ADVANCED ROBOTICS/KINEMATICS

Tomas Pajdla

2021

pajdla@cvut.cz

AAG – Applied Algebra & Geometry

Basic & Applied Research, 15 Members

Tomas Pajdla
Group Leader
Vision
Robotics
Mathematics

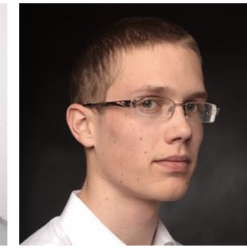
Mircea Cimpoi
Postdoc
Machine
Learning

Ludovic Magerand
Postdoc
3D
Vision

Federica Arrigoni
Postdoc
3D
Vision

Čeněk Albl
PhD Student
Rolling
Shutter

Michal Polic
PhD Student
3D
Reconstruction



Pavel Trutman
PhD Student
Polynomial
Optimization

Stanislav Steidl
PhD Student
Autonomous
Driving

Research

We apply elements of

- Algebra
- Geometry
- Statistics
- Optimization

in

- Computer Vision
- Robotics
- Machine Learning

Teaching

We teach Geometry of

- Computer Vision
- Robotics

at

CTU CZECH TECHNICAL UNIVERSITY IN PRAGUE
CHARLES UNIVERSITY IN PRAGUE

FEE of the CTU in Prague
MFF of Charles University

Projects

We are funded by

LADIO H2020 EU
3D Reconstruction for Movies

IMPACT OP VaVPI
Intelligent Machine Perception

R4I OP VaVPI
Robotics for Industry 4.0

Industry

We collaborate with

Omni-Vision

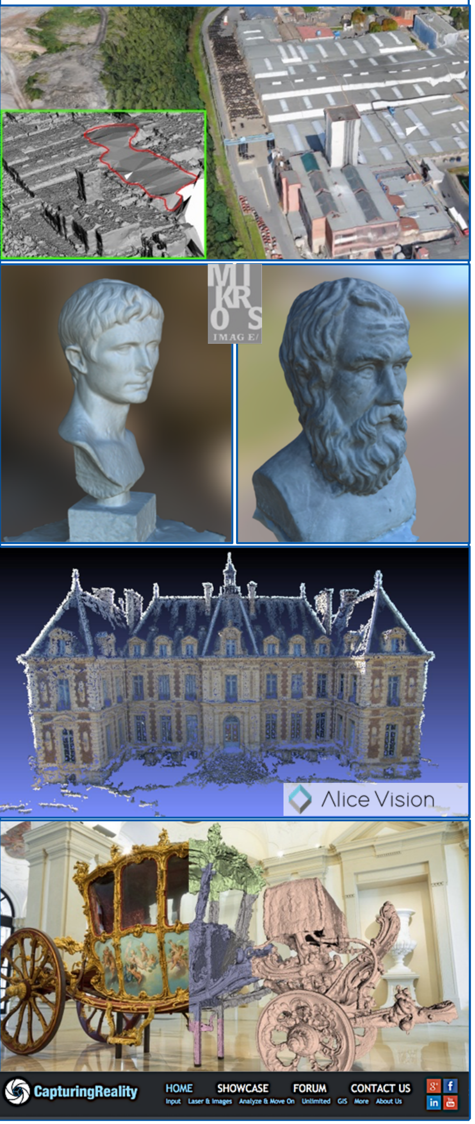
Leica
Photogrammetry

Magik Eye
3D sensing

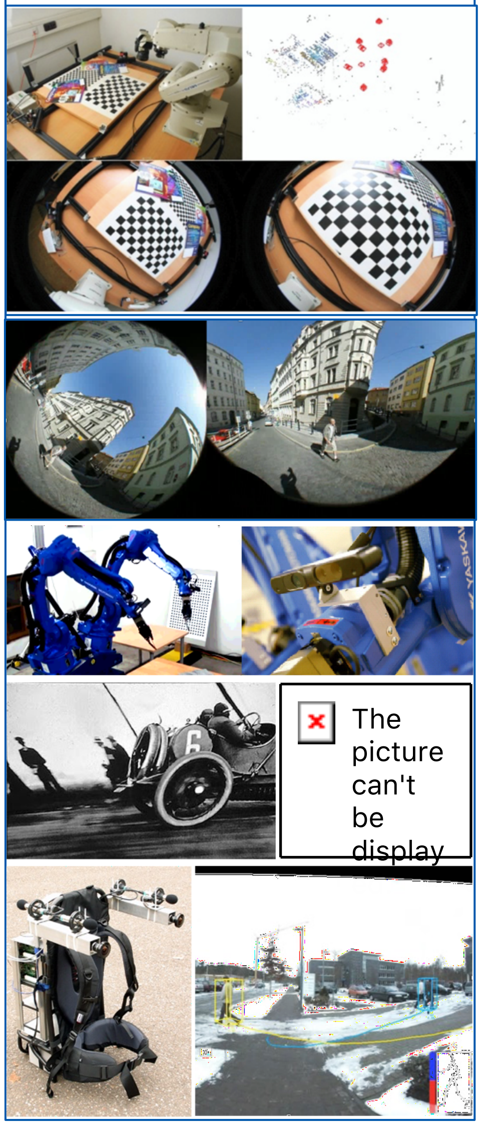
Continental
Camera calibration

AAG – Research & Applications

3D Reconstruction



Camera Geometry



Visual localization

Applications

DAIMLER
Continental
Magik Eye
Leica
gisat
AZOS cz
zentity a touch of innovation

Czech Technical University in Prague

CIIRC Czech Institute of Informatics Robotics and Cybernetics

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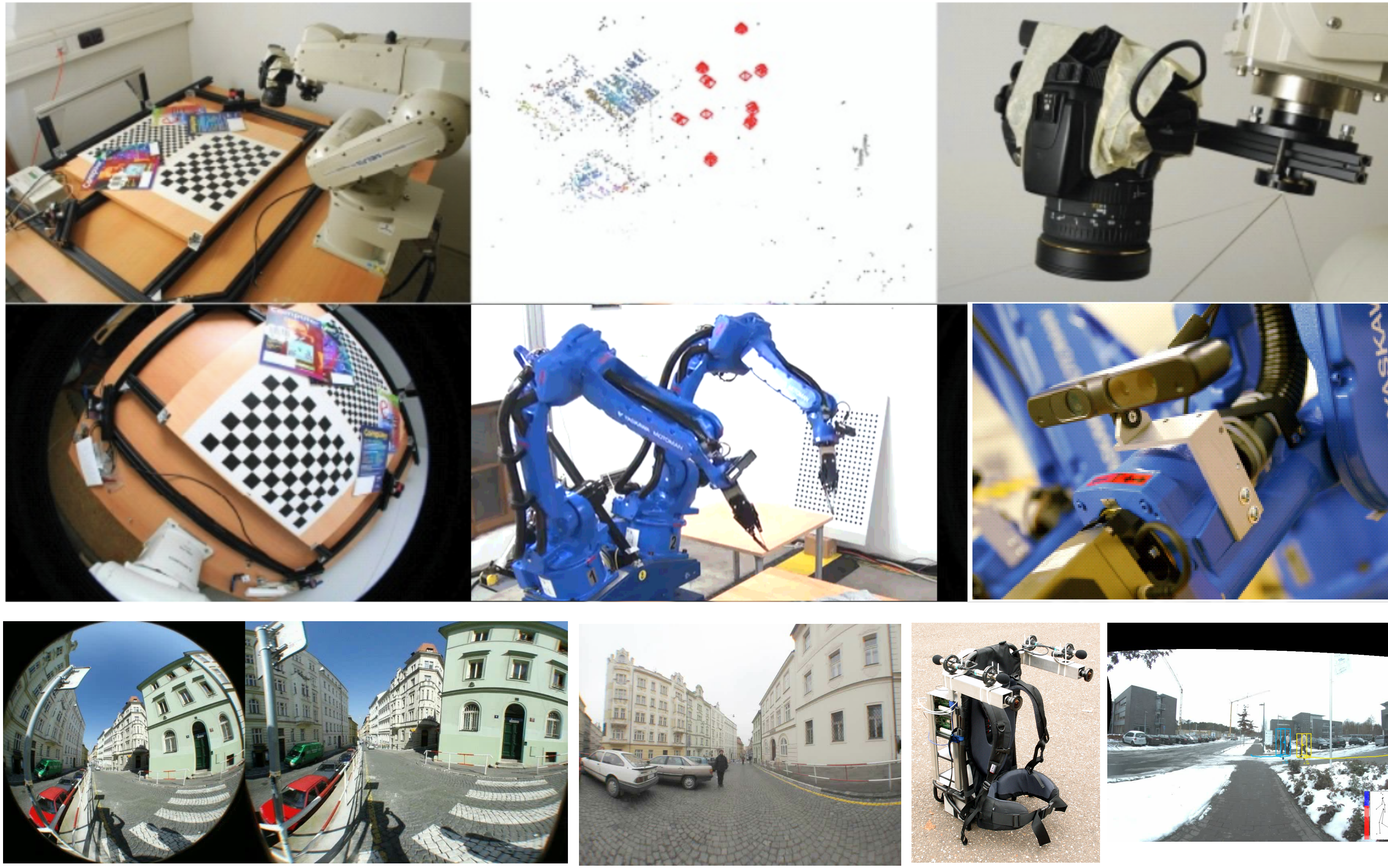
AAG - Geometry of Cameras and Robots



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Advanced Robotics

Lecture 1

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We will study more advanced robot kinematics problems, e.g.,

1. solving inverse kinematics of a general 6 DOF manipulator
2. finding singular poses of a manipulator

with more advanced mathematical tools, such as

1. space rotation and motion and
2. solving algebraic equations

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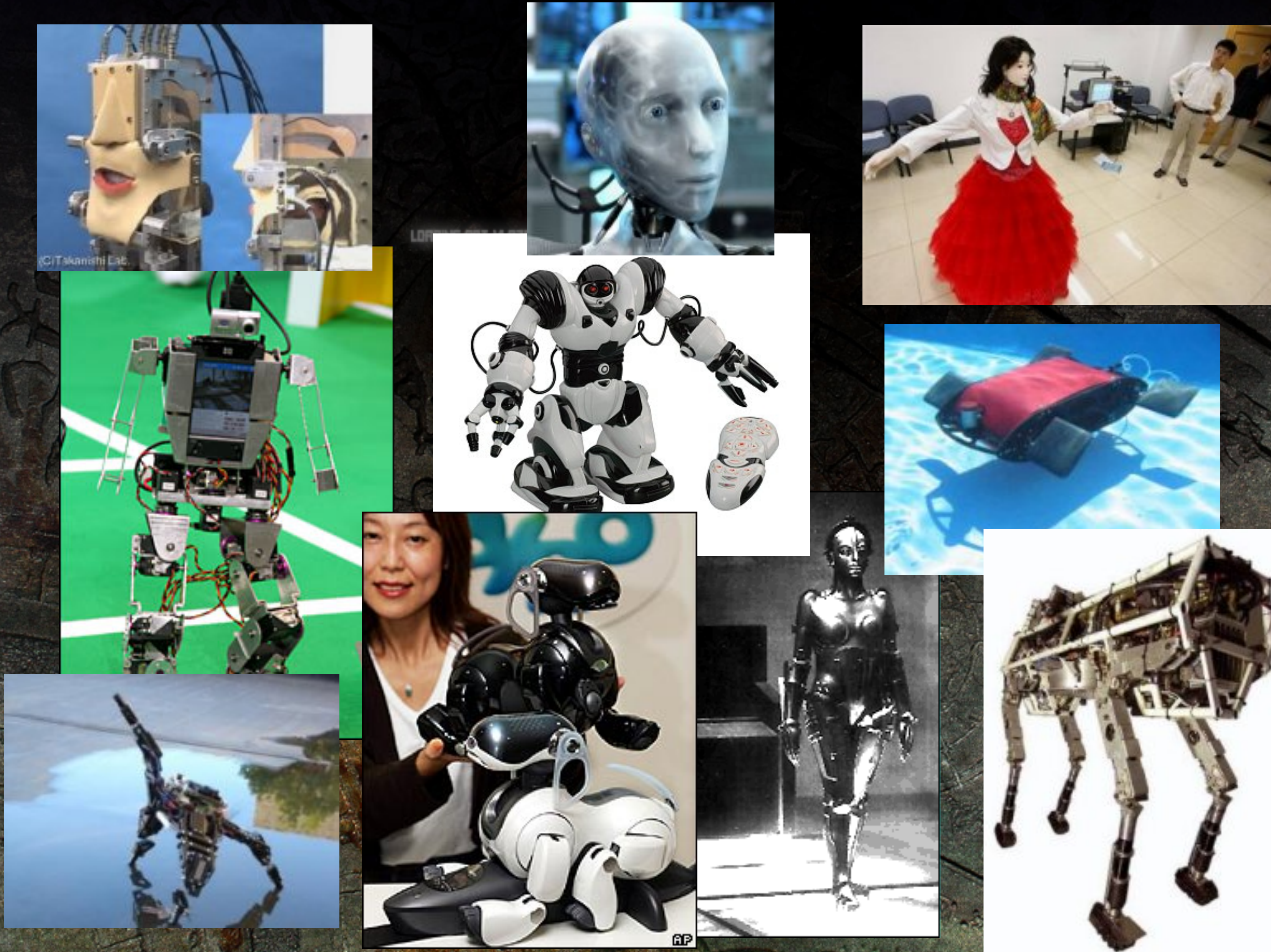
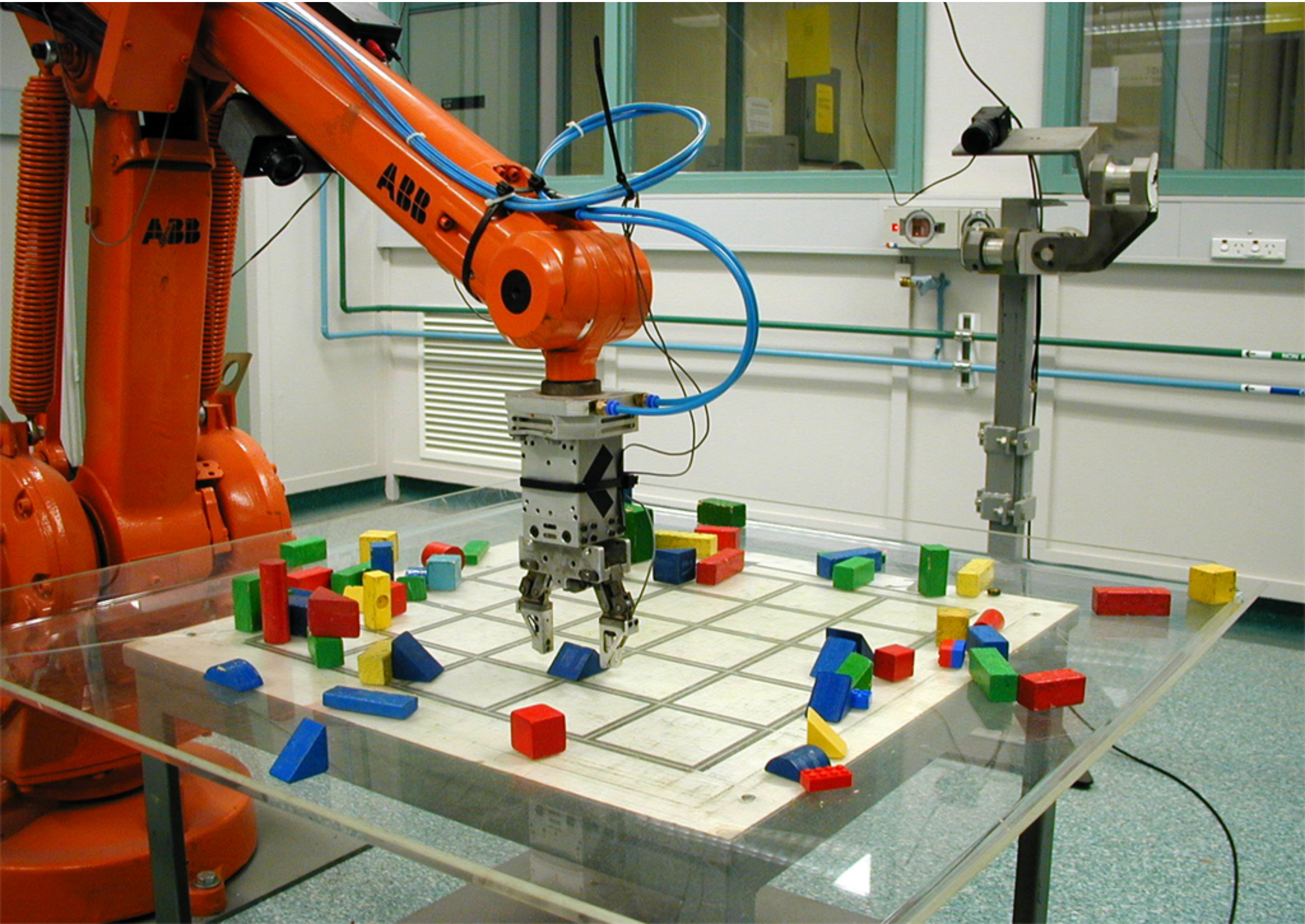

(C)Takanishi Lab.
AP

ROBOT = A GENERAL MANIPULATOR



Industrial Robotic Applications

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- ⊟ Application areas
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  - Assembly
  - Foundry applications
  - Gluing and Sealing
  - ⊞ Material handling and Machine Tending
  - Packing
  - Palletizing
  - Picking
  - Painting and coating
  - Spot welding
  - Waterjet cutting

Application areas

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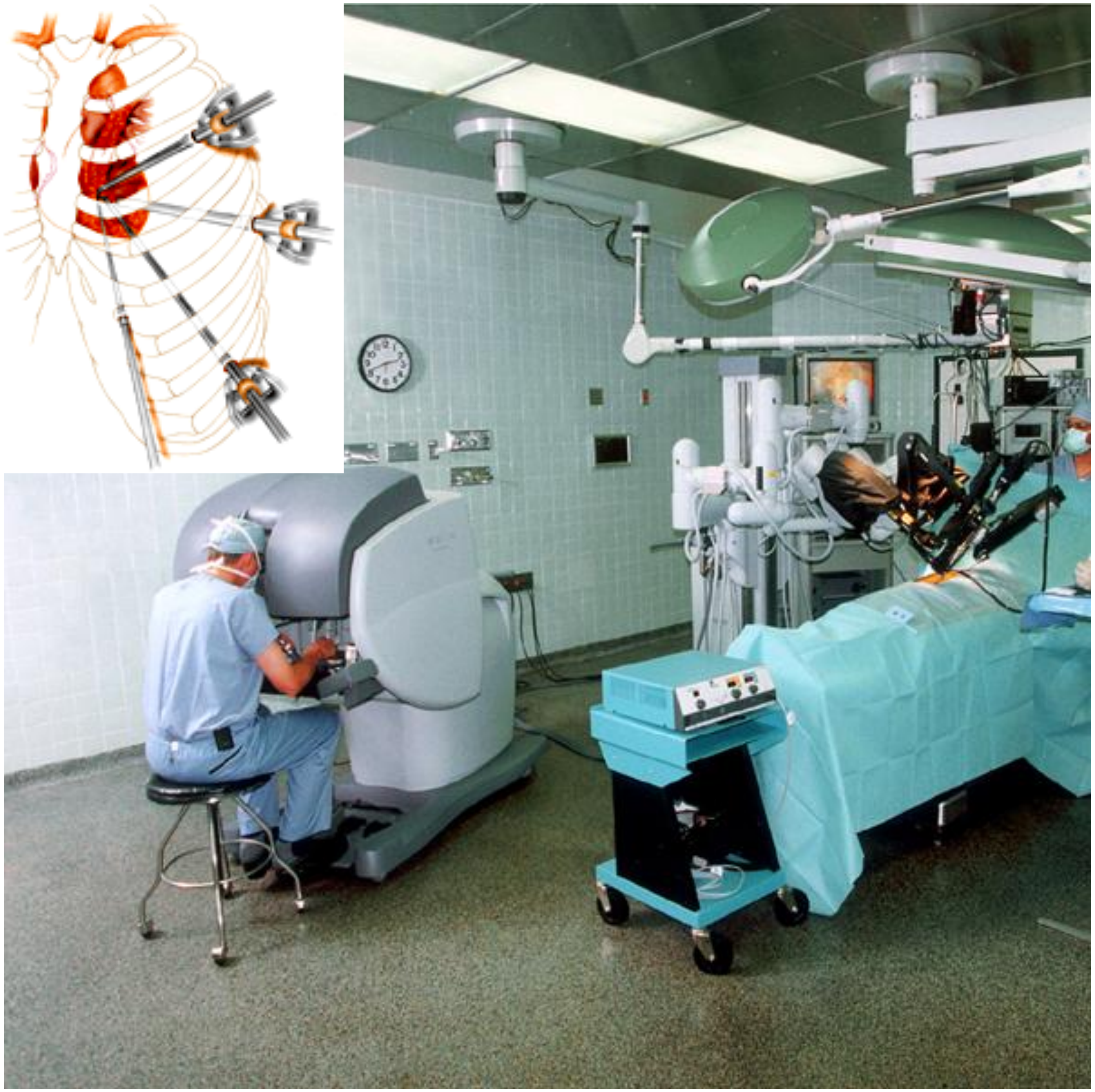
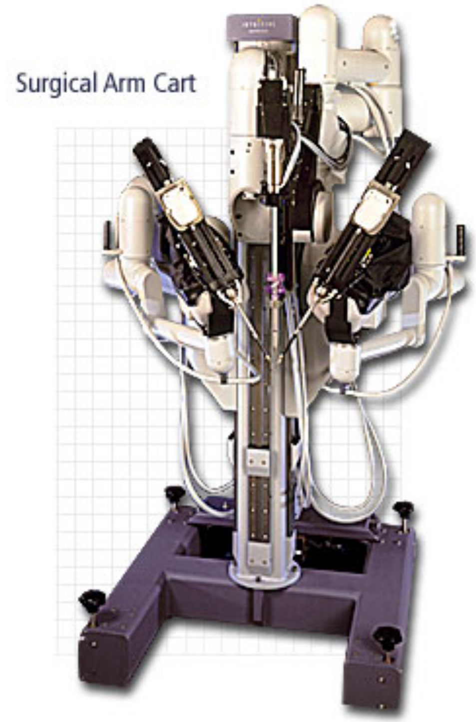
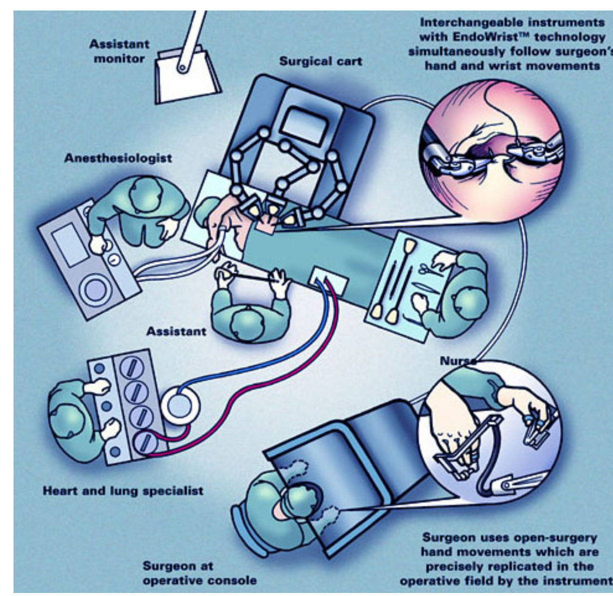
Precision for robotic surgery







http://www.cts.usc.edu/rsi-article-robotputsuscatforefront.html

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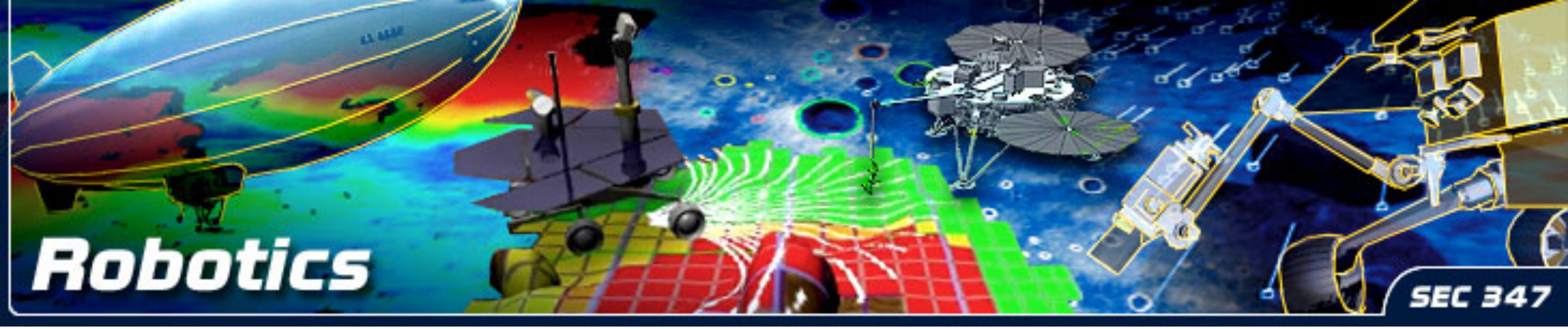
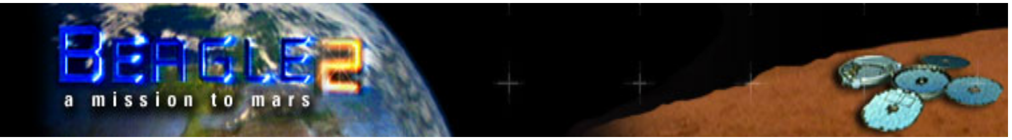
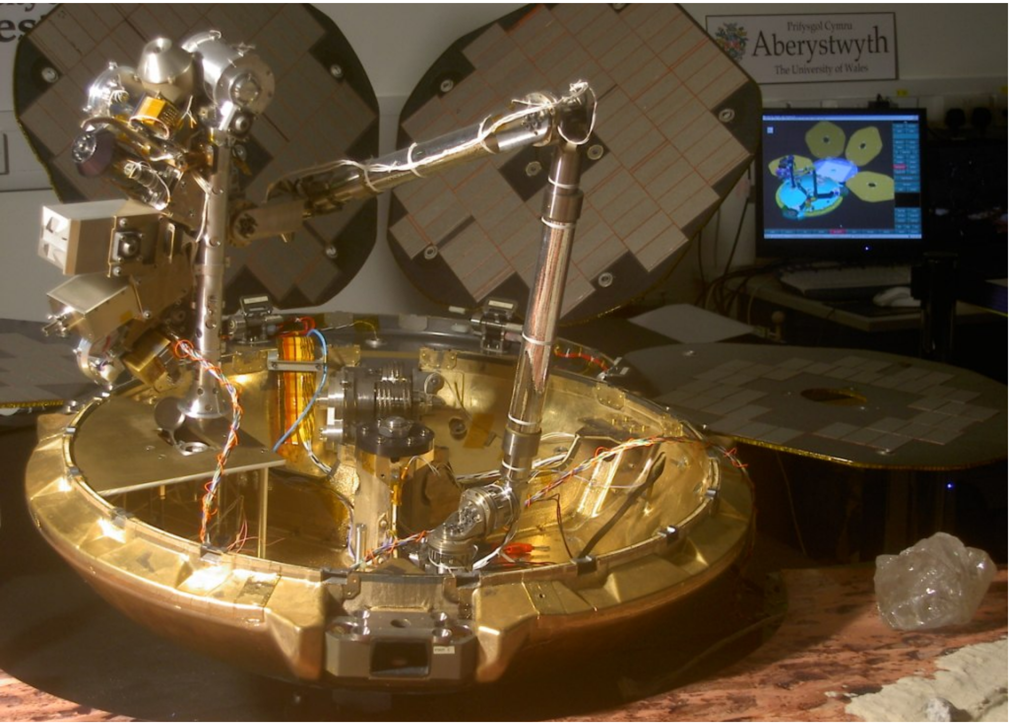
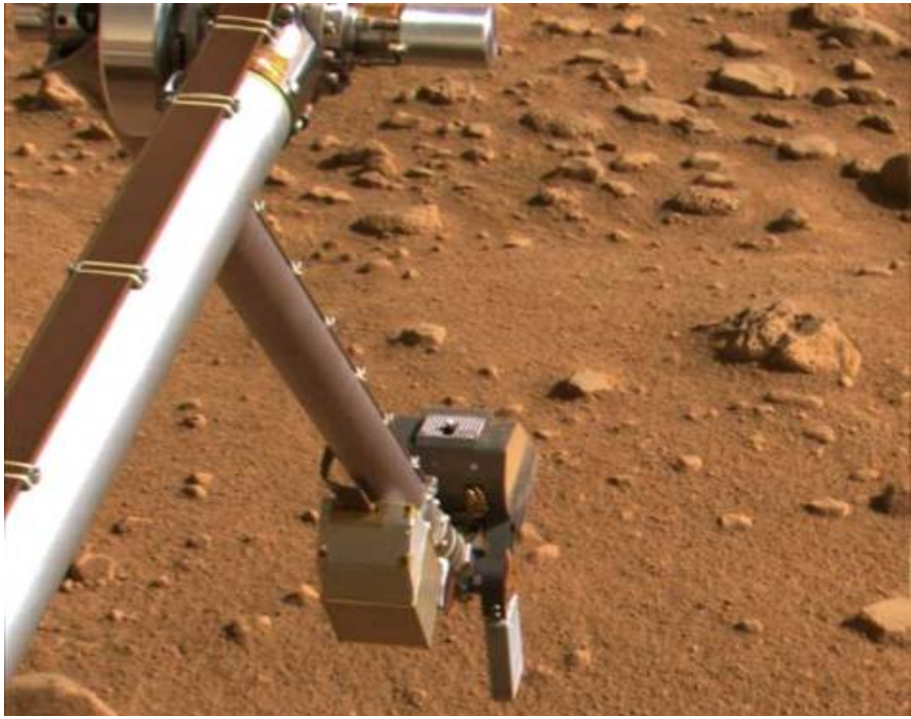

Robotics
SEC 347


BEAGLE2
a mission to mars


PHOENIX MARS MISSION
NASA


Aberystwyth



Two kinds of manipulators

1. Serial manipulators

2. Parallel manipulators

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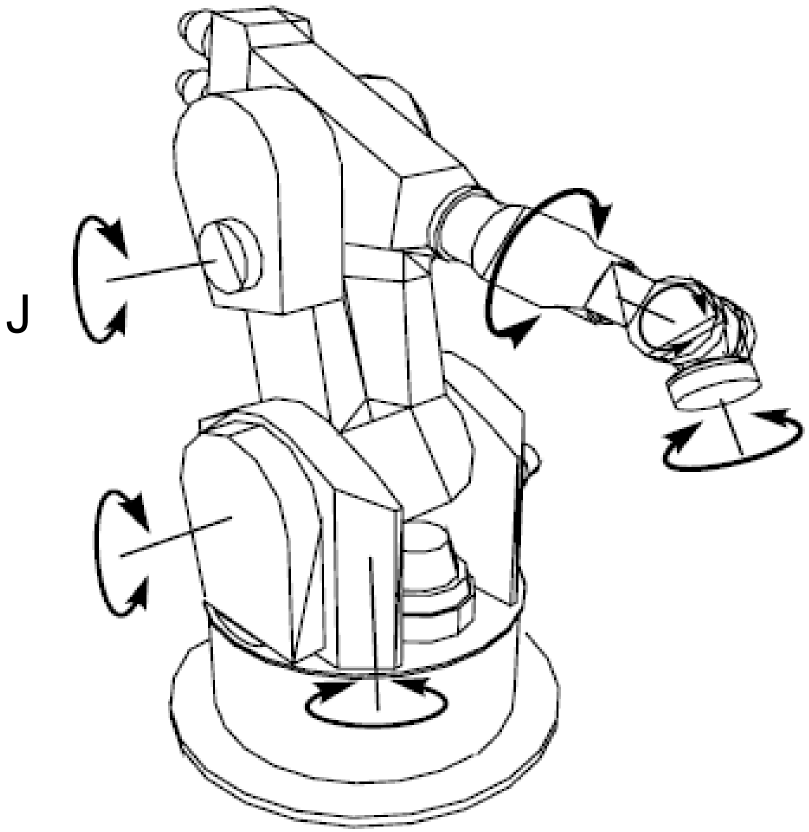
Serial manipulators



KUKA manipulator

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Serial manipulators

Stäubli (courtesy Neovision s.r.o.)

Mitsubishi (courtesy Neovision s.r.o.)

1. Direct kinematic task – easy
2. Inverse kinematic task – difficult

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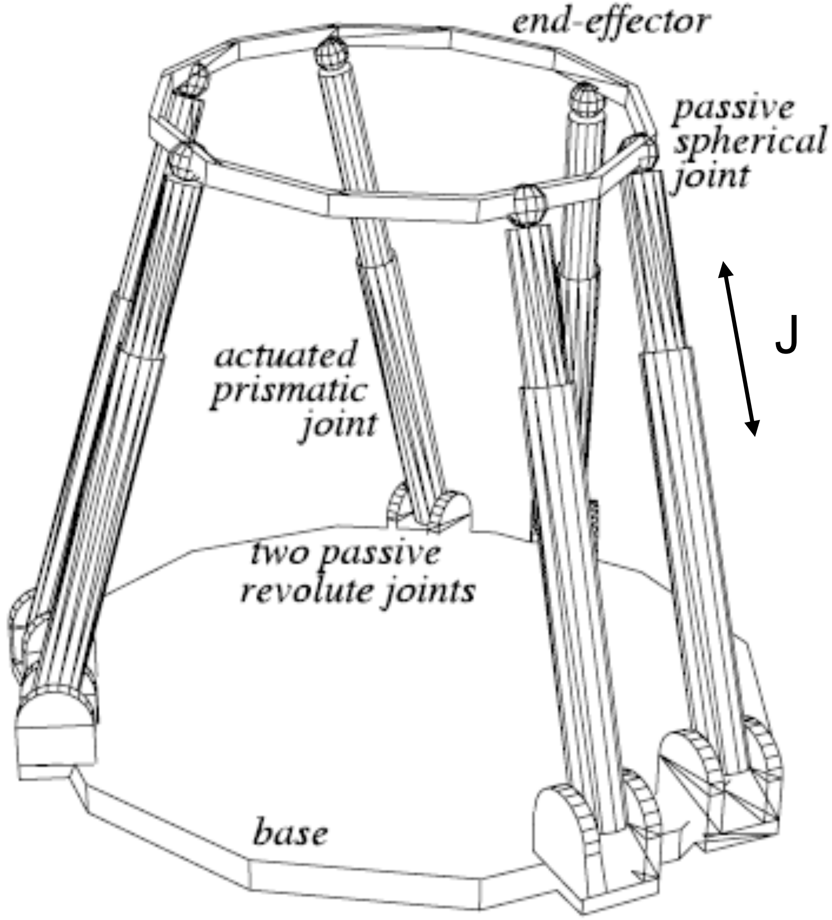
Parallel manipulators



Stewart-Gough Platform

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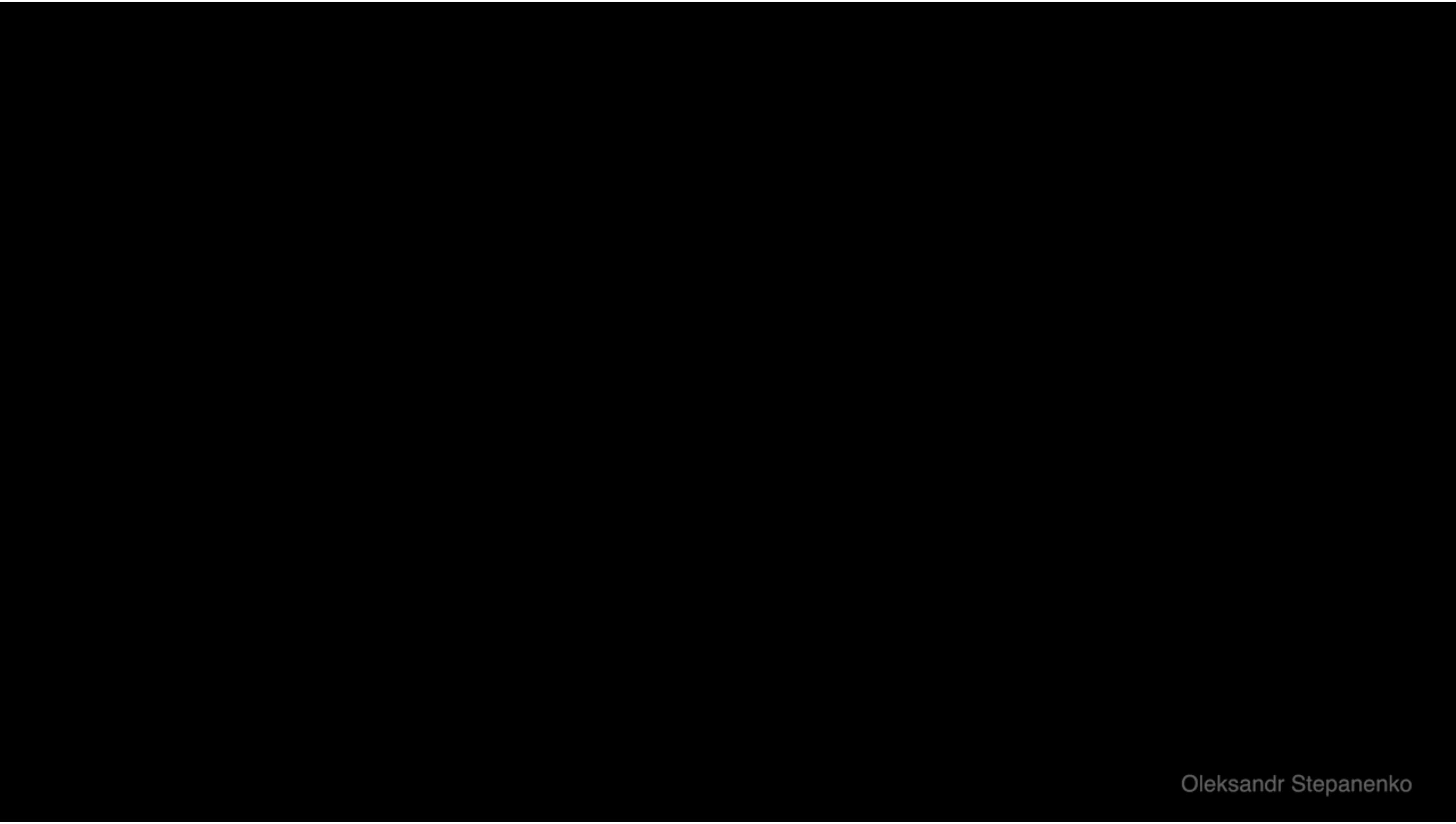

https://youtu.be/xiECumcaEx0

Hexapod (Gough-Stewart platform) 6-axis parallel robot

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Three main problems

1. Direct kinematic task (přímá kinematická úloha)
2. Inverse kinematic task (inverzní kinematická úloha)
3. Manipulator singularity analysis

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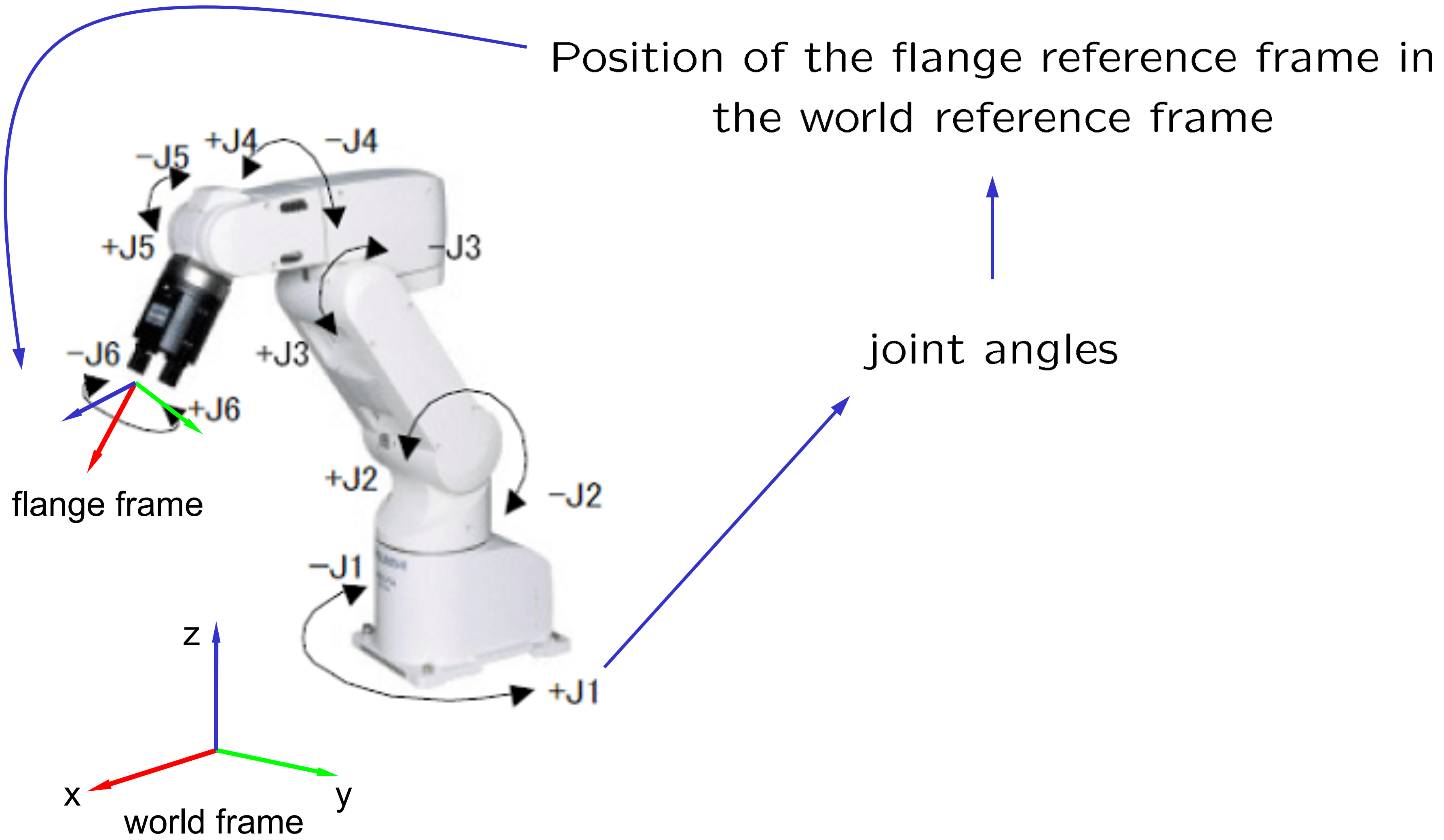
Direct kinematic task



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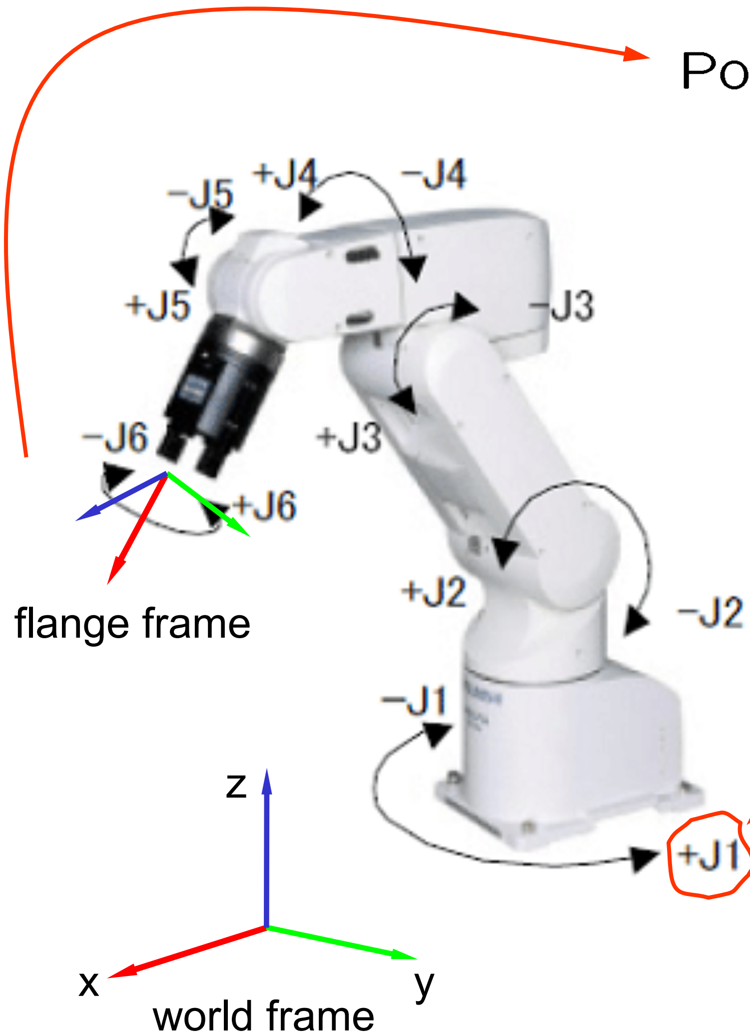
Inverse kinematic task

Position of the flange reference frame in the world reference frame

↓

joint angles



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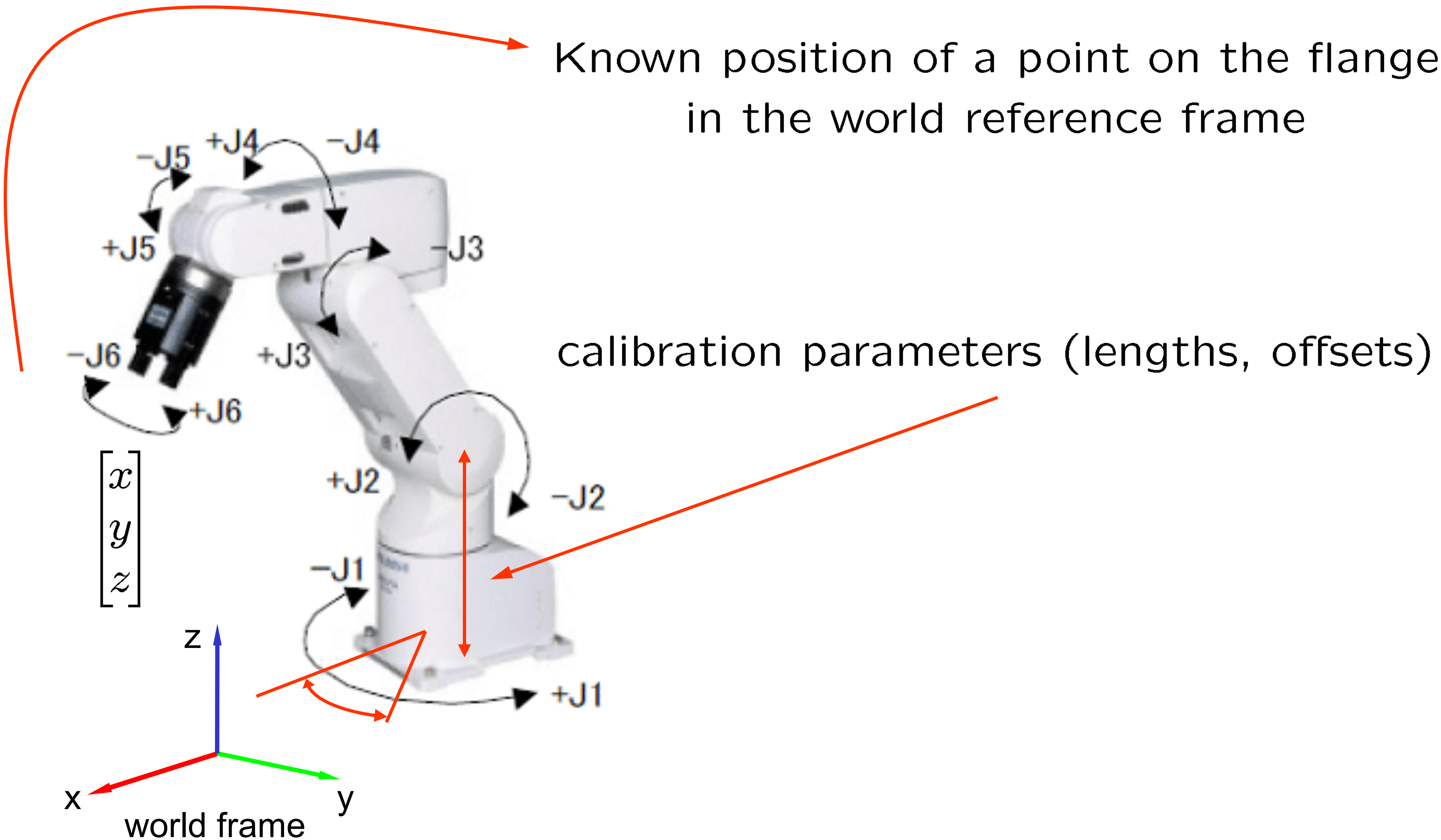
Kinematic calibration



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Singularities of Manipulators

www.mecademic.com/en/what-are-singularities-in-a-six-axis-robot-arm

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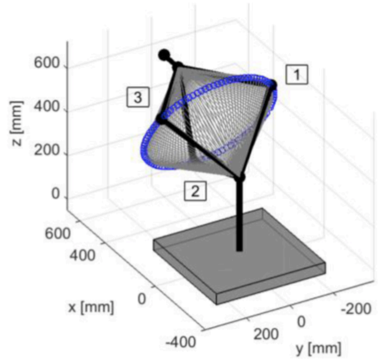
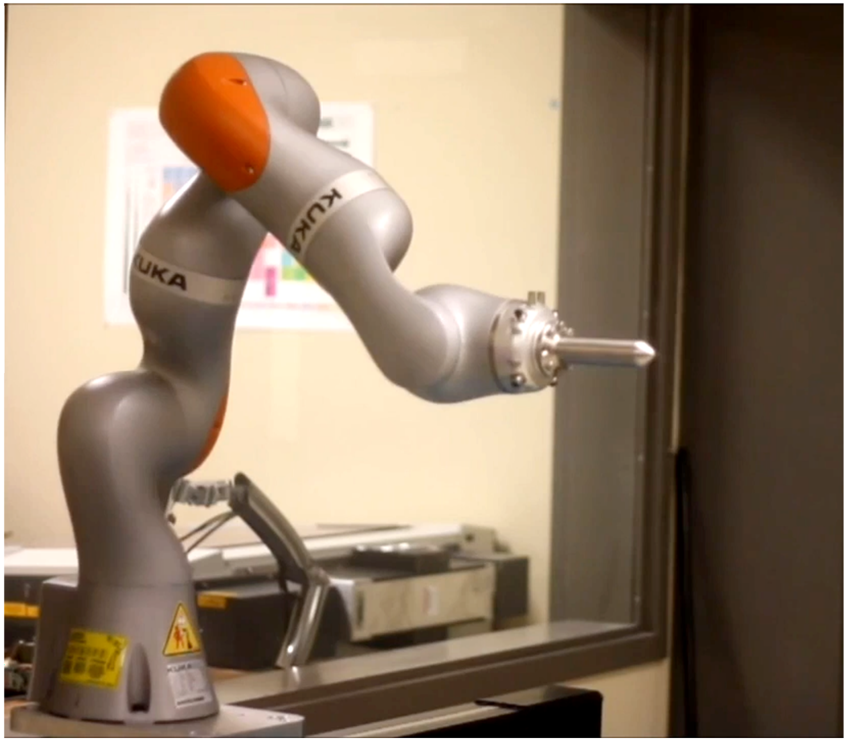
KUKA LBR iiwa



- 7 revolute joints → 7 DOF
- Solutions of IKT can be parametrized by one parameter
- Task: to find an optimal solution



Nicholas Nadeau (https://www.youtube.com/watch?v=JhCzt1KAWMY)

Figure: Manipulator KUKA LBR iiwa.

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Solving kinematic tasks

TASK

$\downarrow$

GEOMETRY

$\downarrow$

COORDINATE SYSTEMS

$\downarrow$

KINEMATIC MODEL

$\downarrow$

ALGEBRAIC EQUATIONS

$\downarrow$

RESULT

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Solving kinematic tasks

1968 Donald L. Pieper (Ph.D. thesis)

The inverse kinematics of any serial manipulator with six revolute joints, and with three consecutive joints intersecting, can be solved in closed-form, i.e., analytically.

1989 M. Raghavan, B. Roth. *Kinematic Analysis of the 6R Manipulator of General Geometry.* Int. Symp. Robotics. Research. Pp. 314-320, Tokyo 1989/1990.

A general technique for computing inverse kinematics for any serial manipulator with six revolute joints.

… leads to solving an algebraic equation of degree 16.

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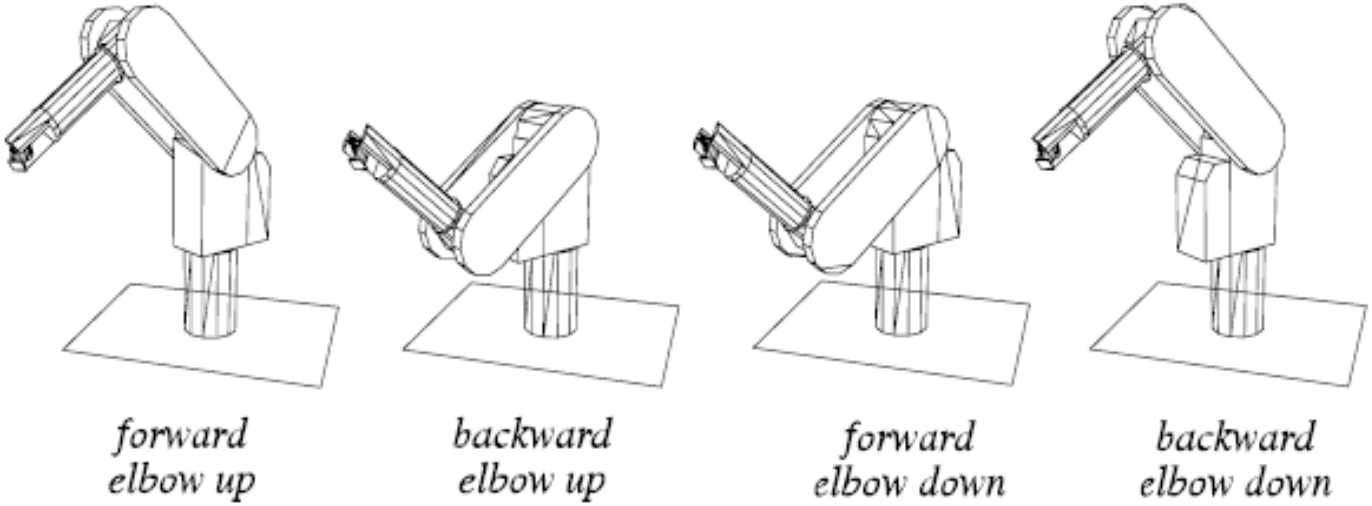
Solving kinematic tasks

Algebraic equation of degree 16 … up to 16 solutions

4 typical solutions



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Solving kinematic tasks

TASK

$\downarrow$

GEOMETRY

$\downarrow$

COORDINATE SYSTEMS

$\downarrow$

KINEMATIC MODEL

$\downarrow$

ALGEBRAIC EQUATIONS

$\downarrow$

RESULT

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Stäubli TX-90 – Geometry

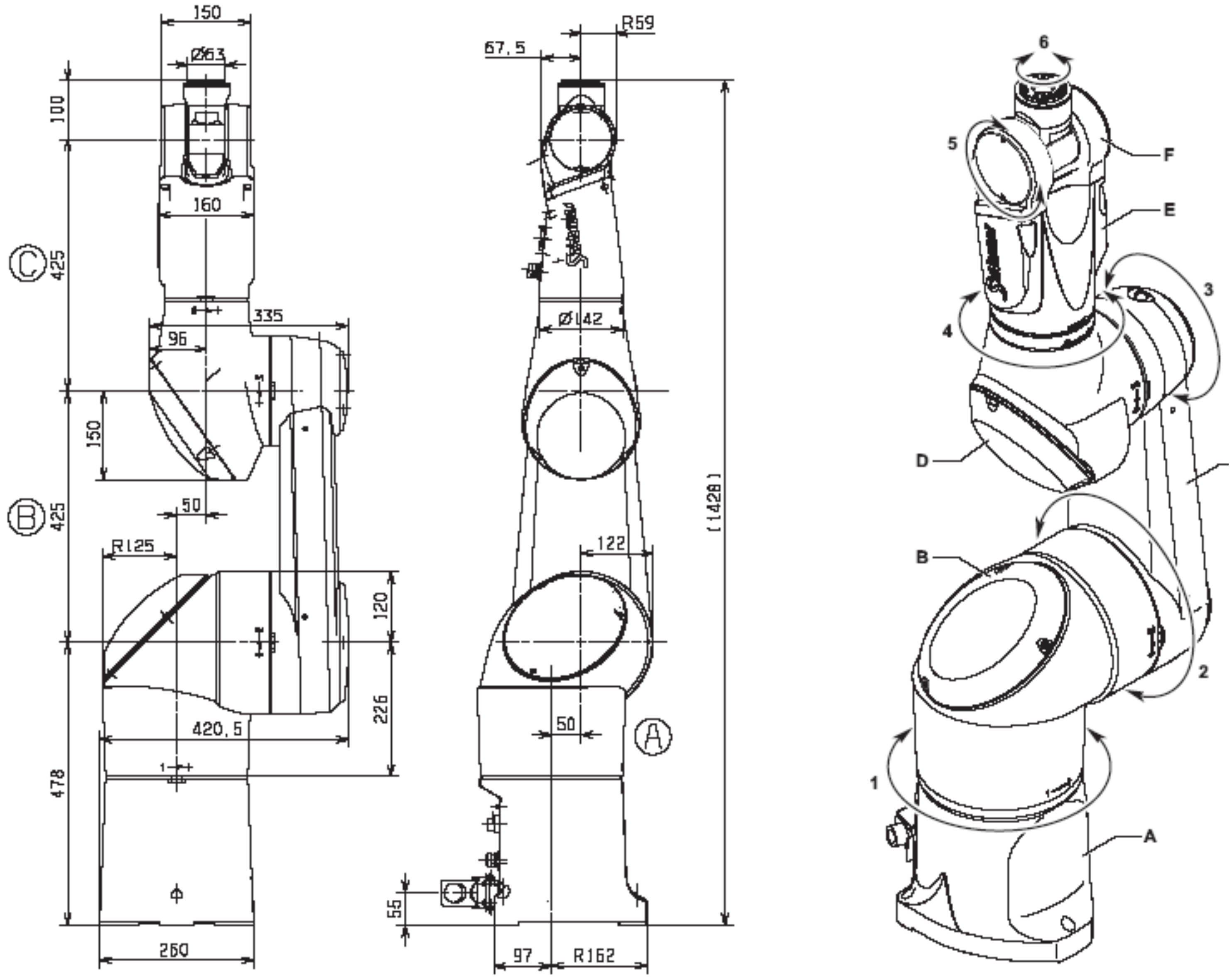
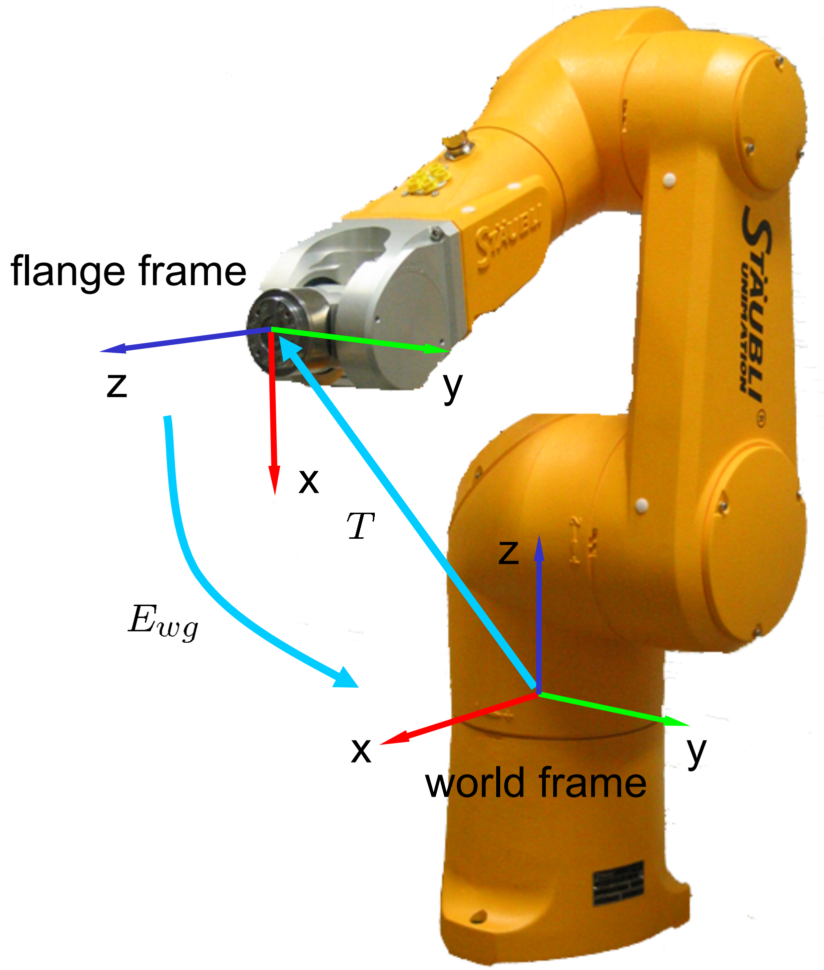


Figure 1.3 - Standard arm

Kinematic model



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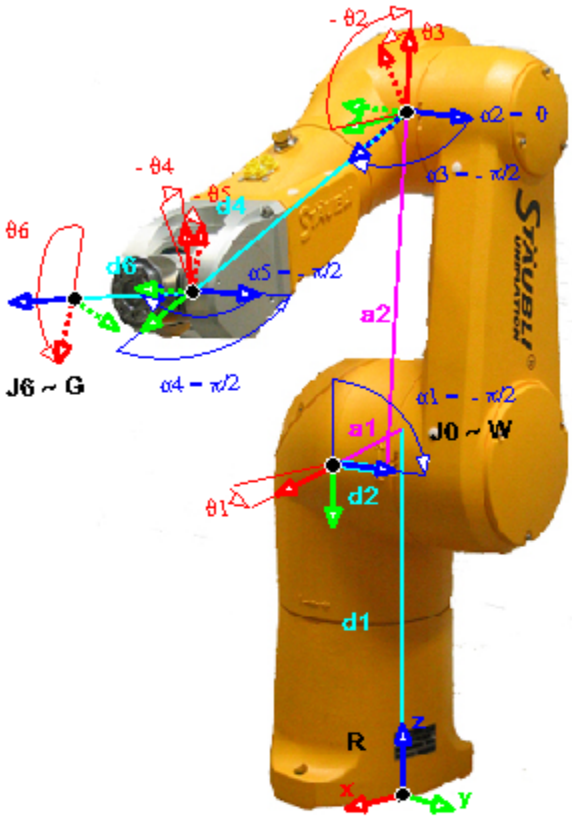
$\alpha_i \mid a_i \mid \theta_i \mid d_i$

$$A_i^{i-1} = \begin{bmatrix} \cos\theta_i & -\sin\theta_i\cos\alpha_i & \sin\theta_i\sin\alpha_i & a_i\cos\theta_i \\ \sin\theta_i & \cos\theta_i\cos\alpha_i & -\cos\theta_i\sin\alpha_i & a_i\sin\theta_i \\ 0 & \sin\alpha_i & \cos\alpha_i & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$G = A_1^0 A_2^1 A_3^2 A_4^3 A_5^4 A_6^5$$



offset $= [0,\ -\frac{\pi}{2},\ -\frac{\pi}{2},\ 0,\ 0,\ -\pi]$

α_1	a_1	θ_1	d_1
$-\frac{\pi}{2}$	a_1	θ_1	0

$$A_1^0 = \begin{bmatrix} \cos\theta_1 & 0 & -\sin\theta_1 & a_1 cos\theta_1 \\ \sin\theta_1 & 0 & \cos\theta_1 & a_1 sin\theta_1 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

α_2	a_2	θ_2	d_2
0	a_2	θ_2	d_2

$$A_2^1 = \begin{bmatrix} \cos\theta_2 & -\sin\theta_2 & 0 & a_2\cos\theta_2 \\ \sin\theta_2 & \cos\theta_2 & 0 & a_2\sin\theta_2 \\ 0 & 0 & 1 & d_2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

α_3	a_3	θ_3	d_3
$-\frac{\pi}{2}$	0	θ_3	0

$$A_3^2 = \begin{bmatrix} \cos\theta_3 & 0 & -\sin\theta_3 & 0 \\ \sin\theta_3 & 0 & \cos\theta_3 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

α_4	a_4	θ_4	d_4
$\frac{\pi}{2}$	0	θ_4	d_4

$$A_4^3 = \begin{bmatrix} \cos\theta_4 & 0 & \sin\theta_4 & 0 \\ \sin\theta_4 & 0 & -\cos\theta_4 & 0 \\ 0 & 1 & 0 & d_4 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

α_5	a_5	θ_5	d_5
$\frac{-\pi}{2}$	0	θ_5	0

$$A_5^4 = \begin{bmatrix} \cos\theta_5 & 0 & -\sin\theta_5 & 0 \\ \sin\theta_5 & 0 & \cos\theta_5 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

α_6	a_6	θ_6	d_6
0	0	θ_6	d_6

$$A_6^5 = \begin{bmatrix} \cos\theta_6 & -\sin\theta_6 & 0 & 0 \\ \sin\theta_6 & \cos\theta_6 & 0 & 0 \\ 0 & 0 & 1 & d_6 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

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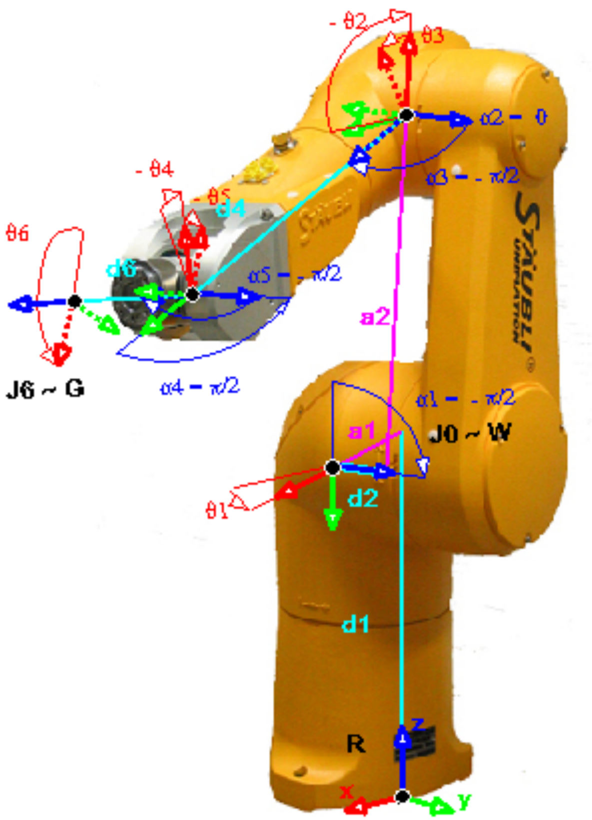
The Standard Kinematic model in Denavit-Hartenberg Convention

Stäubli TX 90



TX-90 (6 axis, RRRRRR) [Staubli]

α	a	θ	d
-1.5708	50.0	0.0	350.0
0.0	425.0	0.0	50.0
-1.5708	0.0	0.0	0.0
1.5708	0.0	0.0	425.0
-1.5708	0.0	0.0	0.0
0.0	0.0	0.0	100.0

6 non-trivial parameteres

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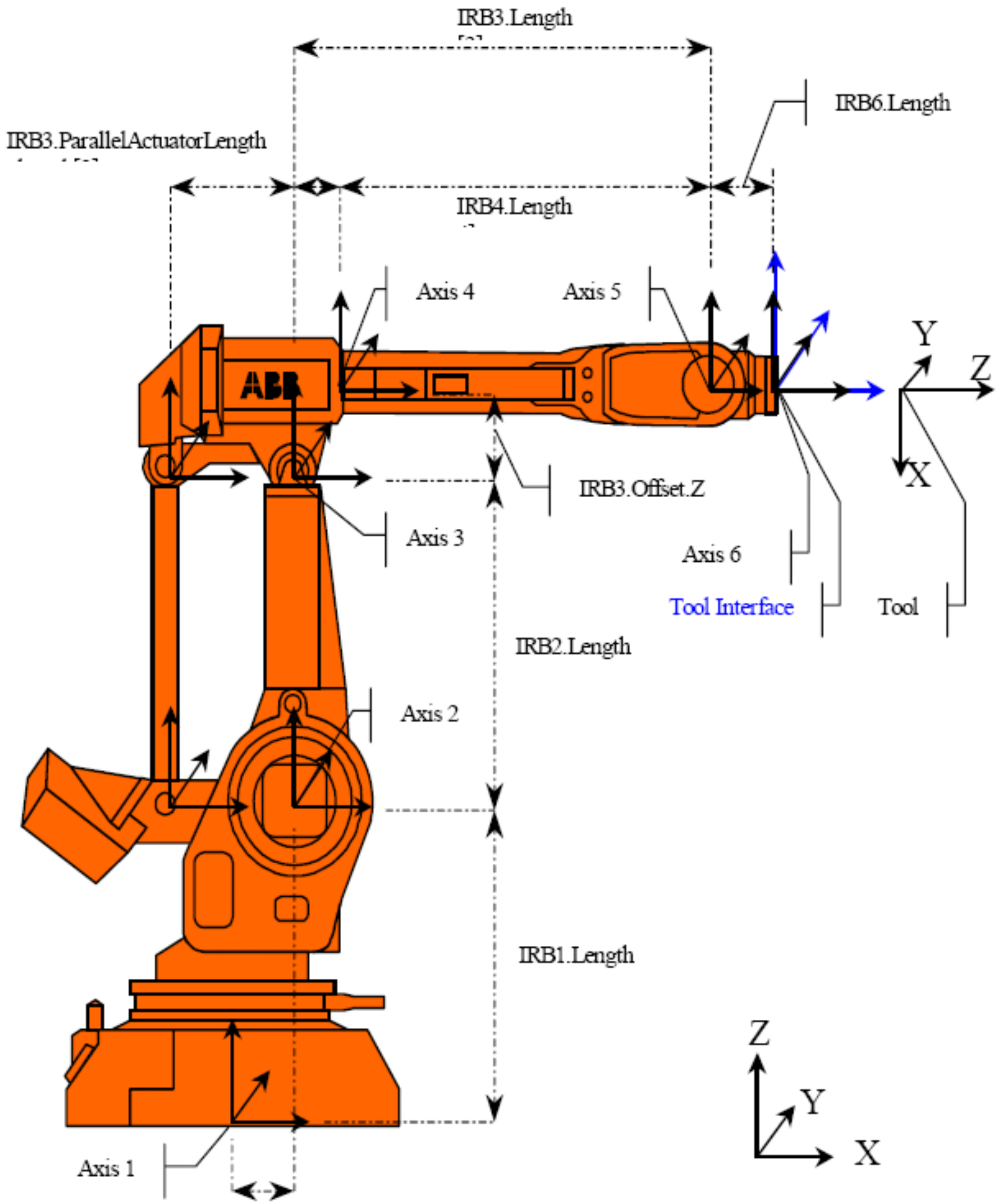
The Standard Kinematic model in Denavit-Hartenberg Convention

ABB IBR 140



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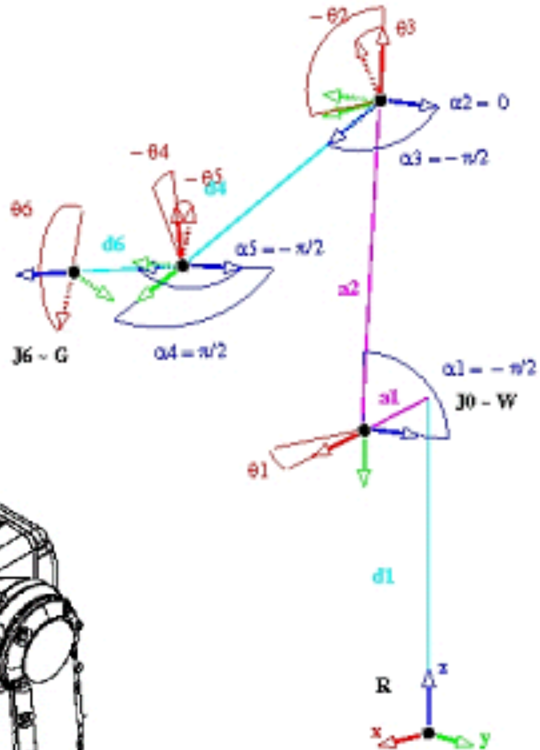
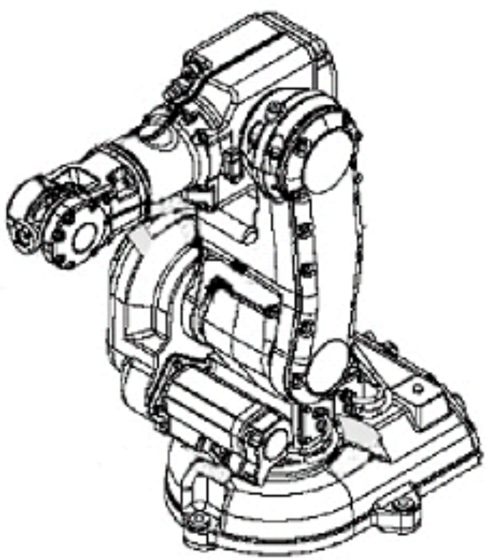
The Standard Kinematic model in Denavit-Hartenberg Convention

ABB IBR 140





IBR-140 (6 axis) [ABB]

α	a	θ	d
-1.5708	70.0	0.0	352.0
0.0	360.0	0.0	0.0
-1.5708	0.0	0.0	0.0
1.5708	0.0	0.0	380.0
-1.5708	0.0	0.0	0.0
0.0	0.0	0.0	65.0

5 non-trivial parameteres

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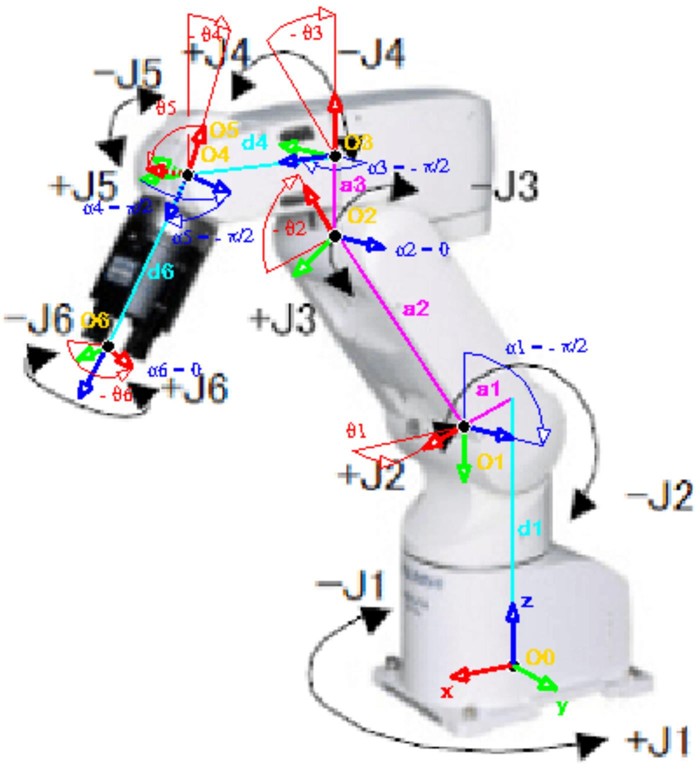
The Standard Kinematic model in Denavit-Hartenberg Convention

Stäubli TX 90



RV-6S (6 axis, RRRRRR) [Mitsubishi]

α	a	θ	d
-1.5708	85.0	0.0	350.0
0.0	280.0	0.0	0.0
-1.5708	100.0	0.0	0.0
1.5708	0.0	0.0	315.0
-1.5708	0.0	0.0	0.0
0.0	0.0	0.0	85.0

6 non-trivial parameteres

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Literature

Linear algebra

P. Pták. *Introduction to Linear Algebra*. Vydavatelství ČVUT, Praha, 2006.

Numerical linear algebra

E. Krajník. *Maticový počet*. Vydavatelství ČVUT, Praha, 2000.

The solution

M. Raghavan, B. Roth. *Kinematic Analysis of the 6R Manipulator of General Geometry.* Int. Symp. Robotics. Research. Pp. 314-320, Tokyo 1989/1990.

The numerical solution

D. Manocha, J. Canny. *Efficient Inverse Kinematics for General 6R Manipulators*. Robotics and Automation 1994.

The pedagogical solution will be developed using

D. Cox, J. Little, D. O'Shea. *Ideals, Varieties, and Algorithms*. Springer 1998.

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Software

Matlab: www.matworks.com

Maple: www.maplesoft.com

Python: www.python.org

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One algebraic equation in one variable

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SOLVING 1 ALGEBRAIC EQUATION

1 equation, 1 variable $\rightarrow$ companion matrix $\rightarrow$ eigenvalues

$$f(x) = x^3 + 4\,x^2 + x - 6 \;=\; -6 + 1\,x + 4\,x^2 + 1\,x^3$$

$$\mathtt{M}_x = \begin{bmatrix} 0 & 0 & 6 \\ 1 & 0 & -1 \\ 0 & 1 & -4 \end{bmatrix}$$

... a simple rule

```
>> e=eig(Mx)
```

$$\mathtt{e} \;=\; \begin{bmatrix} 1 \\ -2 \\ -3 \end{bmatrix} \qquad x_1 = 1,\; x_2 = -2,\; x_3 = -3$$

It works when eig works, i.e. order 100 in Matlab is often OK.

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SOLVING 1 ALGEBRAIC EQUATION

Linear maping $\mathtt{M} \in \mathbb{R}^{n\times n}$

Eigenvalues $\mathtt{M}\,\mathbf{x} = \lambda\,\mathbf{x}$

$$\Updownarrow$$

$$\mathtt{M}\,\mathbf{x} - \lambda\,\mathbf{x} = 0$$

$$\Updownarrow$$

$$\mathtt{M}\,\mathbf{x} - \lambda\,\mathtt{I}\,\mathbf{x} = 0$$

$$\Updownarrow$$

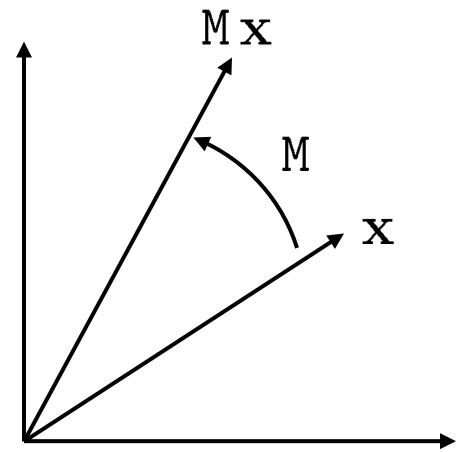
$$(\mathtt{M} - \lambda\,\mathtt{I})\,\mathbf{x} = 0$$

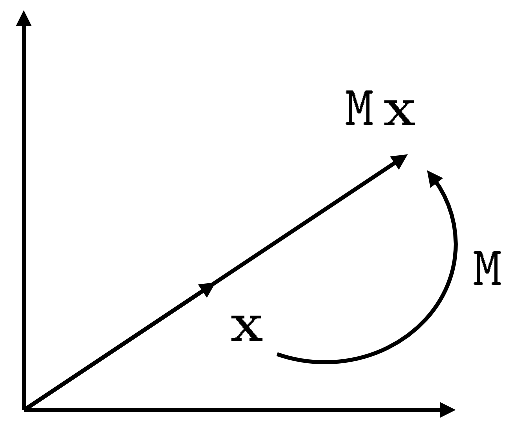
$$\mathbf{x} \neq 0 \;\Rightarrow\; \Updownarrow$$

$$\mathrm{rank}(\mathtt{M} - \lambda\,\mathtt{I}) < n$$

$$\Updownarrow$$

$$\det(\mathtt{M} - \lambda\,\mathtt{I}) = 0$$





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SOLVING 1 ALGEBRAIC EQUATION

algebraic equation

$$f(x) = x^4 + a_3 x^3 + a_2 x^2 + a_1 x + a_0 = \det(-\mathtt{M} + x\,\mathtt{I})$$

$$-\mathtt{M} + x\,\mathtt{I} = \begin{bmatrix} x & & & a_0 \\ -1 & x & & a_1 \\ & -1 & x & a_2 \\ & & -1 & x + a_3 \end{bmatrix}$$

$$f(x) = x^4 + a_3 x^3 + a_2 x^2 + a_1 x + a_0$$

Numerical solution to f(x) is obtained by

```
>> x = eig(M);
```


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