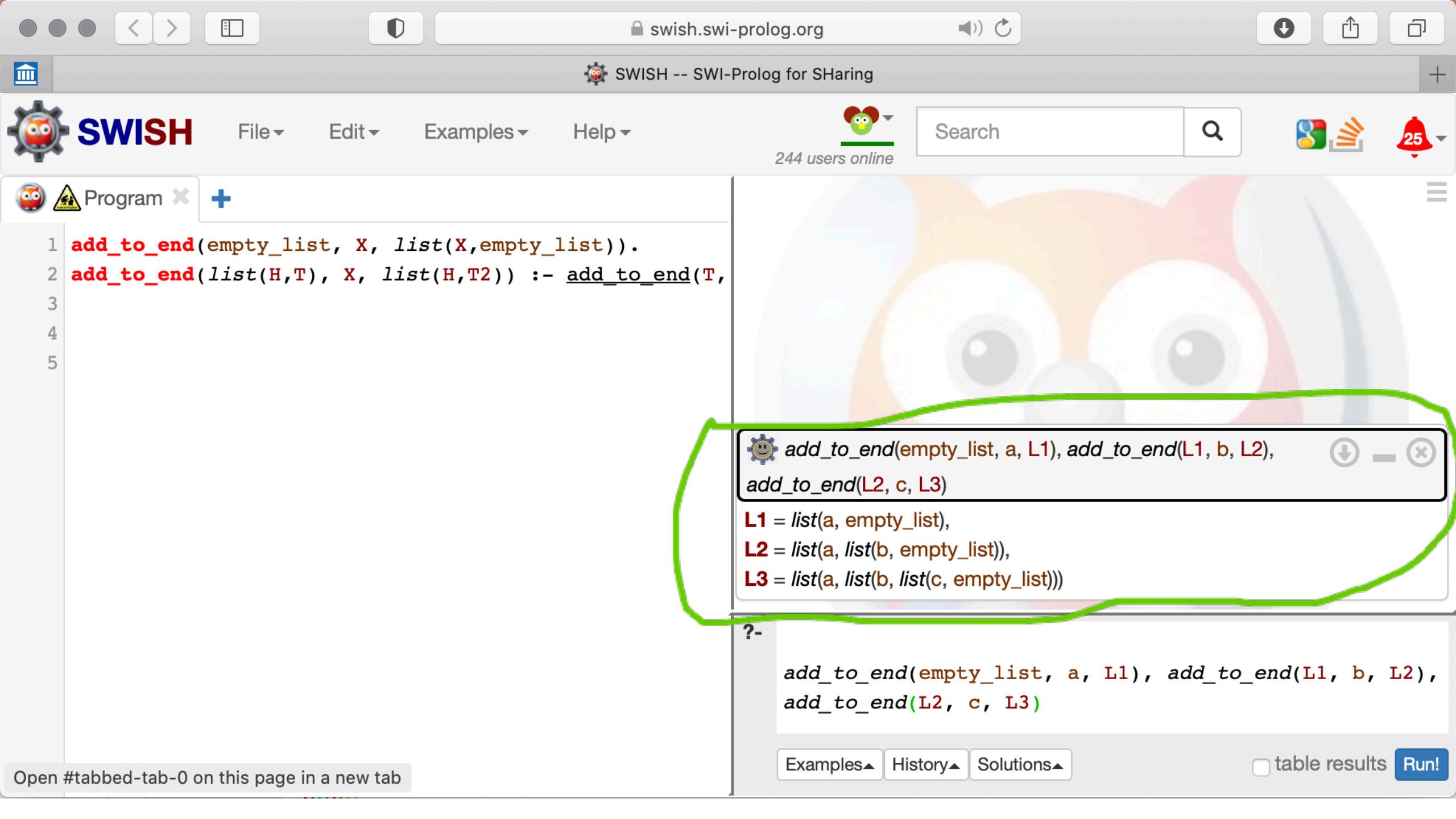
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Open #tabbed-tab-0 on this page in a new tab

Removing last element

We had:

```
add_to_end(empty_list, X, list(X,empty_list)).
add_to_end(list(H,T), X, list(H,T2)) :- add_to_end(T,X,T2).
```

We can use:

```
remove_last(List,NewList) :- add_to_end(NewList,_,List).
```

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add_to_end(empty_list, X, list(X,empty_list)).
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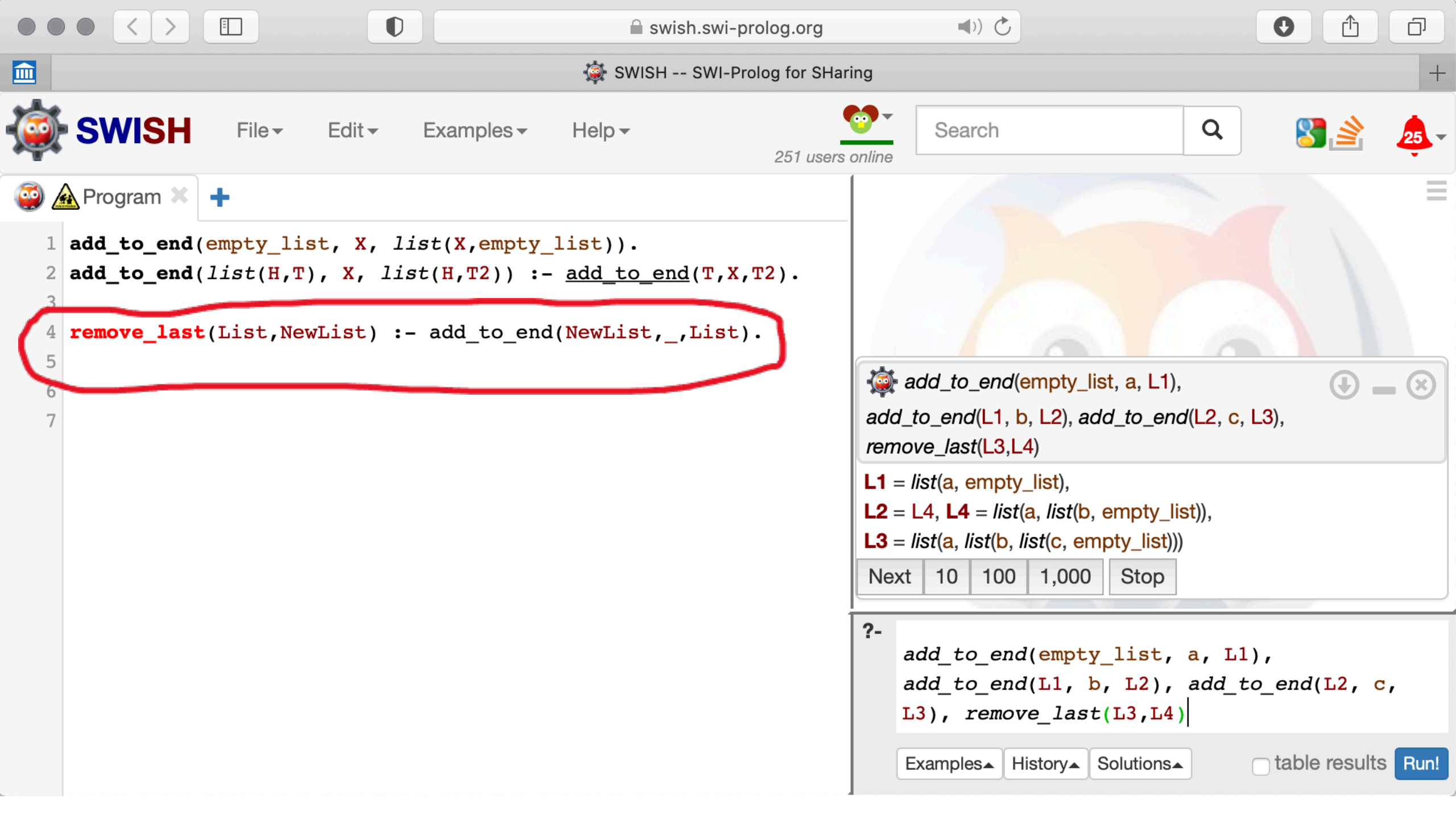
```

in out

We can use:

```
remove_last(List,NewList) :- add_to_end(NewList,_,List).
```

out in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1 add_to_end(empty_list, X, list(X,empty_list)).
2 add_to_end(list(H,T), X, list(H,T2)) :- add_to_end(T,X,T2).
3
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5
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7
add_to_end(empty_list, a, L1),
add_to_end(L1, b, L2), add_to_end(L2, c, L3),
remove_last(L3,L4)
L1 = list(a, empty_list),
L2 = L4, L4 = list(a, list(b, empty_list)),
L3 = list(a, list(b, list(c, empty_list)))
Next
10
100
1,000
Stop
?-
add_to_end(empty_list, a, L1),
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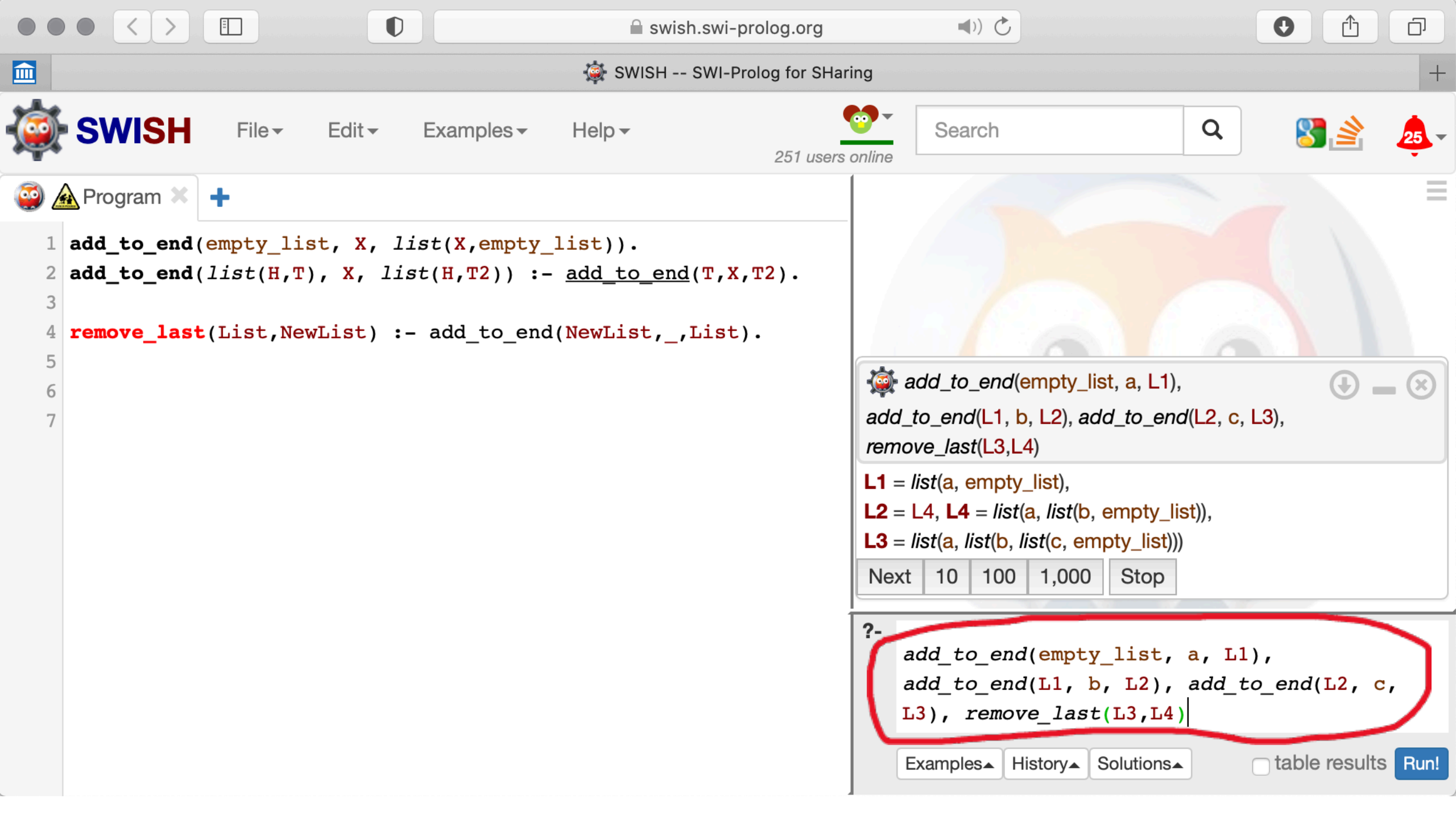
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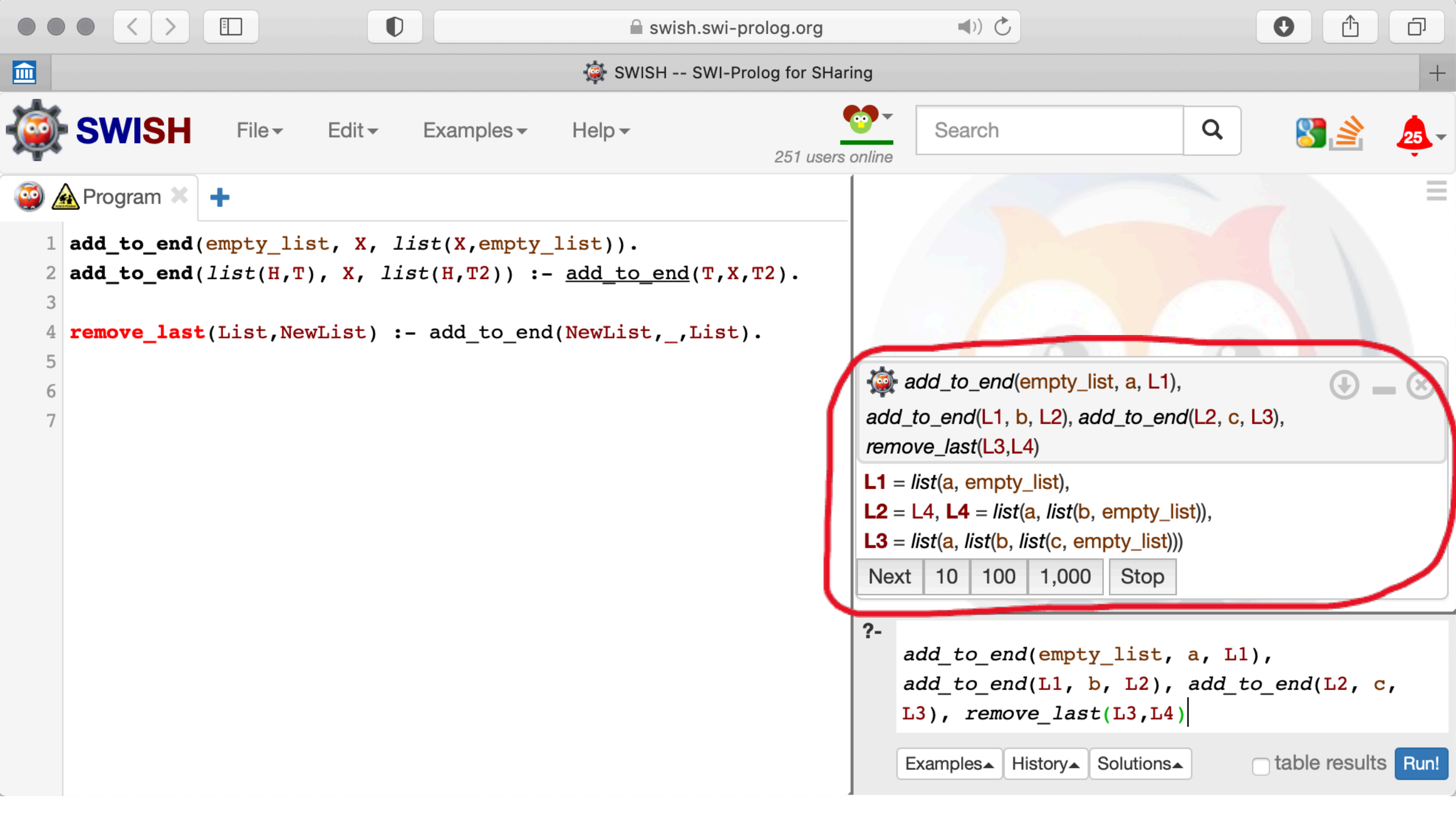
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Built-in Lists in Prolog

Examples of lists:

[], `[a,b,c]`, `[a,X,c,d]`, …

Instead of writing `list(H,T)`, we write [H|T] in Prolog.

```
add_to_end(empty_list, X, list(X,empty_list)).
add_to_end(list(H,T), X, list(H,T2)) :- add_to_end(T,X,T2).

add_to_end([], X, [X]).
add_to_end([H|T], X, [H|T2]) :- add_to_end(T,X,T2).
```

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Program

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1 add_to_end([], X, [X]).
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3
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5 remove_last(List,NewList) :- add_to_end(NewList,_,List).
6
7
8
```

add_to_end(L1, b, L2), *add_to_end*(L2, c, L3),
remove_last(L3,L4)

L1 = *list*(a, empty_list),
L2 = L4, **L4** = *list*(a, *list*(b, empty_list)),
L3 = *list*(a, *list*(b, *list*(c, empty_list)))

add_to_end([], a, L1), *add_to_end*(L1, b, L2),
add_to_end(L2, c, L3), *remove_last*(L3,L4)

L1 = [a],
L2 = L4, **L4** = [a, b],
L3 = [a, b, c]

Next | 10 | 100 | 1,000 | Stop

?-

```
add_to_end([], a, L1), add_to_end(L1, b,
L2), add_to_end(L2, c, L3),
remove_last(L3,L4)
```

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```

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SLD Resolution in Detail

Unification

Recap from the last lecture… Substitutions

- **Example:**

```
likes(X,Y) :- dogPerson(X), dog(Y).
```

Recap from the last lecture… Substitutions

- **Example:**

```
likes(X,Y) :- dogPerson(X), dog(Y).

S = {X/alice, Y/barney}
```

Recap from the last lecture… Substitutions

- **Example:**

```
likes(X,Y) :- dogPerson(X), dog(Y).

S = {X/alice, Y/barney}

likes(alice, barney) :- dogPerson(alice), dog(barney)
```

Unification

- (We need unification to perform resolution.)

- **Unifier:** Given two atoms or terms *A* and *B* with disjoint (!) sets of variables, their unifier is a substitution *Θ* such that AΘ = BΘ.

Unification

- (We need unification to perform resolution.)

- **Unifier:** Given two atoms or terms *A* and *B* with disjoint (!) sets of variables, their unifier is a substitution *Θ* such that AΘ = BΘ.
- A unifier *Θ* is more general (*technically speaking, we should be saying more or equally general*) than a unifier *Θ'* iff there exists a substitution *σ* such that *Θ' = Θ σ.*

Unification - Example

- …A unifier *Θ* is more general (*technically speaking, we should be saying more or equally general*) than a unifier *Θ'* iff there exists a substitution *σ* such that *Θ' = Θ σ*. If *Θ* is more general than *Θ* and not vice versa, then *Θ* is strictly more general than *Θ'.*

Example:

A = `studies(X, Y)`

B = `studies(alice, Z)`

Θ = *{*X/`alice`, Y/Z*}*

Θ' = *{*X/`alice`, Y/`bob` , Z/`bob`*}*

Θ' = Θ σ, where *σ* = *{*Z/`bob`*}*

Unification – Most General Unifier

- **Most General Unifier:** Given two atoms or terms *A* and *B* with disjoint sets of variables, their most general unifier (MGU) is a unfifier *Θ* such that there is no strictly more general unifier of A and B.
- **Example:**

A = `studies(X, Y)`

B = `studies(alice, Z)`

Θ = {X/`alice`, Y/Z}

Here, *Θ* is an MGU of A and B.

The Hebrand Unification Algorithm

(Adapted from Pierre M. Nugues: Language Processing with Perl and Prolog)

- **Initialization:** `Θ = {}, failure = false`, **push** `A = B` on the stack
- **Loop:**

1: **Pop** `x = y` from the stack

2: **If** x and y are constants and `x == y` **then continue**.

3: **Else if** x is a variable and x does not appear in y **then** /* "**Occurs check**" */

4: Replace x with y in the stack and in Θ. Add the substitution x / y to Θ.

5: **Else if** x is a variable and `x == y` **then continue.**

6: **Else if** y is a variable and `x` is not a variable **then push** `y = x` on the stack.

7: **Else if** x and y are compound and x is f(t1,…,tk) and y is f(u1,…,uk) **then**

8: **Push** on the stack: `t1 = u1, t2 = u2, …, tk = uk.`

9: **Else set** `failure = true, Θ = {}` and **return**.

Until stack is empty.

Example

`f(g(X, h(X, b)), Z) = f(g(a, Z), Y)`

(Adapted from Pierre M. Nugues: Language Processing with Perl and Prolog)

Stack:

`f(g(X, h(X, b)), Z) = f(g(a, Z), Y)` */* In Prolog, "=" means "unify" */*

Θ = **{}**

Example

`f(g(X, h(X, b)), Z) = f(g(a, Z), Y)`

(Adapted from Pierre M. Nugues: Language Processing with Perl and Prolog)

Stack:

`g(X, h(X, b)) = g(a, Z)`

`Z = Y`

Θ = **{}**

Example

`f(g(X, h(X, b)), Z) = f(g(a, Z), Y)`

(Adapted from Pierre M. Nugues: Language Processing with Perl and Prolog)

Stack:

`X = a`

`h(X, b) = Z`

`Z = Y`

Θ = {}

Example

`f(g(X, h(X, b)), Z) = f(g(a, Z), Y)`

(Adapted from Pierre M. Nugues: Language Processing with Perl and Prolog)

Stack:

`h(a, b) = Z`

`Z = Y`

`Θ = {X/a}`

Example

`f(g(X, h(X, b)), Z) = f(g(a, Z), Y)`

(Adapted from Pierre M. Nugues: Language Processing with Perl and Prolog)

Stack:

`Z = h(a, b)`

`Z = Y`

`Θ = {X/a}`

Example

`f(g(X, h(X, b)), Z) = f(g(a, Z), Y)`

(Adapted from Pierre M. Nugues: Language Processing with Perl and Prolog)

Stack:

`h(a,b) = Y`

`Θ = {X/a, Z/h(a,b)}`

Example

`f(g(X, h(X, b)), Z) = f(g(a, Z), Y)`

(Adapted from Pierre M. Nugues: Language Processing with Perl and Prolog)

Stack:

EMPTY

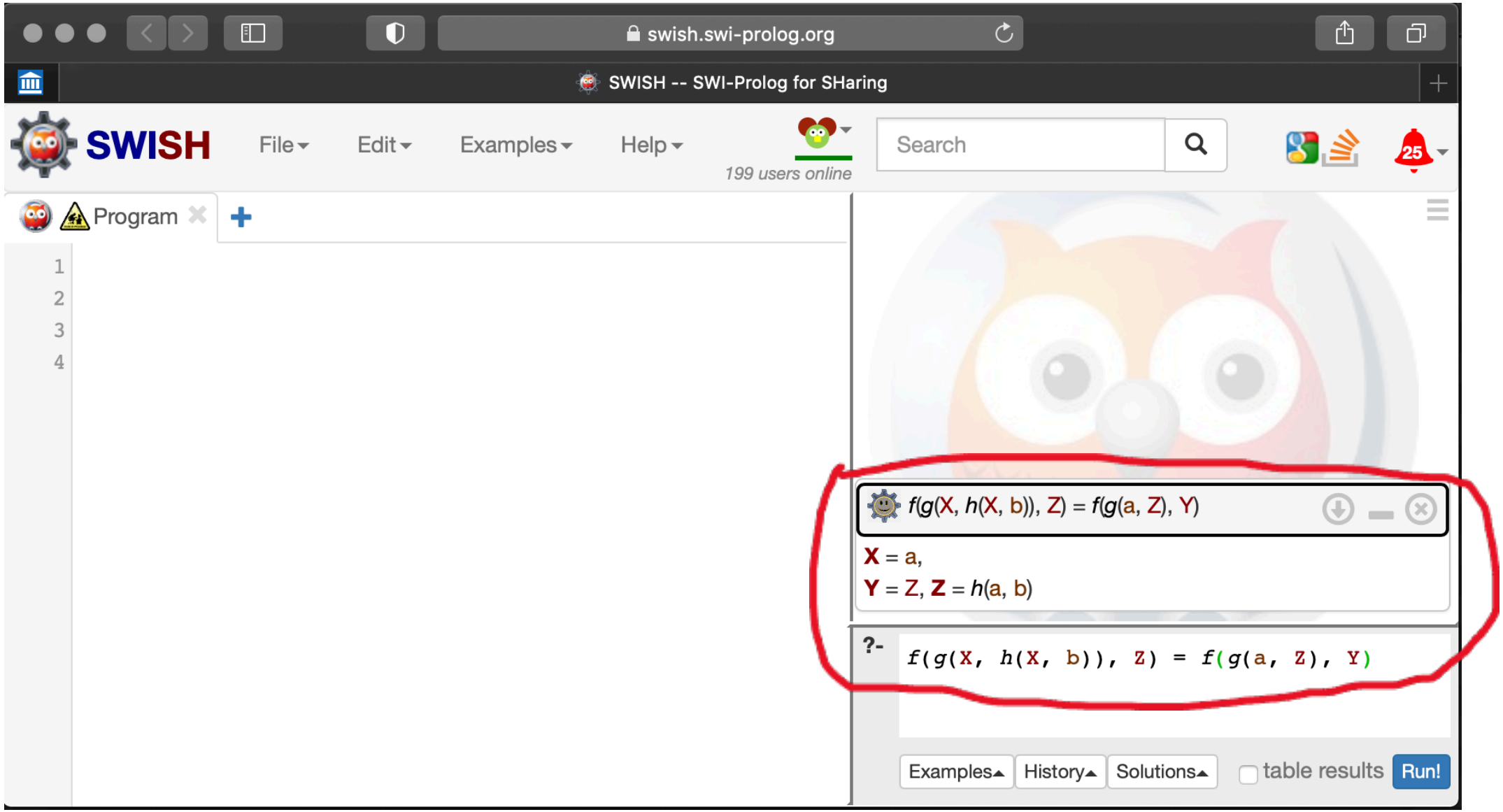
Θ = {X/a, Z/h(a,b), Y/h(a,b)}

Result of unification - MGU:

f(g(a, h(a, b)), h(a, b))

Example

```
f(g(X, h(X, b)), Z) = f(g(a, Z), Y)
```

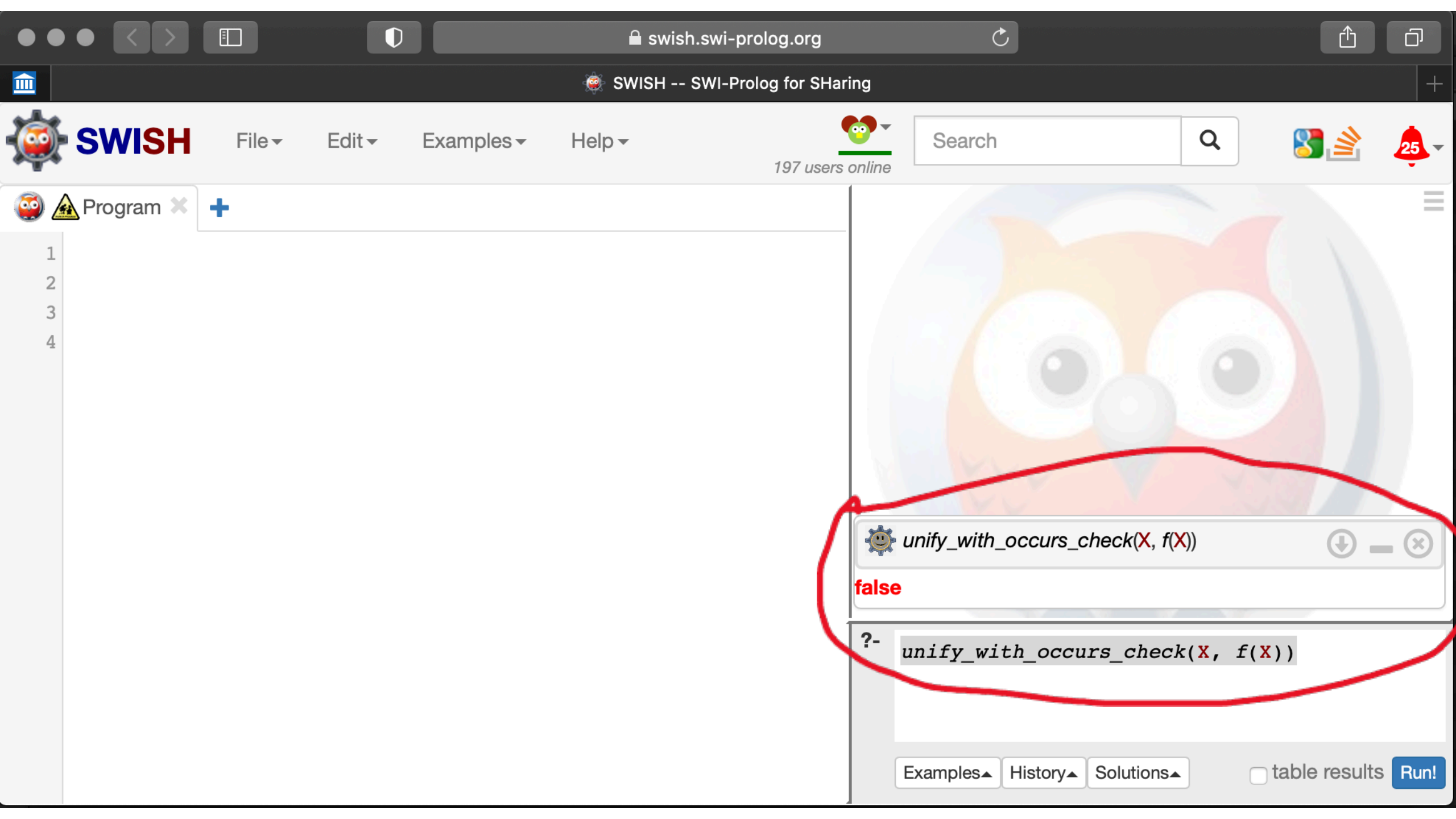


The “Occurs Check”

- Line 3: **Else if** x is a variable and x does not appear in y **then** …

- This is not implemented in many Prologs for efficiency reasons and programmers have to take care of it themselves. Without occours check, Prolog inference is not really sound (this is a tradeoff for efficiency).

- If you want sound unification, use unify_with_occurs_check/2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unify_with_occurs_check(X, f(X))
false
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unify_with_occurs_check(X, f(X))
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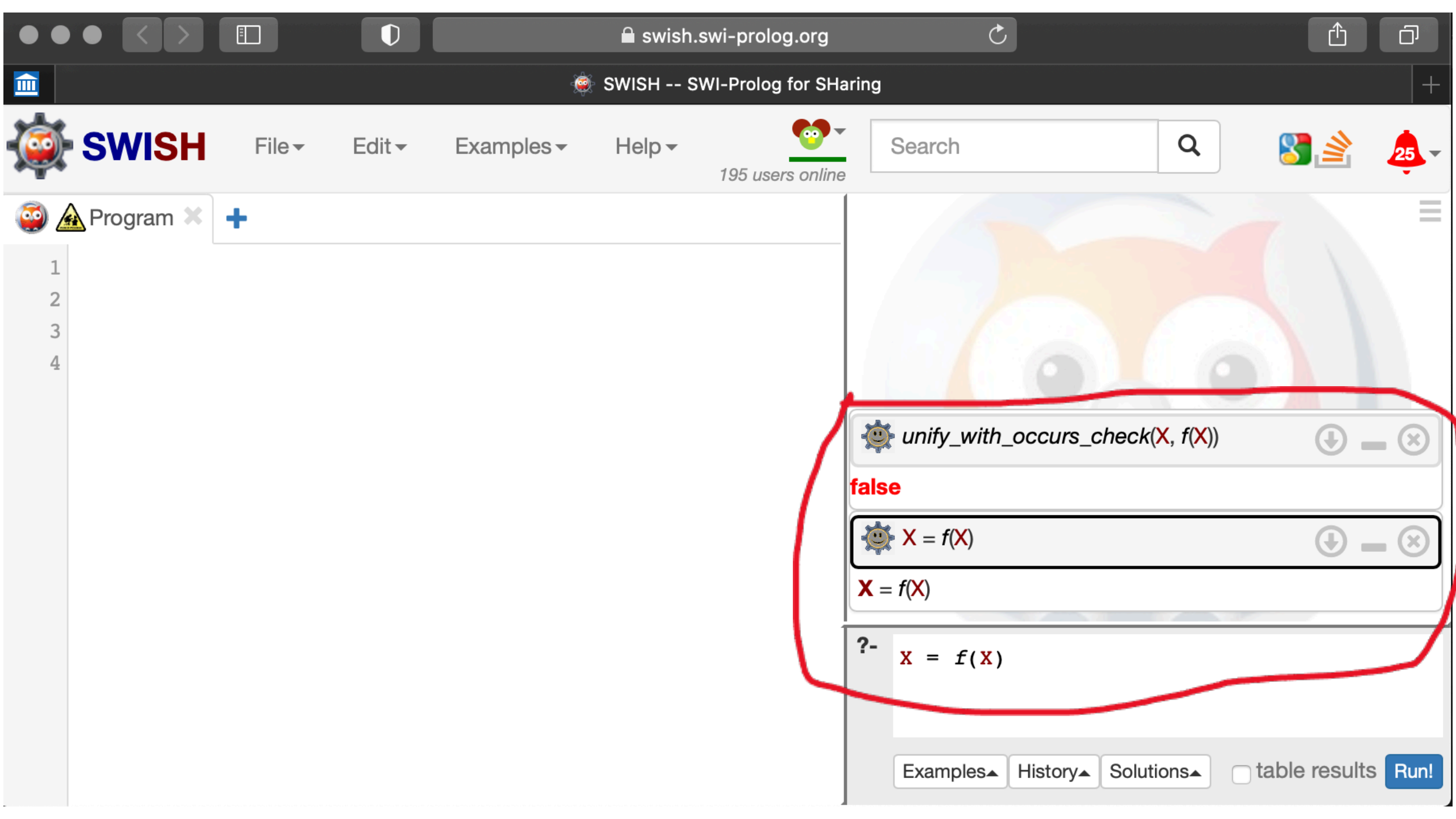
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1
2
3
4
unify_with_occurs_check(X, f(X))
false
X = f(X)
X = f(X)
?-
X = f(X)
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Next week… SLD trees