* 1. We take an undirected cycle and we assign randomly direction to each edge. What is the relation between the number of roots and number of leaves in the resulting directed graph?

* 2. A complete bipartite graph $K_{m,n}$ is Hamiltonian. What can be said about values m and n?

* 3. We take an undirected cycle with 20 nodes and add another three edges in such way that the resulting graph contains an Eulerian cycle. We do not add any node. How many different (non-isomorphic) graphs can be produced this way?

* 4. We say that two directed graphs are weakly equivalent iff their respective condensations contain equal number of nodes. What is the best possible asymptotic complexity of verification whether two graphs are weakly equivalent?

* 5. You have to find and write out all paths of length 3 in a simple (no parallel edges) directed acyclic graph. What is the maximum possible number of these paths relatively to the number of nodes in the graph? What is the asymptotic complexity of your algorithm?

6. Assign direction to each edge of undirected cycle with 7 nodes. How many non-isomorfic directed graphs can be produced this way?

7. We are given a directed graph G with n nodes and m edges. The task is to produce a strongly connected graph by adding some edges (no nodes) into G. The number of added edges should be minimal. Suggest an effective algorithm which solves the task and find its asymptotic complexity.

8. An undirected graph is directionaly homogenous when the distance from a root to a leaf is the same for each possible pair (root, leaf). Formulate an effective algorithm which decides whether a graph is directionally homogenous. Assess the asymptotic complexity of your algorithm. Additionally, suppose that we know that the graph is acyclic. Can the algorithm be improved to handle acyclic graphs more effectively?

9. Find a graph in which the indegree and outdegree of each node is at least 1 and also there is a node in the graph which is not part of any cycle.

10. The graph in the picture does not contain a Hamiltonian cycle. Present a complete and exhaustive explanation of this fact. Decide whether the graph contains a Hamiltonian path. Can you add a single edge to the graph so that the result will contain a Hamiltonian cycle?
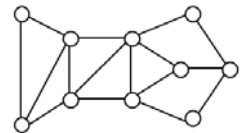
11. Undirected graph of type (r) is produced as follows. Choose two sets of nodes A = $\{a_1, a_2, ... a_r\}$, B = $\{b_1, b_2, ... b_r\}$. Create a complete graph G1 which node set is A and another complete graph G2 which node set is B. Next, create a final graph by joinig G1 and G2 by edges $(a_1, b_1), (a_2, b_2) ..., (a_r, b_r)$. You have to decide for which values of r the resulting graph of type (r) will be an Euler graph.

12. We are given an acyclic weighted graph and the task is to find the most expensive path from some root to some leaf both of which we can choose. Formulate the algorithm, explain its asymptotical complexity and decide which graph representation is best suited for the task.

13. A distance from a strong component A to another strong component B in a directed graph is defined as a minimum number of such strong components not equal to A or B which intersetion with the shortest path from A to B is unempty. If no path from A to B exists then the distance from A to B is defined as positive infinity. Find an algorithm which input is a pair of nodes x, y and the output is the distance from component A to the component B, where x $\in$ A, y $\in$ B. What is the asymptotic complexity of the algorithm?

14. Suppose that a directed graph is represented by an unordered list of edges and no other representation is available and no other representation can be created additionally. We have to produce a new graph which represents the condensation of the original one. You may freely choose the representation of the new graph. What would be the asymptotic complexity of your algorithm?

15. A weakly connected directed graph G with n nodes and m edges contains $c1$ roots and $c2$ leaves. The constants n, $c1$ and $c2$ are fixed, the number of edges m may vary. How can you choose the value of m to be certain that G is acyclic? What is the maximum possible value of m depending on n, $c1$ and $c2$?