# LP-based Heuristics for Cost-optimal Classical Planning

Florian Pommerening Gabriele Röger Malte Helmert

Based on: ICAPS 2015 Tutorial

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# Linear Programs

# Linear Programs and Integer Programs

## Linear Program

A linear program (LP) consists of:

- a finite set of real-valued variables $V$
- a finite set of linear inequalities (constraints) over $V$
- an objective function, which is a linear combination of $V$
- which should be minimized or maximized.

Integer program (IP): ditto, but with integer-valued variables

# Linear Program: Example

Example:

$$\text{maximize} \quad 2x - 3y + z \quad \text{subject to}$$

$$\begin{aligned} x + 2y + z &\leq 10 \\ x \phantom{+ 2y} - z &\leq 0 \end{aligned}$$

$$x \geq 0, \quad y \geq 0, \quad z \geq 0$$

⇝ unique optimal solution:
$x = 5$, $y = 0$, $z = 5$ (objective value 15)

# Solving Linear Programs and Integer Programs

Complexity:

- LP solving is a polynomial-time problem.
- Finding solutions for IPs is NP-complete.

Common idea:

- Approximate IP solution with corresponding LP (LP relaxation).

# Three Key Ideas

# Cost Partitioning

## Idea 1: Cost Partitioning

- create copies $\Pi_1, \ldots, \Pi_n$ of planning task $\Pi$
- each has its own operator cost function $cost_i$ (otherwise identical to $\Pi$)
- for all $o$: require $cost_1(o) + \cdots + cost_n(o) \leq cost(o)$

⇝ sum of solution costs in copies is admissible heuristic: $h^*_{\Pi_1} + \cdots + h^*_{\Pi_n} \leq h^*_{\Pi}$

### Motivation:

- method for obtaining additive admissible heuristics
- very general and powerful

# Operator Counting Constraints

## Idea 2: Operator Counting Constraints

- linear constraints whose variables denote number of occurrences of a given operator
- must be satisfied by every plan that solves the task

Examples:

- $Y_{o_1} + Y_{o_2} \geq 1$ "must use $o_1$ or $o_2$ at least once"
- $Y_{o_1} - Y_{o_3} \leq 0$ "cannot use $o_1$ more often than $o_3$"

Motivation:

- declarative way to represent knowledge about solutions
- allows reasoning about solutions to derive heuristic estimates

# Potential Heuristics

## Idea 3: Potential Heuristics

Heuristic design as an optimization problem:

- Define simple numerical state features $f_1, \dots, f_n$.
- Consider heuristics that are linear combinations of features:

$$h(s) = w_1 f_1(s) + \dots + w_n f_n(s)$$

  with weights (potentials) $w_i \in \mathbb{R}$
- Find potentials for which $h$ is admissible and well-informed.

Motivation:

- declarative approach to heuristic design
- heuristic very fast to compute if features are fast to compute

# Connections

## Three unrelated ideas?

- No! It turns out they are closely connected.

# Lecture Structure

1. ~~Introduction and Overview~~
2. Cost Partitioning
3. Operator Counting
4. Potential Heuristics

# Cost Partitioning

# Cost Partitioning

## Idea 1: Cost Partitioning

- create copies $\Pi_1, \ldots, \Pi_n$ of planning task $\Pi$
- each has its own operator cost function $cost_i : \mathcal{O} \to \mathbb{R}_0^+$ (otherwise identical to $\Pi$)
- for all $o$: require $cost_1(o) + \cdots + cost_n(o) \leq cost(o)$

$\rightsquigarrow$ sum of solution costs in copies is admissible heuristic: $h^*_{\Pi_1} + \cdots + h^*_{\Pi_n} \leq h^*_{\Pi}$

# Cost Partitioning

- for admissible heuristics $h_1, \ldots, h_n$,
  $h(s) = h_{1,\Pi_1}(s) + \cdots + h_{n,\Pi_n}(s)$
  is an admissible estimate
- $h(s)$ can be better or worse than any $h_{i,\Pi}(s)$
  $\rightarrow$ depending on cost partitioning
- strategies for defining cost-functions
  - uniform: $cost_i(o) = cost(o)/n$
  - zero-one: full operator cost in one copy, zero in all others
  - ...

Can we find an optimal cost partitioning?

# Optimal CP

# Optimal Cost Partitioning

Optimal Cost Partitioning with LPs

- Use variables for cost of each operator in each task copy
- Express heuristic values with linear constraints
- Maximize sum of heuristic values subject to these constraints

LPs known for

- abstraction heuristics
- landmark heuristic

# Caution

A word of warning

- optimization for every state gives best-possible cost partitioning
- but takes time

Better heuristic guidance often does not outweigh the overhead.

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# Operator-counting

# Operator Counting

Reminder:

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Examples:

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Motivation:

- declarative way to represent knowledge about solutions
- allows reasoning about solutions to derive heuristic estimates

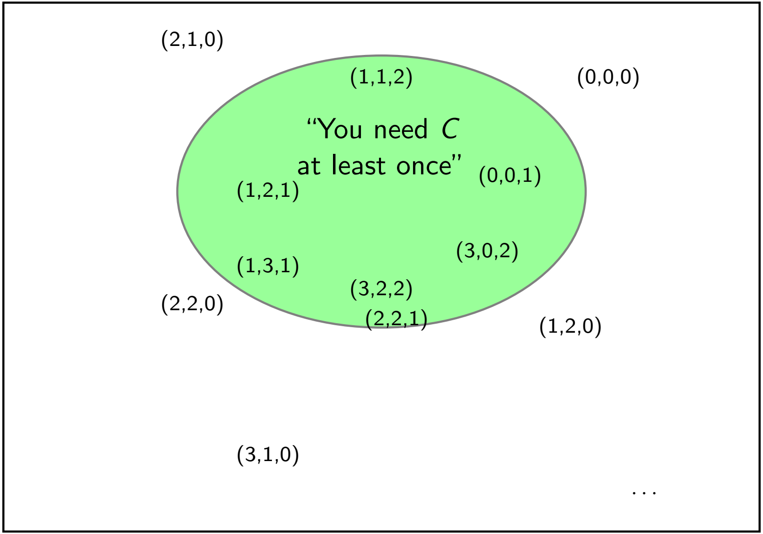
# Operator Counting Heuristics

Operator occurrences in potential plans

(2,1,0)
(1,1,2)
(0,0,0)
(0,0,1)
(1,2,1)
(3,0,2)
(1,3,1)
(3,2,2)
(2,2,0)
(2,2,1)
(1,2,0)
(3,1,0)
...

# Operator Counting Heuristics

Operator occurrences in potential plans



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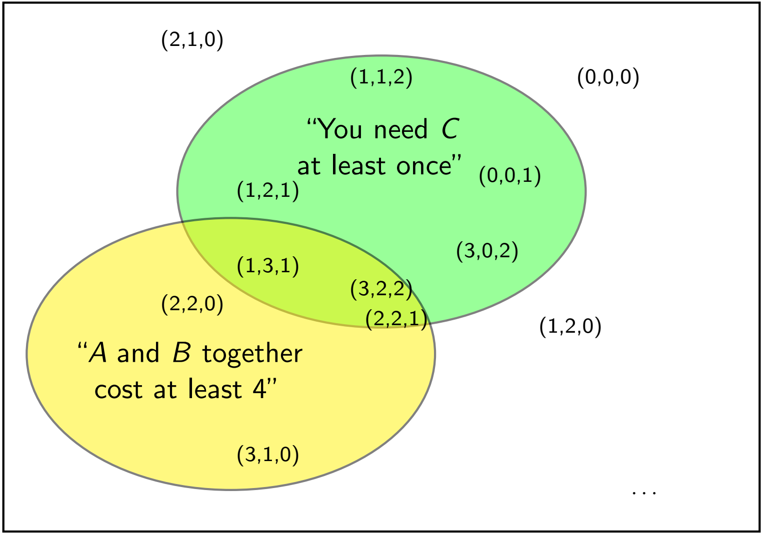

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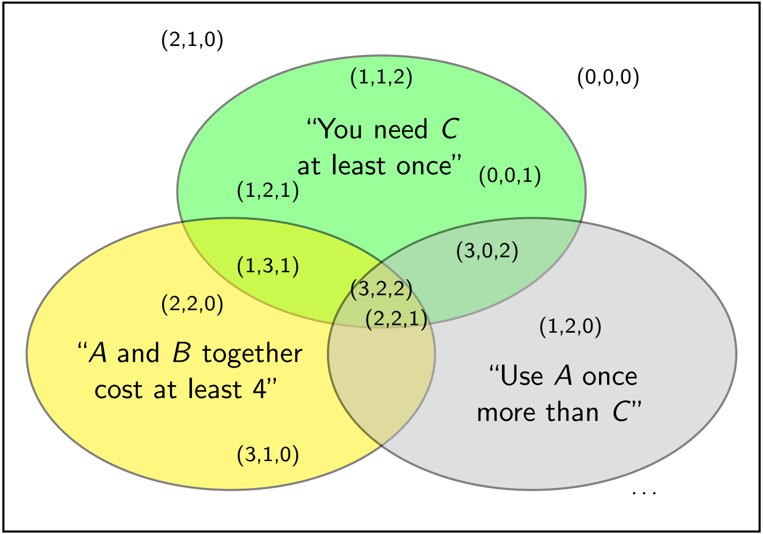

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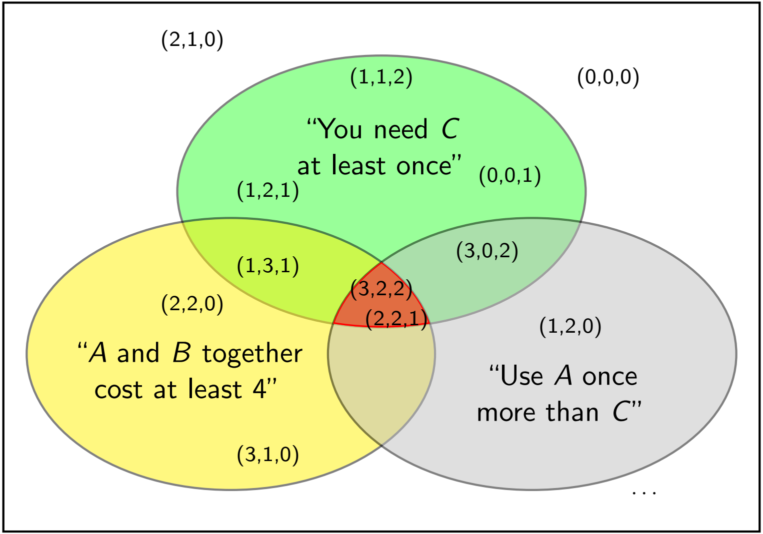

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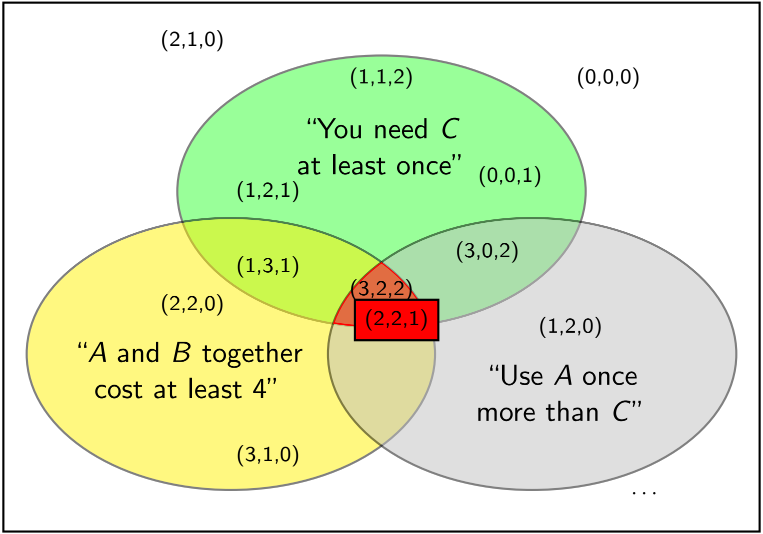
Operator occurrences in potential plans



# Operator-counting Heuristics

## Operator-counting IP/LP Heuristic

$$\text{Minimize } \sum_{o} Y_o \cdot cost(o) \text{ subject to}$$

$Y_o \geq 0$ and some operator-counting constraints

## Operator-counting constraint

- Set of linear inequalities
- For every plan $\pi$ there is an LP-solution where $Y_o$ is the number of occurrences of $o$ in $\pi$ .

# Properties of Operator-counting Heuristics

## Admissibility

Operator-counting (IP and LP) heuristics are admissible.

## Computation time

Operator-counting LP heuristics are solvable in polynomial time.

## Adding constraints

Adding constraints can only make the heuristic more informed.

# State-equation Heuristic

# State-equation Heuristic (SEQ)

Main idea:

- Facts can be produced (made true) or consumed (made false) by an operator
- Number of producing and consuming operators must balance out for each fact

# State-equation Heuristic

**Net-change constraint for fact $f$**

$$G(f) - S(f) = \sum_{f \in eff(o)} Y_o - \sum_{f \in pre(o)} Y_o$$

**Net-change constraint for fact $f$**

$$G(f) - S(f) = \sum_{o \text{ produces } f} Y_o - \sum_{o \text{ consumes } f} Y_o$$

Remark:

- Assumes transition normal form (not a limitation)
  - Operator mentions same variables in precondition and effect
  - There is only one goal state which is defined over all variables
  - General form of constraints more complicated

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# Potential Heuristics

# Potential Heuristics

Reminder:

**Idea 3: Potential Heuristics**

Heuristic design as an optimization problem:

- Define simple numerical state features $f_1, \dots, f_n$.
- Consider heuristics that are linear combinations of features:

$$h(s) = w_1 f_1(s) + \cdots + w_n f_n(s)$$

  with weights (potentials) $w_i \in \mathbb{R}$
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Motivation:

- declarative approach to heuristic design
- heuristic very fast to compute if features are

# Comparison to Previous Parts (1)

What is the same as in operator-counting constraints:

- We again use LPs to compute (admissible) heuristic values (spoiler alert!)

# Comparison to Previous Parts (2)

What is different from operator-counting constraints (computationally):

- With potential heuristics, solving one LP defines the entire heuristic function, not just the estimate for a single state.
- Hence we only need one LP solver call, making LP solving much less time-critical.

# Comparison to Previous Parts (3)

What is different from operator-counting constraints (conceptually):

- axiomatic approach for defining heuristics:
  - What should a heuristic look like mathematically?
  - Which properties should it have?
- define a space of interesting heuristics
- use optimization to pick a good representative

# Features

**Definition (feature)**

A (state) feature for a planning task is a numerical function defined on the states of the task: $f : S \to \mathbb{R}$.

# Potential Heuristics

**Definition (potential heuristic)**

A potential heuristic for a set of features $\mathcal{F} = \{f_1, \dots, f_n\}$ is a heuristic function $h$ defined as a linear combination of the features:

$$h(s) = w_1 f_1(s) + \dots + w_n f_n(s)$$

with weights (potentials) $w_i \in \mathbb{R}$.

$\leadsto$ cf. evaluation functions for board games like chess

# Atomic Potential Heuristics

Atomic features test if some proposition is true in a state:

**Definition (atomic feature)**

Let $X = x$ be an atomic proposition of a planning task.

The atomic feature $f_{X=x}$ is defined as:

$$f_{X=x}(s) = \begin{cases} 1 & \text{if variable } X \text{ has value } x \text{ in state } s \\ 0 & \text{otherwise} \end{cases}$$

- We only consider atomic potential heuristics, which are based on the set of all atomic features.
- Example for a task with state variables $X$ and $Y$:

$$h(s) = 3f_{X=a} + \frac{1}{2}f_{X=b} - 2f_{X=c} + \frac{5}{2}f_{Y=d}$$

# Finding Good Potential Heuristics

# How to Set the Weights?

We want to find good atomic potential heuristics:

- admissible
- consistent
- well-informed

How to achieve this? Linear programming to the rescue!

# Admissible and Consistent Potential Heuristics

Constraints on potentials characterize (= are necessary and sufficient for) admissible and consistent atomic potential heuristics:

## Goal-awareness (i.e., $h(s) = 0$ for goal states)

$$\sum_{\text{goal facts } f} w_f = 0$$

## Consistency

$$\sum_{\substack{f \text{ consumed} \\ \text{by } o}} w_f - \sum_{\substack{f \text{ produced} \\ \text{by } o}} w_f \leq cost(o) \quad \text{for all operators } o$$

Remarks:

- assumes transition normal form (not a limitation)
- goal-aware and consistent = admissible and consistent

# Well-Informed Potential Heuristics

How to find a well-informed potential heuristic?

$\rightsquigarrow$ encode quality metric in the objective function
and use LP solver to find a heuristic maximizing it

Examples:

- maximize heuristic value of a given state (e.g., initial state)
- maximize average heuristic value of all states
(including unreachable ones)
- maximize average heuristic value of some sample states
- minimize estimated search effort

# Connections

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So what does this have to do with what we talked about before?

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**Theorem (Pommerening et al., AAAI 2015)**

*For state $s$, let $h^{\text{maxpot}}(s)$ denote the maximal heuristic value of all admissible and consistent atomic potential heuristics in $s$.*

*Then $h^{\text{maxpot}}(s) = h^{\text{SEQ}}(s) = h^{\text{gOCP}}(s)$.*

- $h^{\text{SEQ}}$: state equation heuristic a.k.a. flow heuristic
- $h^{\text{gOCP}}$: optimal general cost partitioning of atomic projections

proof idea: compare dual of $h^{\text{SEQ}}(s)$ LP
to potential heuristic constraints optimized for state $s$

# What Do We Take From This?

- general cost partitioning, operator-counting constraints and potential heuristics: facets of the same phenomenon
- study of each reinforces understanding of the others
- potential heuristics: fast admissible approximations of $h^{\mathrm{SEQ}}$
- clear path towards generalization beyond $h^{\mathrm{SEQ}}$:
  use non-atomic features

# The End

1. ~~Introduction and Overview~~
2. ~~Cost Partitioning~~
3. ~~Operator Counting~~
4. ~~Potential Heuristics~~

Thank you for your attention!