# Curvature-Constrained Data Collection Planning
## Dubins Traveling Salesman Problem with Neighborhoods (DTSPN)
## and
## Dubins Orienteering Problem with Neighborhoods (DOPN)

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Lecture 07

B4M36UIR – Artificial Intelligence in Robotics

# Overview of the Lecture

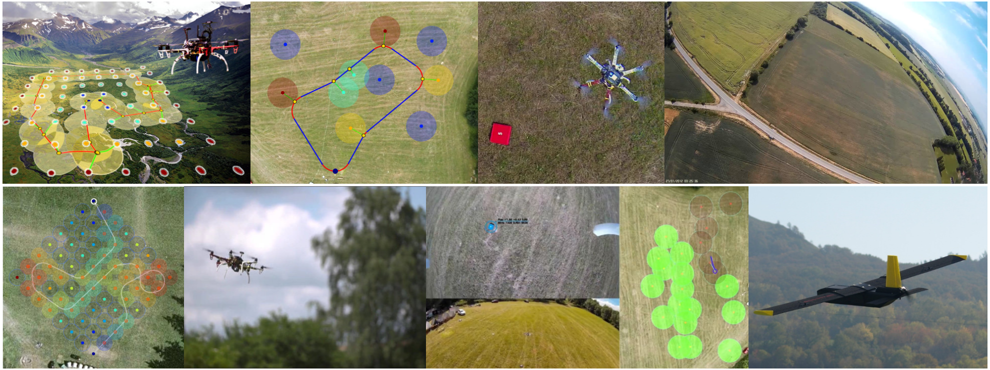
- Part 1 – Curvature-Constrained Data Collection Planning
  - Dubins Vehicle and Dubins Planning
  - Dubins Touring Problem (DTP)
  - Dubins Traveling Salesman Problem
  - Dubins Traveling Salesman Problem with Neighborhoods
  - Dubins Orienteering Problem
  - Dubins Orienteering Problem with Neighborhoods
  - Planning in 3D – Examples and Motivations

# Part I

## Part 1 – Curvature-Constrained Data Collection Planning

# Motivation – Surveillance Missions with Aerial Vehicles

- Provide **curvature-constrained** path to collect the most valuable measurements with shortest possible path/time or under limited travel budget.



- Formulated as routing problems with Dubins vehicle
  - **Dubins Traveling Salesman Problem with Neighborhoods**
  - **Dubins Orienteering Problem with Neighborhoods**

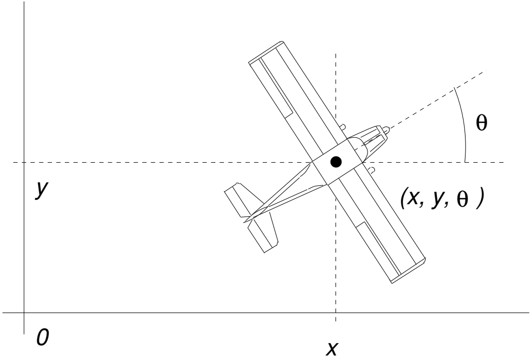
# Dubins Vehicle

- Non-holonomic vehicle such as car-like or aircraft can be modeled as the Dubins vehicle:
  - Constant forward velocity;
  - Limited minimal turning radius $\rho$;
  - Vehicle state is represented by a triplet $q = (x, y, \theta)$, where
  - Position is $(x, y) \in \mathbb{R}^2$, vehicle heading is $\theta \in \mathbb{S}^2$, and thus $q \in SE(2)$.

The vehicle motion can be described by the equation

$$\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\theta} \end{bmatrix} = v \begin{bmatrix} \cos\theta \\ \sin\theta \\ \frac{u}{\rho} \end{bmatrix}, \quad |u| \leq 1,$$

where $u$ is the control input.



# Optimal Maneuvers for Dubins Vehicle

- For two states $q_1 \in SE(2)$ and $q_2 \in SE(2)$ in the environment without obstacles $\mathcal{W} = \mathbb{R}^2$, the optimal path connecting $q_1$ with $q_2$ can be characterized as one of two main types
  - **CCC** type: LRL, RLR;
  - **CSC** type: LSL, LSR, RSL, RSR;

  where S – straight line arc, C – circular arc oriented to left (L) or right (R).

  *L. E. Dubins (1957) – American Journal of Mathematics*

- The optimal paths are called **Dubins maneuvers**.
  - Constant velocity: $v(t) = v$ and turning radius $\rho$.
  - **Six** types of trajectories connecting any configuration in $SE(2)$. *(Without obstacles)*
  - The control $u$ is according to $C$ and $S$ type one of three possible values $u \in \{-1, 0, 1\}$.

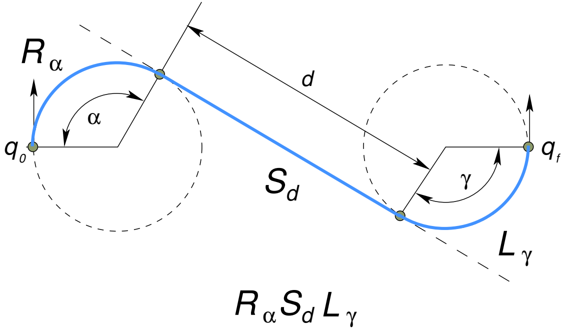
# Parametrization of Dubins Maneuvers

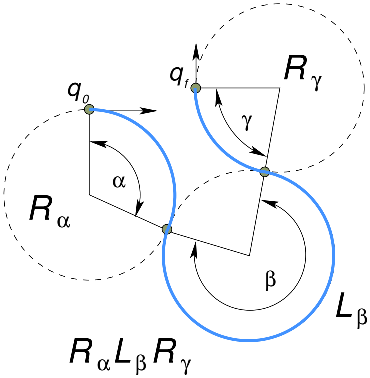
- Parametrization of each trajectory phase:

$$\{L_\alpha R_\beta L_\gamma,\ R_\alpha L_\beta R_\gamma,\ L_\alpha S_d L_\gamma,\ L_\alpha S_d R_\gamma,\ R_\alpha S_d L_\gamma,\ R_\alpha S_d R_\gamma\}$$

for $\alpha \in [0, 2\pi)$, $\beta \in (\pi, 2\pi)$, $d \geq 0$.

*Notice the prescribed orientation at $q_0$ and $q_f$.*

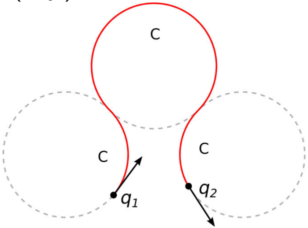


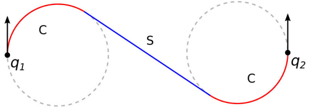



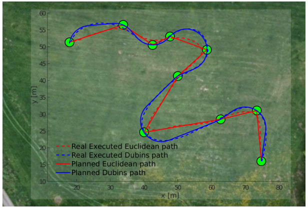
# Multi-goal Dubins Path

- Minimal turning radius $\rho$ and constant forward velocity $v$.
- State of the Dubins vehicle is $q = (x, y, \theta)$, $q \in SE(2)$, $(x, y) \in \mathbb{R}^2$ and $\theta \in \mathbb{S}^1$.

$$\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\theta} \end{bmatrix} = v \begin{bmatrix} \cos\theta \\ \sin\theta \\ \frac{u}{\rho} \end{bmatrix}$$



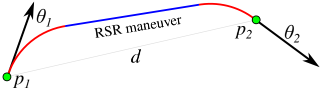

Smooth Dubins path connecting a sequence of locations is also suitable for multi-rotor aerial vehicle.

- Optimal path connecting $q_1 \in SE(2)$ and $q_2 \in SE(2)$ consists only of straight line arcs and arcs with the maximal curvature, i.e., two types of maneuvers CCC and CSC and the solution can be found analytically. (Dubins, 1957)
- In **multi-goal Dubins path planning**, we need to solve the underlying TSP.

# Difficulty of Dubins Vehicle in the Solution of the TSP

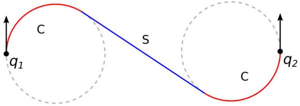
- For the minimal turning radius $\rho$, the **optimal path** connecting $\boldsymbol{q}_1 \in SE(2)$ and $\boldsymbol{q}_2 \in SE(2)$ can be found analytically.

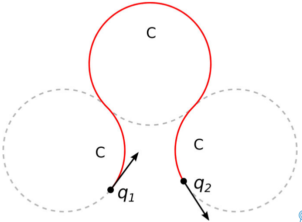
L. E. Dubins (1957) – American Journal of Mathematics

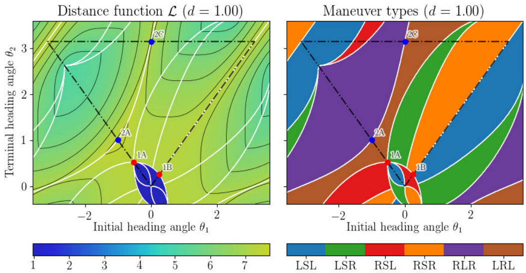



- Two types of optimal Dubins maneuvers: CSC and CCC.
- The length of the optimal maneuver $\mathcal{L}$ has a closed-form solution.
  - It is **piecewise-continuous function**;
  - (continuous for $\|(\boldsymbol{p}_1, \boldsymbol{p}_2)\| > 4\rho$).

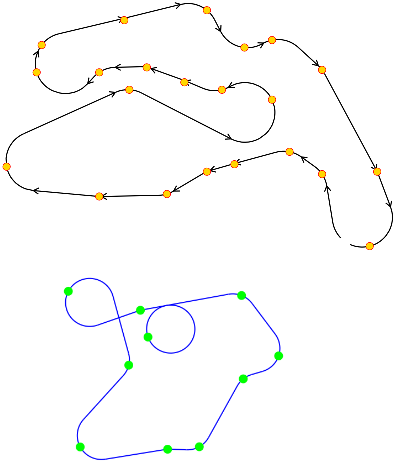
*Can be computed in less than* 0.5 $\mu s$



# Dubins Traveling Salesman Problem (DTSP)



- Determine (closed) shortest Dubins path visiting each $\boldsymbol{p}_i \in \mathbb{R}^2$ of the given set of $n$ locations $P = \{\boldsymbol{p}_1, \ldots, \boldsymbol{p}_n\}$.

1. Permutation $\Sigma = (\sigma_1, \ldots, \sigma_n)$ of visits (sequencing).

   ***Combinatorial optimization***

2. *Headings* $\Theta = \{\theta_{\sigma_1}, \theta_{\sigma_2}, \ldots, \theta_{\sigma_n}\}$, $\theta_i \in [0, 2\pi)$, *for* $\boldsymbol{p}_{\sigma_i} \in P$.

   ***Continuous optimization***

- **DTSP** is an optimization problem over all possible **sequences** $\Sigma$ and **headings** $\Theta$ at the states $(\boldsymbol{q}_{\sigma_1}, \boldsymbol{q}_{\sigma_2}, \ldots, \boldsymbol{q}_{\sigma_n})$ such that $\boldsymbol{q}_{\sigma_i} = (\boldsymbol{p}_{\sigma_i}, \theta_{\sigma_i})$, $\boldsymbol{p}_{\sigma_i} \in P$

$$\text{minimize}_{\Sigma,\Theta} \quad \sum_{i=1}^{n-1} \mathcal{L}(\boldsymbol{q}_{\sigma_i}, \boldsymbol{q}_{\sigma_{i+1}}) + \mathcal{L}(\boldsymbol{q}_{\sigma_n}, \boldsymbol{q}_{\sigma_1}) \qquad \text{where}$$

$$\text{subject to} \quad \boldsymbol{q}_i = (\boldsymbol{p}_i, \theta_i)\ i = 1, \ldots, n,$$

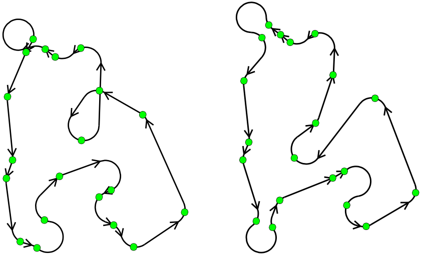
$\mathcal{L}(\boldsymbol{q}_{\sigma_i}, \boldsymbol{q}_{\sigma_j})$ is the length of Dubins path between $\boldsymbol{q}_{\sigma_i}$ and $\boldsymbol{q}_{\sigma_j}$.

The continuous domain of the heading angles is simular to the regions in the TSPN-like problem formulations.

# Challenges of the Dubins Traveling Salesman Problem

- The key difficulty of the DTSP is that the path length mutually depends on
  - Order of the visits to the locations;
  - Headings at the target locations.

  *We need the sequence to determine headings, but headings may influence the sequence.*
- The Dubins TSP is **sequence dependent problem.**
- Two fundamental approaches can be found in literature.



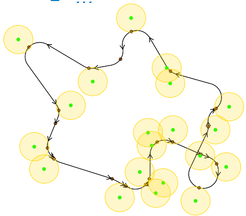
1. **Decoupled** approach based on a given sequence of the locations, e.g., found by a solution of the Euclidean TSP.
2. **Sampling-based** approach with sampling of the headings at the locations into discrete sets of values and considering the problem as the variant of the **Generalized TSP**.

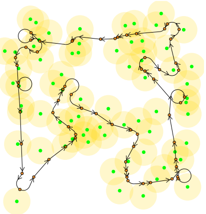
Besides, further approaches are

- Genetic and memetic techniques (evolutionary algorithms);
- Unsupervised learning based approaches.

# Existing Approaches to the DTSP(N)

- Heuristic (decoupled & evolutionary) approaches
  - *Savla et al., 2005*
  - *Ma and Castanon, 2006*
  - *Macharet et al., 2011*
  - *Macharet et al., 2012*
  - *Ny et al., 2012*
  - *Yu and Hang, 2012*
  - *Macharet et al., 2013*
  - *Zhant et al., 2014*
  - *Macharet and Campost, 2014*
  - *Váňa and Faigl, 2015*
  - *Isaiah and Shima, 2015*
  - ...





- Sampling-based approaches
  - *Obermeyer, 2009*
  - *Oberlin et al., 2010*
  - *Macharet et al., 2016*
- Convex optimization
  - (Only if the locations are far enough)
  - *Goac et al., 2013*
- **Lower bound for the DTSP**
  - Dubins Interval Problem (DIP)
  - *Manyam et al., 2016*
  - DIP-based inform sampling
  - *Váňa and Faigl, 2017*
- **Lower bound for the DTSPN**
  - Using Generalized DIP (GDIP)
  - *Váňa and Faigl, 2018, 2020*

# Planning with Dubins Vehicle – Summary

- The optimal path connecting two configurations can be found analytically.

  *E.g., for UAVs that usually operates in environment without obstacles.*

- The Dubins maneuvers can also be used in randomized-sampling based motion planners, such as RRT, in the control based sampling.
- Dubins vehicle model can be considered in the multi-goal path planning such as surveillance, inspection or monitoring missions to periodically visits given target locations (areas).
- **Dubins Touring Problem** (**DTP**)

  Given a sequence of locations, what is the shortest path visting the locations, i.e., what are the headings of the vehicle at the locations.

- **Dubins Traveling Salesman Problem** (**DTSP**)

  Given a set of locations, what is the shortest Dubins path that visits each location exactly once and returns to the origin location.

- **Dubins Orienteering Problem** (**DOP**)

  Given a set of locations, each with associated reward, what is the Dubins path visiting the most rewarding locations and not exceeding the given travel budget.

# Dubins Touring Problem – DTP

- For a sequence of the $n$ waypoint locations $P = (p_1, \dots p_n)$, $p_i \in \mathbb{R}^2$, the **Dubins Touring Problem** (**DTP**) stands to determine the **optimal headings** $T = \{\theta_1, \dots, \theta_n\}$ at the waypoints $q_i$ such that

$$\begin{aligned} \text{minimize}_{T} \quad & \mathcal{L}(T, P) = \sum_{i=1}^{n-1} \mathcal{L}(q_i, q_{i+1}) + \mathcal{L}(q_n, q_1) \\ \text{subject to} \quad & q_i = (p_i, \theta_i),\ \ \theta_i \in [0, 2\pi),\ \ p_i \in P, \end{aligned}$$

  where $\mathcal{L}(q_i, q_j)$ is the length of the Dubins maneuver connecting $q_i$ with $q_j$.
- The DTP is a continuous optimization problem.
- The term $\mathcal{L}(q_n, q_1)$ is for possibly closed tour that can be for example requested in the TSP with Dubins vehicle, a.k.a. DTSP.

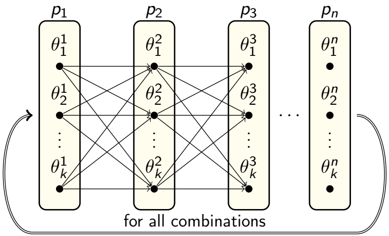
  *On the other, the DTP can also be utilized for open paths such as solutions of the OP with Dubins vehicle.*
- In some cases, it may be suitable to relax the heading at the first/last locations in finding closed tours (i.e., solving DTSP).

# Sampling-based Solution of the DTP

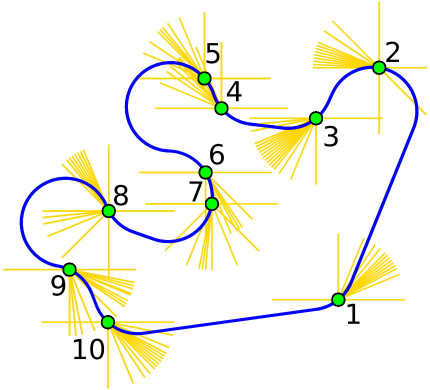
- For a closed sequence of the waypoint locations

$$P = (p_1, \ldots, p_n).$$

- We can sample possible heading values at each location $i$ into a discrete set of $k$ headings, i.e., $\Theta^i = \{\theta^i_1, \ldots, \theta^i_k\}$ and create a graph of all possible Dubins maneuvers.




- For a set of heading samples, the optimal solution can be found by a forward search of the graph in $O(nk^3)$.

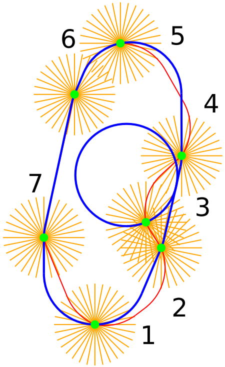



For open sequence we do not need to evalute all possible initial headings, and the complexity is $O(nk^2)$.

- **The problem is to determine the most suitable heading samples.**

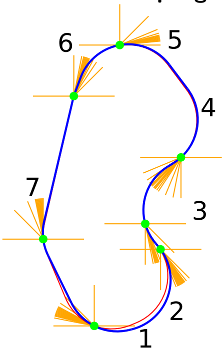
# Example of Heading Sampling – Uniform vs. Informed

**Uniform sampling**




$N = 224$, $T_{cpu} = 128$ ms
$\mathcal{L} = 19.8$, $\mathcal{L}_U = 13.8$

**Informed sampling**

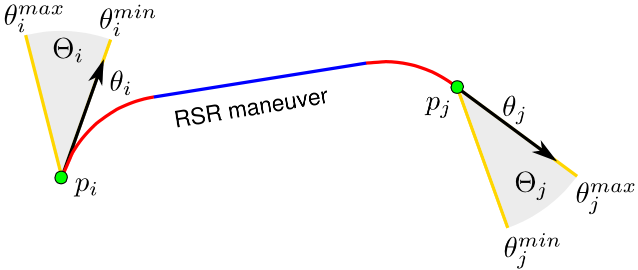



$N = 128$, $T_{cpu} = 76$ ms
$\mathcal{L} = 14.4$, $\mathcal{L}_U = 14.2$.

- $N$ is the total number of samples, i.e., 32 samples per waypoint for uniform sampling.
- $\mathcal{L}$ is the length of the tour (blue) and $\mathcal{L}_U$ is the lower bound (red) determined as a solution of the **Dubins Interval Problem** (**DIP**).

# Dubins Interval Problem (DIP)

- **Dubins Interval Problem (DIP)** is a generalization of Dubins maneuvers to the shortest path connecting two points $p_i$ and $p_j$.
- In the DIP, the leaving interval $\Theta_i$ at $p_i$ and the arrival interval $\Theta_j$ at $p_j$ are consider (not a single heading value).
- The optimal solution can be found analytically. *Manyam et al. (2015)*

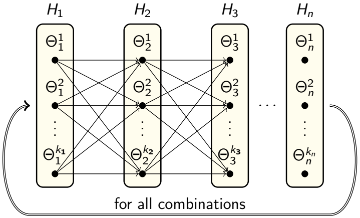


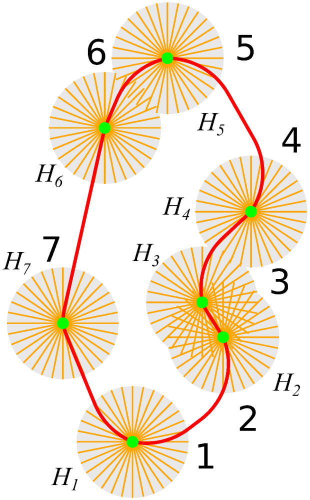

- Solution of the DIP is a tight lower bound for the DTP.
- Solution of the DIP is not a feasible solution of the DTP.

*Notice, for $\Theta_i = \Theta_j = \langle 0, 2\pi)$ the optimal maneuver for DIP is a straight line segment.*

# Lower Bound of the DTP

- For a discrete set of heading intervals $\mathcal{H} = \{H_1, \ldots, H_n\}$, where $H_i = \{\Theta_i^1, \Theta_i^2, \ldots, \Theta_i^{k_i}\}$, a similar graph as for the DTP can be constructed with the edge cost determined by the solution of the associated DIP.



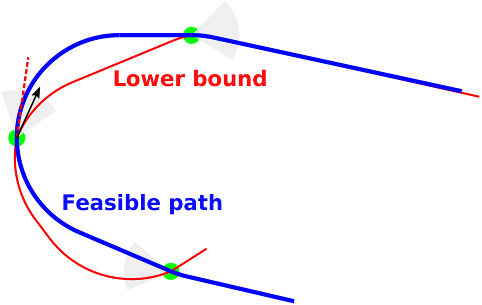




- The forward search of the graph with dense samples provides a tight lower bound of the DTP. *Manyam and Rathinam, 2015*

# Lower Bound and Feasible Solution of the DTP

- The arrival and departure angles may not be the same.

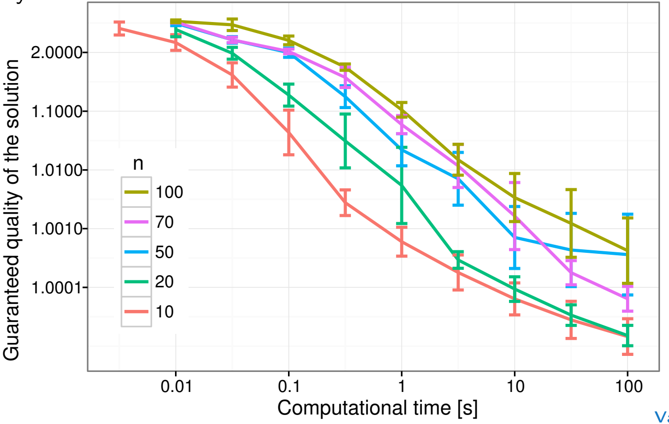
*The lower bound solution is not a feasible solution of the DTP.*




- DTP solution – use any particular heading of each interval in the lower bound solution.

# The DIP-based Sampling of Headings in the DTP

- Using heading intervals for a sequence of waypoints and a solution of the DIP, we can determine **lower bound** of the DTP using the forward search graph as for the DTP.
- The ratio between the lower bound and feasible solution of the DTP provides an estimation of the solution quality.




Váňa and Faigl (2016)

# Iteratively-Refined Informed Sampling (IRIS) of Headings in the Solution of the DTP

- Iterative refinement of the heading intervals $\mathcal{H}$ up to the angular resolution $\epsilon_{req}$.
- The angular resolution is gradually decreased for the most promising intervals.
- `refineDTP` – divide the intervals of the lower bound solution.
- `solveDTP` – solve DTP using the heading from the refined intervals.

**Algorithm 1:** Iterative Informed Sampling-based DTP Algorithm

**Input**: $P$ – Target locations to be visited
**Input**: $\epsilon_{req}$ – Requested angular resolution
**Input**: $\alpha_{req}$ – Requested quality of the solution
**Output**: $T$ – A tour visiting the targets

```
ε ← 2π                          // initial angular resolution;
H ← createIntervals(P, ε)       // initial intervals;
L_L ← 0                         // init lower bound;
L_U ← ∞                         // init upper bound;
while ε > ε_req and L_U/L_L > α_req do
    ε ← ε/2;
    (H, L_L) ← refineDTP(P, ε, H);
    (T, L_U) ← solveDTP(P, H);
end
return T;
```

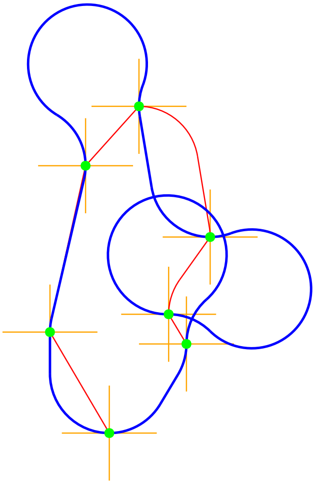
Faigl, J., Váňa, P., Saska, M., Báča, T., and Spurný, V.: *On solution of the Dubins touring problem*, **ECMR**, 2017.

- It simultaneously provides **feasible** and **lower bound** solutions of the DTP.

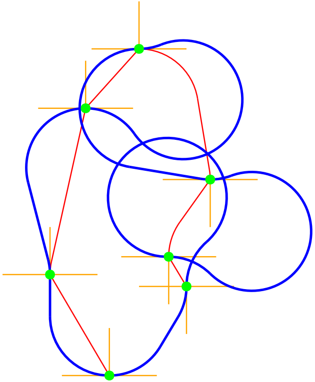
  *The lower bound provides a tight estimation of the solution quality.*

- The first solution is provided very quickly – **any-time algorithm**.

# Uniform vs Informed Sampling



$\epsilon = 2\pi/4$, $N = 28$, $T_{CPU} = 8$ ms
$\mathcal{L} = 27.9$, $\mathcal{L}_U = 13.2$



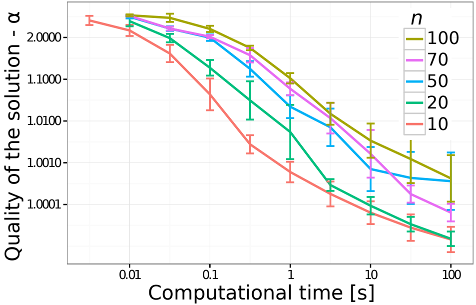
$\epsilon = 2\pi/4$, $N = 21$, $T_{CPU} = 8$ ms
$\mathcal{L} = 29.9$, $\mathcal{L}_U = 13.2$

# Results and Comparison with Uniform Sampling

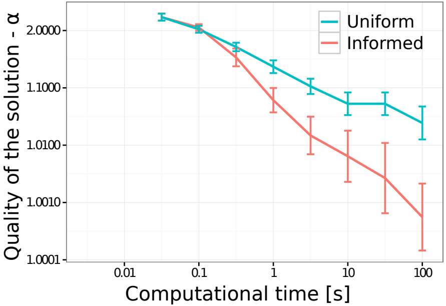
- Random instances of the DTSP with a sequence of visits to the targets determined as a solution of the Euclidean TSP.
- The waypoints placed in a squared bounding box with the side $s = (\rho\sqrt{n})/d$ for the $\rho = 1$ and density $d = 0.5$.

It matters on the density of targets!

Quality of solution for increasing $n$




Comparision with the uniform sampling



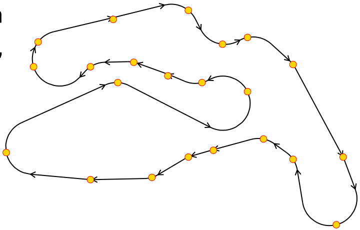

- The informed sampling-based approach provides solutions up to 0.01% from the optima.
- A solution of the DTP is a fundamental bulding block for **routing problems with Dubins vehicle**.

# Dubins Traveling Salesman Problem (DTSP)

1. Determine a closed shortest Dubins path visiting each location $p_i \in P$ of the given set of $n$ locations $P = \{p_1, \ldots, p_n\}$, $p_i \in \mathbb{R}^2$.
2. Permutation $\Sigma = (\sigma_1, \ldots, \sigma_n)$ of visits.

   *Sequencing part of the problem*
3. Headings $\Theta = \{\theta_{\sigma_1}, \theta_{\sigma_2}, \ldots, \theta_{\sigma_n}\}$ for $p_{\sigma_i} \in P$.

   *Continuous optimization*



- **DTSP** is an optimization problem over all possible **permutations** $\Sigma$ and **headings** $\Theta$ in the states $(q_{\sigma_1}, q_{\sigma_2}, \ldots, q_{\sigma_n})$ such that $q_{\sigma_i} = (p_{\sigma_i}, \theta_{\sigma_i})$

$$\text{minimize}_{\Sigma,\Theta} \quad \sum_{i=1}^{n-1} \mathcal{L}(q_{\sigma_i}, q_{\sigma_{i+1}}) + \mathcal{L}(q_{\sigma_n}, q_{\sigma_1})$$

$$\text{subject to} \quad q_i = (p_i, \theta_i) \; i = 1, \ldots, n,$$

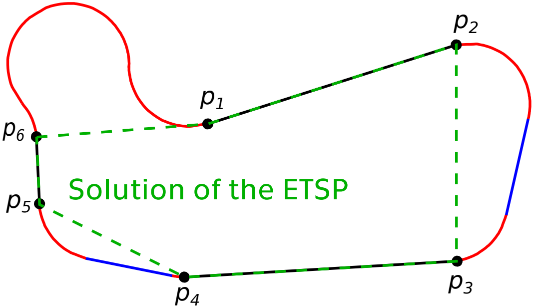
where $\mathcal{L}(q_{\sigma_i}, q_{\sigma_j})$ is the length of Dubins path between $q_{\sigma_i}$ and $q_{\sigma_j}$.

# Decoupled Solution of the DTSP – Alternating Algorithm

Alternating Algorithm (AA) provides a solution of the DTSP for an even number of targets $n$.

Savla, K., Frazzoli, E., Bullo, F.: *On the point-to-point and traveling salesperson problems for Dubins'vehicle*, IEE American Control Conference, 2005.

1. Solve the related Euclidean TSP.
   *Relaxed motion constraints*
2. Establish headings for even edges using straight line segments.
3. Determine optimal maneuvers for odd edges using the analytical form for Dubins maneuvers.
   *Headings are known.*




*Courtesy of P. Váňa*

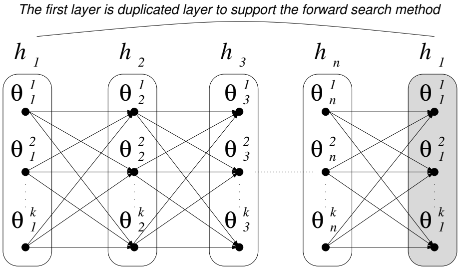
# DTSP with the Given Sequence of the Visits to the Targets

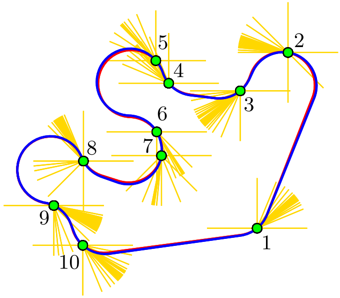
- If the sequence of the visits $\Sigma$ to the target locations is given.
- the problem is to determine the optimal heading at each location.
- and the problem becomes the **Dubins Touring Problem** (**DTP**).

- Let for each location $g_i \in G$ sample possible heading to $k$ values, i.e., for each $g_i$ the set of headings be $h_i = \{\theta_1^1, \ldots, \theta_1^k\}$.

- Since $\Sigma$ is given, we can construct a graph connecting two consecutive locations in the sequence by all possible headings.
- For such a graph and particular headings $\{h_1, \ldots, h_n\}$, we can find an optimal headings and thus, the optimal solution of the DTP.

# DTSP as a Solution of the DTP




- The edge cost corresponds to the length of Dubins maneuver.
- Better solution of the DTP can be found for more samples, which will also improve the DTSP but only for the given sequence.

*Two questions arise for a practical solution of the DTP.*

- How to sample the headings? More samples makes finding solution more demanding.

  *We need to sample the headings in a "smart" way, i.e., guided sampling using lower bound of the DTP?*

- What is the solution quality? Is there a tight lower bound?

  *Yes, the lower bound can be computed as a solution of the Dubins Interval Problem (DIP).*

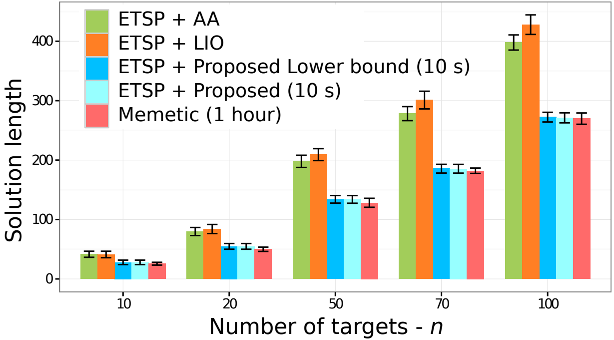
# DTP Solver in Solution of the DTSP

- The solution of the DTP can be used to solve DTSP for the given sequence of the waypoints.

  *E.g., determined as a solution of the Euclidean TSP as in the Alternating Algorithm.*

- Comparision with the Alternating Algorithm (AA), Local Iterative Optimization (LIO), and Memetic algorithm.

  AA – Savla et al., 2005, LIO – Váňa & Faigl, 2015, Memetic – Zhang et al. 2014



# DTSP – Sampling-based Approach

- Sampled heading values can be directly utilized to find the sequence as a solution of the **Generalized Traveling Salesman Problem** (**GTSP**).

*Notice For Dubins vehicle, it is the Generalized Asymmetric TSP (GATSP).*

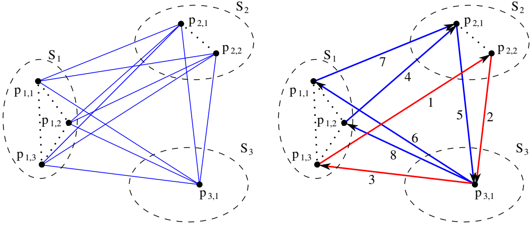
> The problem is to determine a shortest tour in a graph that visits all specified subsets of the graph's vertices.

*The TSP is a special case of the GTSP when each subset to be visited consists just a single vertex.*

- GATSP → ATSP;

*Noon and Bean (1991)*

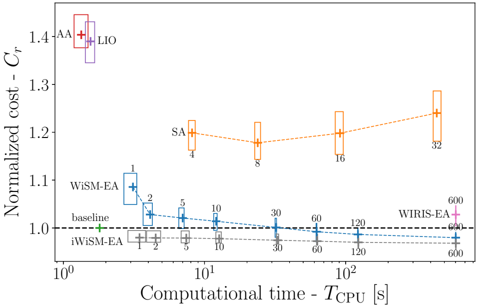
- ATSP can be solved by LKH;
- ATSP → TSP, which can be solved optimally, e.g., by Concorde.



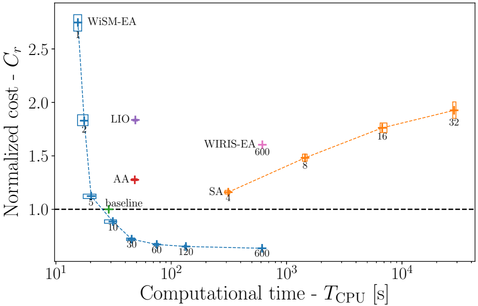

GTSP GATSP

# DTSP – Evolutionary Approach with Surrogate Model

- Use standard genetic operators with tournament selection and OX1 crossover method.
- The population is evaluated using learned surrogate model based on multi-layer perceptron.
- The surrogate model estimates solution cost of candidate sequences (instances of the DTP).
- Massive speedup of the evaluation yields improved solutions and scalability.




Instances with low density $d$ and $n = 100$ target locations




Instances with high density $d$ and $n = 500$ target locations

Drchal, J., Váňa, P., and Faigl, J.: *WiSM: Windowing Surrogate Model for Evaluation of Curvature-Constrained Tours with Dubins vehicle*, IEEE Transactions on Cybernetics, 2020.

# Dubins Traveling Salesman Problem with Neighborhoods

- In surveillance planning, it may be required to visit a set of target regions $\boldsymbol{G} = \{R_1, \ldots, R_n\}$ by the Dubins vehicle.
- Then, for each target region $R_i$, we have to determine a particular point of the visit $p_i \in R_i$ and DTSP becomes the **Dubins Traveling Salesman Problem with Neighborhoods** (**DTSPN**).

*In addition to $\Sigma$ and headings $\Theta$, waypoint locations $P$ have to be determined.*

- DTSPN is an optimization problem over all permutations $\Sigma$, headings $\Theta = \{\theta_{\sigma_1}, \ldots, \theta_{\sigma_n}\}$ and points $P = (p_{\sigma_1}, \ldots, p_{\sigma_n})$ for the states $(q_{\sigma_1}, \ldots, q_{\sigma_n})$ such that $q_{\sigma_i} = (p_{\sigma_i}, \theta_{\sigma_i})$ and $p_{\sigma_i} \in R_{\sigma_i}$:

$$\begin{aligned} \text{minimize}_{\Sigma,\Theta,P} \quad & \sum_{i=1}^{n-1} \mathcal{L}(q_{\sigma_i}, q_{\sigma_{i+1}}) + \mathcal{L}(q_{\sigma_n}, q_{\sigma_1}) \\ \text{subject to} \quad & q_i = (p_i, \theta_i), p_i \in R_i \;\; i = 1, \ldots, n. \end{aligned}$$

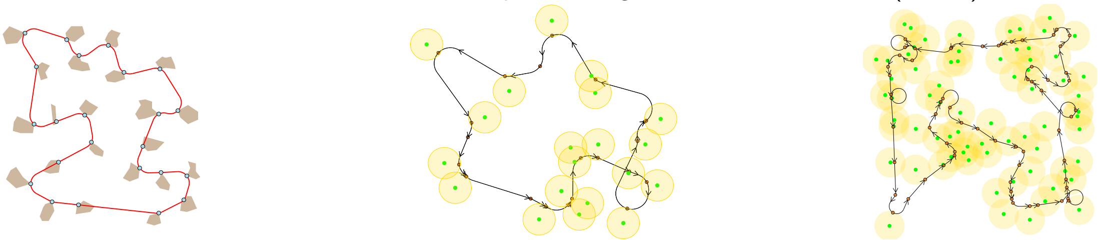
- $\mathcal{L}(q_{\sigma_i}, q_{\sigma_j})$ is the length of the shortest possible Dubins maneuver connecting the states $q_{\sigma_i}$ and $q_{\sigma_j}$.

# DTSPN – Approches and Examples of Solution

- Similarly to the DTSP, also the DTSPN can be addressed by
  - **Decoupled approaches** for which a sequence of visits to the regions can be found as a solution of the ETSP(N);
  - **Sampling-based approaches** and formulation as the GATSP.
    - Clusters of sampled waypoint locations each with sampled possible heading values.
  - **Soft-computing** techniques such as memetic algorithms.
  - **Unsupervised learning** techniques.

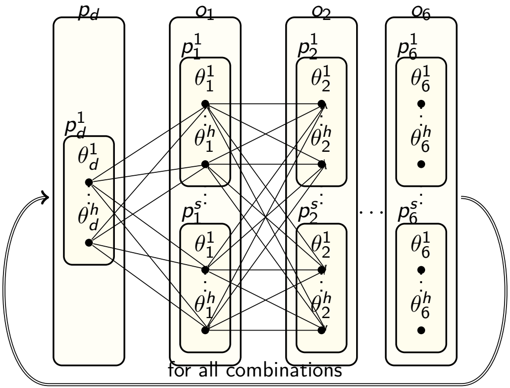
Váňa and Faigl (IROS 2015), Faigl and Váňa (ICANN 2016, IJCNN 2017)

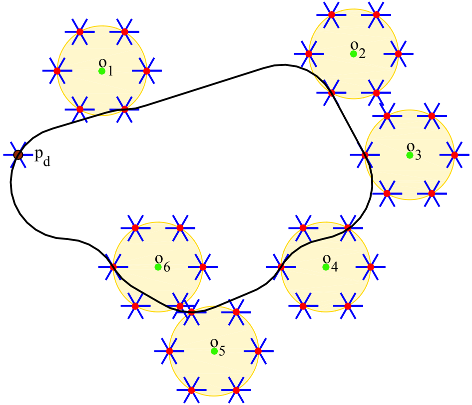
- Similarly to the lower bound of the DTSP based on the **Dubins Interval Problem** (DIP) a lower bound for the DTSPN can be computed using the **Generalized DIP** (**GDIP**).



# DTSPN – Decoupled Approach

1. Determine a sequence of visits to the $n$ target regions as the solution of the ETSP.
2. Sample possible waypoint locations and for each such a location sample possible heading values, e.g., $s$ locations per each region and $h$ heading per each location.
3. Construct a search graph and determine a solution in $O(n(sh)^3)$.
4. An example of the search graph for $n = 6$, $s = 6$, and $h = 6$.



# DTSPN – Local Iterative Optimization (LIO)

- Instead of sampling into a discrete set of waypoint locations each with sampled possible headings, we can perform local optimization, e.g., hill-climbing technique.
- At each waypoint location $p_i$, the heading can be $\theta_i \in [0, 2\pi)$.
- A waypoint location $p_i$ can be parametrized as a point on the bounday of the respective region $R_i$, i.e., as a parameter $\alpha \in [0, 1)$ measuring a normalized distance on the boundary of $R_i$.
- The multi-variable optimization is treated independently for each particular variable $\theta_i$ and $\alpha_i$ iteratively.

**Algorithm 2:** Local Iterative Optimization (LIO) for the DTSPN

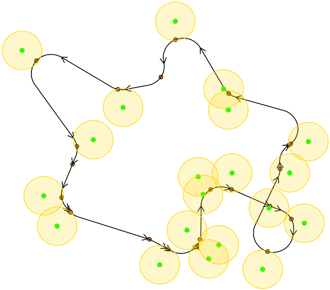
**Data:** Input sequence of the goal regions $\boldsymbol{G} = (R_{\sigma_1}, \ldots, R_{\sigma_n})$, for the permutation $\Sigma$
**Result:** Waypoints $(q_{\sigma_1}, \ldots, q_n)$, $q_i = (p_i, \theta_i)$, $p_i \in \delta R_i$

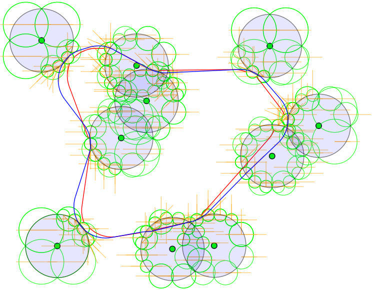
```
initialization()  // random assignment of q_i ∈ δR_i;
while global solution is improving do
    for every R_i ∈ G do
        θ_i := optimizeHeadingLocally(θ_i);
        α_i := optimizePositionLocally(α_i);
        q_i := checkLocalMinima(α_i, θ_i);
    end
end
```

Váňa, P. and Faigl, J.: *On the Dubins Traveling Salesman Problem with Neighborhoods*, **IROS**, 2015, pp. 4029–4034.

# Lower Bound for the DTSP with Neighborhoods Generalized Dubins Interval Problem

- In the DTSPN, we need to determine the **headings** and also the **waypoint locations**.
- The **Dubins Interval Problem** (**DIP**) is not sufficient to provide tight lower-bound.





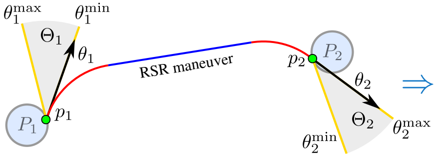
- **Generalized Dubins Interval Problem** (**GDIP**) can be utilized for the DTSPN similarly as the DIP for the DTSP.

Váňa and Faigl: *Optimal Solution of the Generalized Dubins Interval Problem*, **RSS** 2018, **best student paper finalist**.

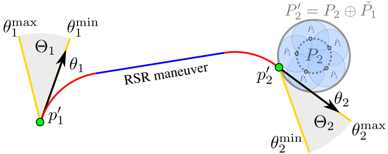
# Generalized Dubins Interval Problem (GDIP)

- Determine the shortest Dubins maneuver connecting $P_i$ and $P_j$ given the angle intervals $\theta_i \in [\theta_i^{min}, \theta_i^{max}]$ and $\theta_j \in [\theta_j^{min}, \theta_j^{max}]$.

**Full problem (GDIP)**

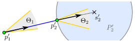



**One-side version (OS-GDIP)**

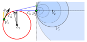



- **Optimal solution** – Closed-form expressions for (1–6) and convex optimization (7).

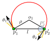
**1) S type**



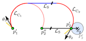
**2) CS type**



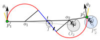
**3) $C_\psi$ type**



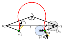
**4) CSC type**



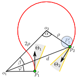
**5) $CS\overline{C}$ type**



**6) $C\overline{C}_\psi C$ type**



**7) $C\overline{C}_\psi$ type**



Average computational time

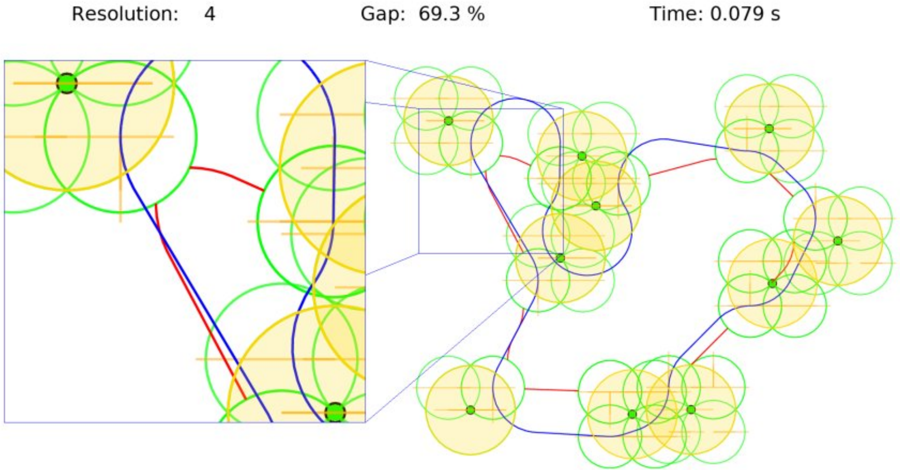
| Problem | Time [$\mu$s] | Ratio |
|---|---|---|
| Dubins maneuver | 0.4 | 1.0 |
| DIP | 1.1 | 3.0 |
| GDIP | 5.4 | 14.5 |

https://github.com/comrob/gdip

Váňa, P. and Faigl, J.: *Optimal Solution of the Generalized Dubins Interval Problem Finding the Shortest Curvature-constrained Path Through a Set of Regions*, **Autonomous Robots**, 44(7):1359-1376, 2020.

# GDIP-based Informed Sampling for the DTSPN

- Iterative refinement of the neighborhood samples and heading samples.



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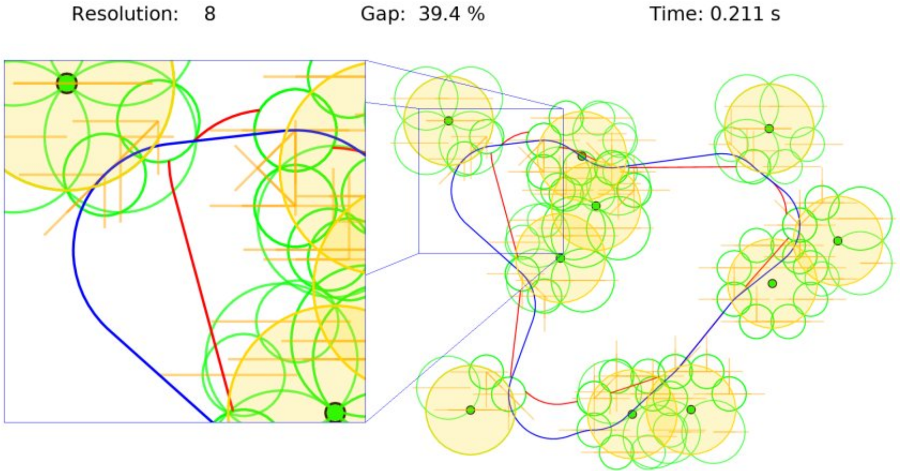

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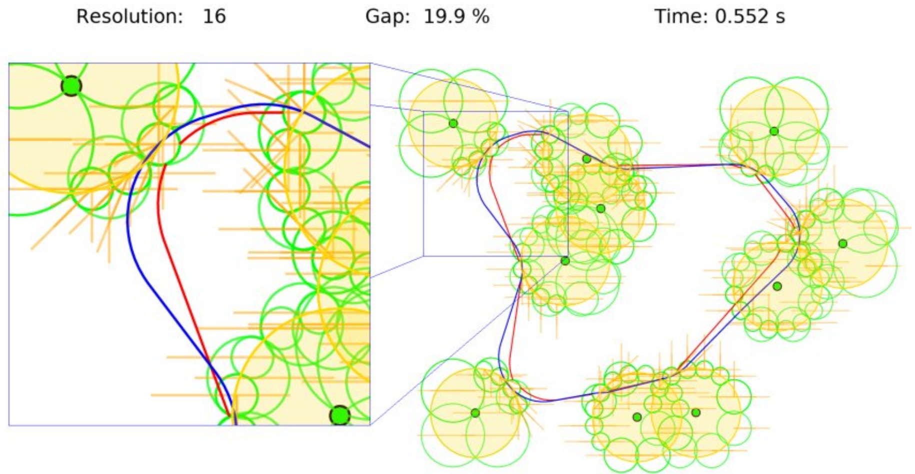

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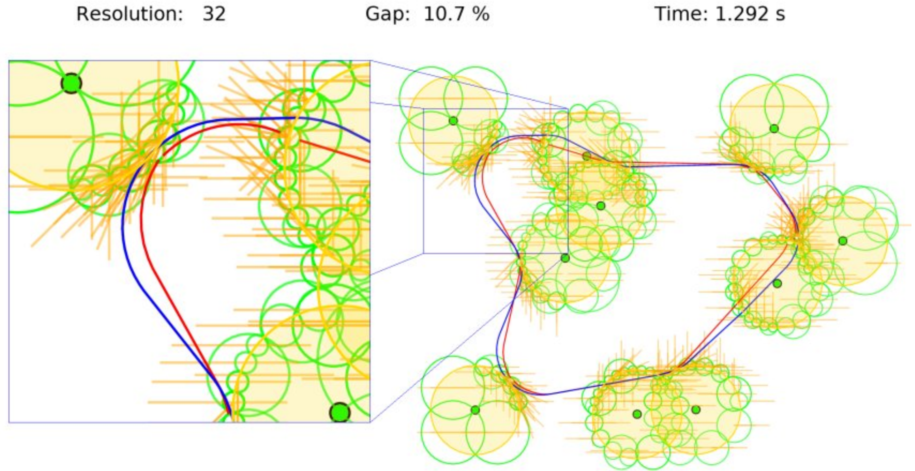

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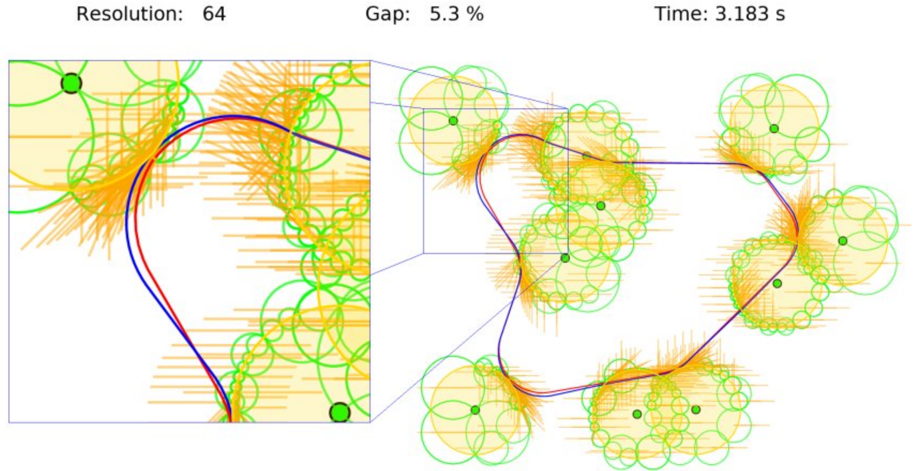

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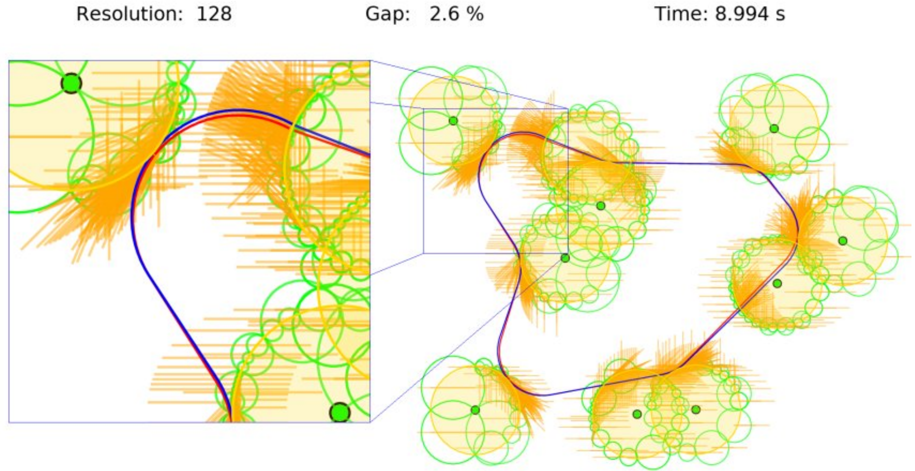

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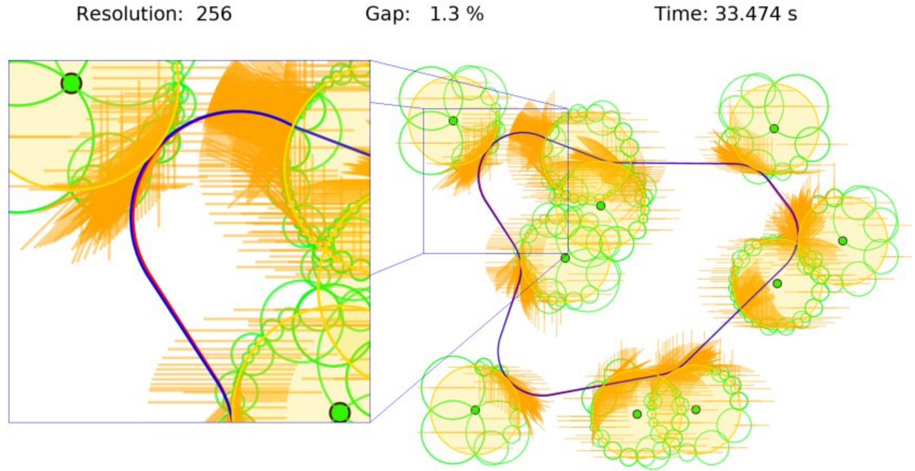
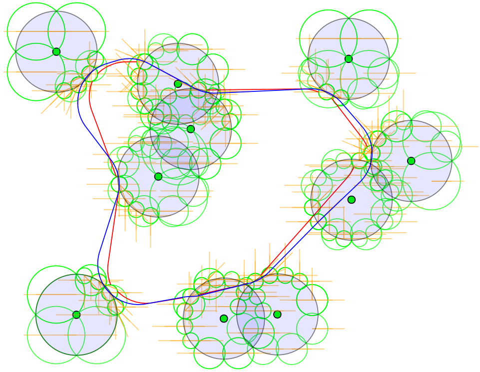
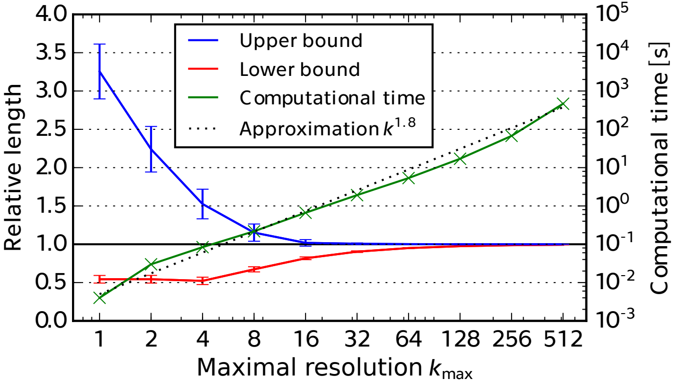


# DTSPN – Convergence to the Optimal Solution

- For a given sequence of visits to the target regions (locations).



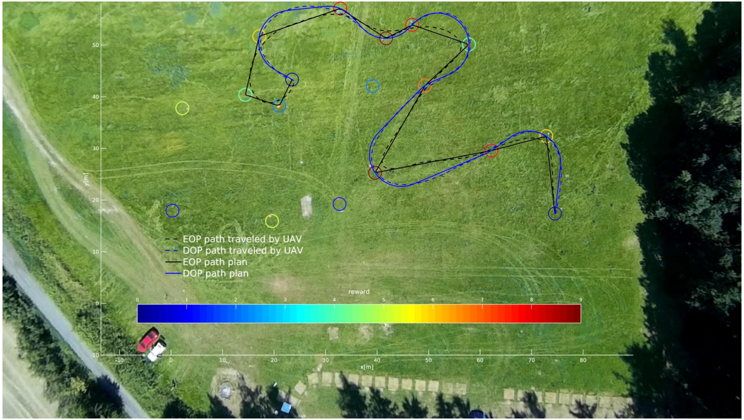



- The algorithm scales linearly with the number of locations.
- Complexity of the algorithm is approximately $\mathcal{O}(nk^{1.8})$.

https://github.com/comrob/gdip

# Data Collection / Surveillance Planning with Travel Budget

- Visit the most important targets because of limited travel budget.
- The problem can be formulated as the **Orienteering Problem** with Dubins vehicle, a.k.a. **Dubins Orienteering Problem** (**DOP**). Robert Pěnička, Jan Faigl, Petr Váňa and Martin Saska, RA-L 2017

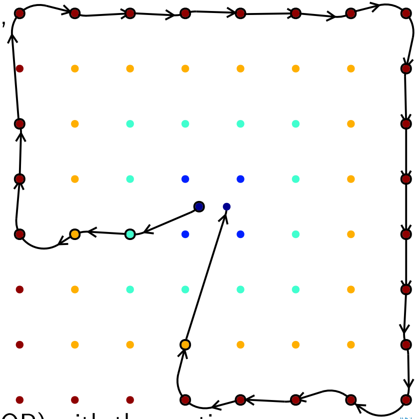



http://mrs.felk.cvut.cz/icra17dop

# Dubins Orienteering Problem

- Curvature-constrained data collection path respecting Dubins vehicle model with the minimal turning radius $\rho$ and constant forward velocity $v$.
- The path is a sequence of waypoints $q_i \in SE(2)$, $q = (s, \theta)$,
- In addition to $S_k, k, \Sigma$ (OP) determine headings $\Theta = (\theta_{\sigma_1}, \ldots, \theta_{\sigma_k})$ such that

$$\begin{aligned} \text{maximize}_{k,S_k,\Sigma} \quad & R = \sum_{i=1}^{k} r_{\sigma_i} \\ \text{subject to} \quad & \sum_{i=2}^{k} \mathcal{L}(q_{\sigma_{i-1}}, q_{\sigma_i}) \leq \mathrm{T}_{\max}, \\ & q_{\sigma_i} = (s_{\sigma_i}, \theta_{\sigma_i}), s_{\sigma_i} \in S, \theta_{\sigma_i} \in \mathbb{S}^1 \\ & s_{\sigma_1} = s_1, s_{\sigma_k} = s_n \end{aligned}.$$



The problem combines discrete combinatorial optimization (OP) with the continuous optimization for determining the vehicle headings.

# Variable Neighborhood Search (VNS)

- **Variable Neighborhood Search** (**VNS**) is a general metaheuristic for combinatorial optimization (routing problems).

  Hansen, P. and Mladenović, N. (2001): **Variable neighborhood search: Principles and applications**. European Journal of Operational Research.

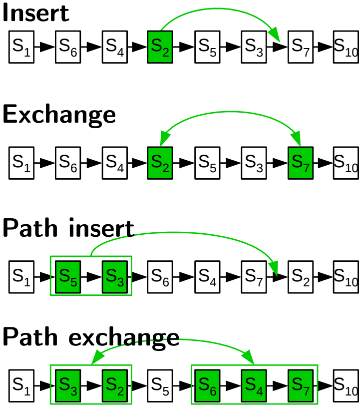
- The VNS is based on **shake** and **local search** procedures.
  - Shake procedure aims to escape from local optima by changing the solution within the neighborhoods $N_{1,\ldots,k_{max}}$. *The neighborhoods are particular operators.*
  - Local search procedure searches fully specific neighborhoods of the solution using $l_{max}$ predefined operators.

# Variable Neighborhood Search (VNS) for the DOP

- The solution is the first *k* locations of the sequence of all target locations satisfying $T_{max}$.

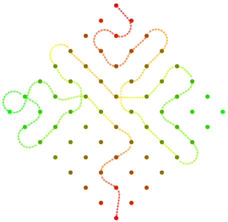
Sevkli Z., Sevilgen F.E.: *Variable Neighborhood Search for the Orienteering Problem*, **SCIS**, 2006.

- It is an improving heuristics, i.e., an initial solution has to be provided.
- A set of predefined neighborhoods are explored to find a better solution.
- **Shake** – explores the configuration space and escape from a local minima using
  - **Insert** – moves one random element;
  - **Exchange** – exchanges two random elements.
- **Local Search** – optimizes the solution using
  - **Path insert** – moves a random sub-sequence;
  - **Path exchange** – exchanges two random sub-sequences.
- **Randomized VNS** – examines only $n^2$ changes in the *Local Search* procedure in each iteration.



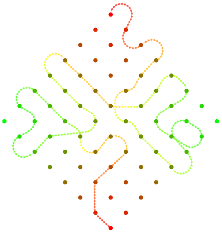
# Evolution of the VNS Solution to the DOP

Initial solution



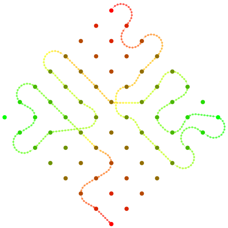
$T_{CPU} = 10.9$ s,
$\mathcal{L} = 79.6,\ R = 960$

4710*th* iteration
(4*th* improvement)



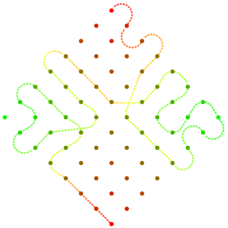
$T_{CPU} = 144.8$ s,
$\mathcal{L} = 79.7,\ R = 990$

4790*th* iteration
(12*th* improvement)



$T_{CPU} = 147.3$ s,
$\mathcal{L} = 79.3,\ R = 1008$

5560*th* iteration
(16*th* improvement)



$T_{CPU} = 170.0$ s,
$\mathcal{L} = 79.1,\ R = 1050$

# Possible Solutions of the Dubins Orienteering Problem

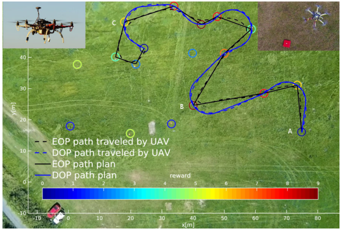
1. Solve the Euclidean OP (EOP) and then determine Dubins path.

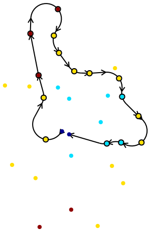
   *The final path may exceed the budget and the vehicle can miss the locations because of motion control.*

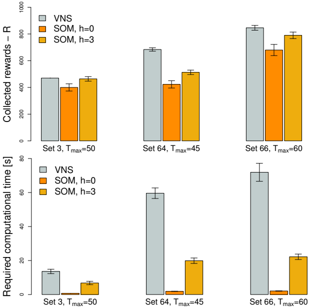
2. Directly solve the **Dubins Orienteering Problem (DOP)** such as
   - Sample possible heading values and use Variable Neighborhood Search (VNS);

     Pěnička, R., Faigl, J., Váňa, P., and Saska, M.: *Dubins Orienteering Problem*, **IEEE Robotics and Automation Letters**, 2(2):1210–1217, 2017.
   - Unsupervised learning based on Self-Organizing Maps (SOM);

     Faigl, J.: *Self-organizing map for orienteering problem with dubins vehicle*, **Advances in Self-Organizing Maps, Learning Vector Quantization, Clustering and Data Visualization**, 2017, pp. 125–132.





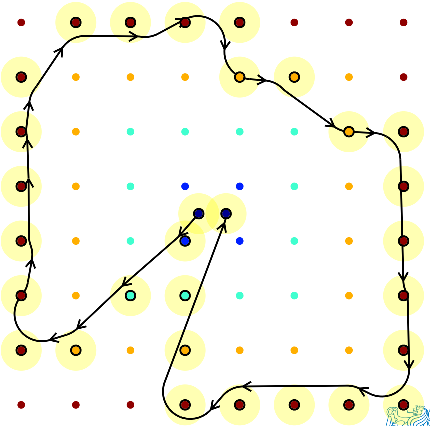

The VNS-based approach provides better solutions than the SOM-based solution, but it tends to be more demanding.

# Dubins Orienteering Problem with Neighborhoods

- Curvature-constrained path respecting Dubins vehicle model.
- Each waypoint consists of location $p \in \mathbb{R}^2$ and the heading $\theta \in \mathbb{S}^1$.

- In addition to $S_k, k, \Sigma$ determine locations $P_k = (p_{\sigma_1}, \ldots, p_{\sigma_k})$ and headings $\Theta = (\theta_{\sigma_1}, \ldots, \theta_{\sigma_k})$ such that

$$
\begin{aligned}
&\text{maximize}_{k,S_k,\Sigma} && R = \sum_{i=1}^{k} r_{\sigma_i} \\
&\text{subject to} && \sum_{i=2}^{k} \mathcal{L}(q_{\sigma_{i-1}}, q_{\sigma_i}) \leq T_{\max}, \\
& && q_{\sigma_i} = (p_{\sigma_i}, \theta_{\sigma_i}), p_{\sigma_i} \in \mathbb{R}^2, \theta_{\sigma_i} \in \mathcal{S}^1 \\
& && ||p_{\sigma_i}, s_{\sigma_i}|| \leq \delta, s_{\sigma_i} \in S_k \\
& && p_{\sigma_1} = s_1, p_{\sigma_k} = s_n
\end{aligned}
.
$$



We need to solve the continuous optimization for determining the vehicle heading at each waypoint and the waypoint locations $P_k = \{p_{\sigma_1}, \ldots, p_{\sigma_k}\}$, $p_{\sigma_i} \in \mathbb{R}^2$.

# Variable Neighborhoods Search (VNS) for the DOPN

**Algorithm 3:** VNS based method for the DOPN

**Input** : $S$ – Set of the target locations
**Input** : $\mathrm{T}_{\mathrm{max}}$ – Maximal allowed budget
**Input** : $o$ – Initial number of position waypoints for each target
**Input** : $m$ – Initial number of heading values for each waypoints
**Input** : $r_i$ – Local waypoint improvement ratio
**Input** : $l_{max}$ – Maximal neighborhood number
**Output**: $P$ – Found data collecting path

$S_r \leftarrow$ getReachableLocations($S$, $\mathrm{T}_{\mathrm{max}}$)
$P \leftarrow$ createInitialPath($S_r$, $\mathrm{T}_{\mathrm{max}}$) // greedy
**while** *Stopping condition is not met* **do**
    $l \leftarrow 1$
    **while** $l \leq l_{max}$ **do**
        $P' \leftarrow$ shake($P$, $l$)
        $P'' \leftarrow$ localSearch($P'$, $l$, $r_i$)
        **if** $\mathcal{L}_d(P'') \leq T_{max}$ **and**
        $[[R(P'') > R(P)]$ **or** $[R(P'') == (P)$ **and** $\mathcal{L}_d(P'') < \mathcal{L}_d(P)\mathcal{L}_d(P'')]]$
        **then**
            $P \leftarrow P''$
            $l \leftarrow 1$
        **else**
            $l \leftarrow l + 1$
        **end**
    **end**
**end**

The particular $l$ for the individual operators of the **shake** procedure are:

- **Waypoint Shake** ($l = 1$);
- **Path Move** ($l = 2$);
- **Path Exchange** ($l = 3$).

The **local search** procedure consists of three operators and the particular $l$ for the individual operators of the **local search** procedure are:

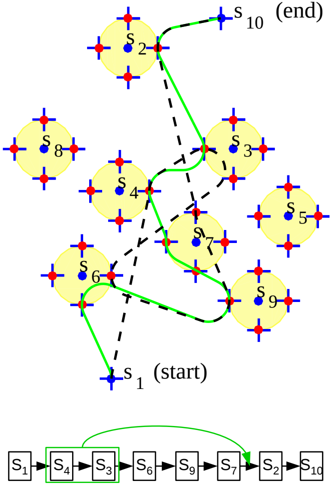
- **Waypoint Improvement** ($l = 1$);
- **One Point Move** ($l = 2$);
- **One Point Exchange** ($l = 3$).

Pěnička, R., Faigl, J., Saska, M., and Váňa, P.: *Data collection planning with non-zero sensing distance for a budget and curvature constrained unmanned aerial vehicle*, **Autonomous Robots**, 43(8):1937–1956, 2019.

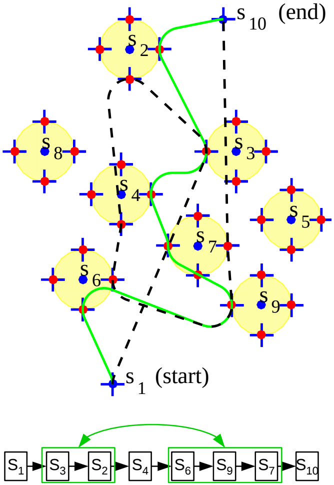
Pěnička, R., Faigl, J., Váňa, P., and Saska, M.: *Dubins Orienteering Problem with Neighborhoods*, **International Conference on Unmanned Aircraft Systems (ICUAS)**, 2017, pp. 1555–1562.

# VNS for DOPN – Example of the Shake Operators

## Path Move




## Path Exchange

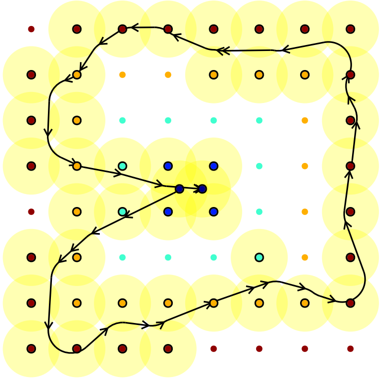


# Comparision of the DOPN Solvers

- VNS-based DOPN solver with $s = 16$ sampled waypoint locations per sensor and $h = 16$ heading samples per waypoint location. Pěnička, Faigl, et al. (ICUAS 2017)
- SOM-based DOPN solver with $h = 3$. Faigl, Pěnička (IROS 2017)
- Aggregate results using average relative percentage error (ARPE) and relative percentage error (RPE) to the reference (best found) solution.

| Problem set | VNS-based | | SOM-based ($h = 3$) | | |
|---|---|---|---|---|---|
| | $\overline{\text{ARPE}}$ | $T_{cpu}$* [s] | $\overline{\text{RPE}}$ | $\overline{\text{ARPE}}$ | $T_{cpu}$ [s] |
| Set 3, $\delta = 0.0$ | 1.0 | 1,178.9 | 3.6 | 7.4 | 7.0 |
| Set 3, $\delta = 0.5$ | 0.9 | 13,273.3 | 6.6 | 10.6 | 7.9 |
| Set 3, $\delta = 1.0$ | 0.5 | 13,304.4 | 5.5 | 9.2 | 8.3 |
| Set 64, $\delta = 0.0$ | 1.9 | 5,272.2 | 17.4 | 23.8 | 17.9 |
| Set 64, $\delta = 0.5$ | 2.8 | 13,595.6 | 18.7 | 24.2 | 20.2 |
| Set 64, $\delta = 1.0$ | 1.3 | 13,792.3 | 9.9 | 15.2 | 22.2 |
| Set 66, $\delta = 0.0$ | 1.5 | 6,546.6 | 3.6 | 9.1 | 22.9 |
| Set 66, $\delta = 0.5$ | 1.4 | 13,650.1 | 6.7 | 11.8 | 25.5 |
| Set 66, $\delta = 1.0$ | 3.2 | 13,824.5 | 16.1 | 21.3 | 26.7 |

*The results have been obtained with a grid Xeon CPUs running at 2.2 GHz to 3.4 GHz due to computational requirements.

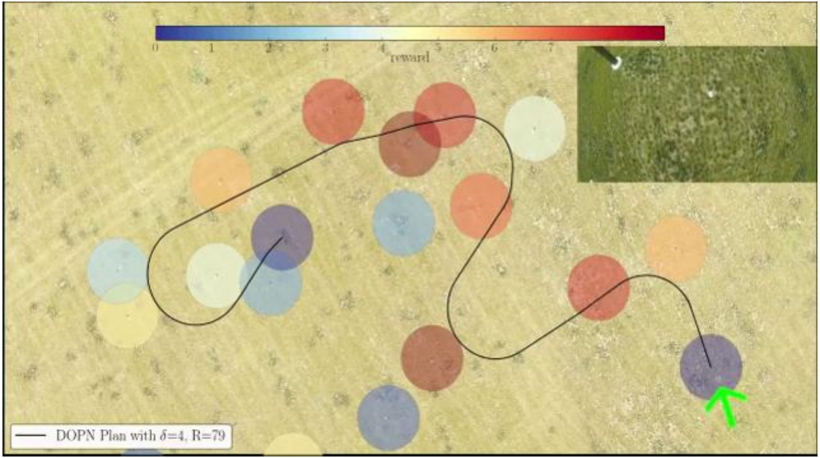


$\rho = 1.0$, $\delta = 1.25$, $R = 1185$

# DOPN – Example of Solution and Practical Deployment

- VNS-based solution of the DOPN.

Pěnička, R., Faigl, J., Váňa, P., and Saska, M.: *Dubins Orienteering Problem with Neighborhoods*, International Conference on Unmanned Aircraft Systems (ICUAS), 2017, pp. 1555–1562.



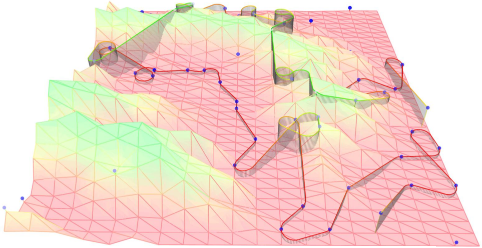

http://mrs.felk.cvut.cz/jint17dopn

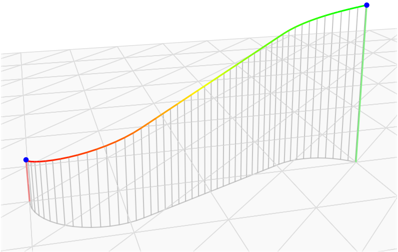
# 3D Data Collection Planning with Dubins Airplane Model

- **Dubins Airplane model** describes the vehicle state $q = (p, \theta, \psi)$, $p \in \mathbb{R}^3$ and $\theta, \psi \in \mathbb{S}^1$ as

$$\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{z} \\ \dot{\theta} \end{bmatrix} = v \begin{bmatrix} \cos\theta \cdot \cos\psi \\ \sin\theta \cdot \cos\psi \\ \sin\psi \\ u_\theta \cdot \rho^{-1} \end{bmatrix}.$$

H. Chitsaz and S. M. LaValle: *Time-optimal paths for a Dubins airplane*, **IEEE Conference on Decision and Control**, 2007, pp. 2379–2384.

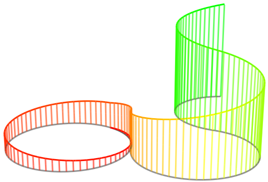




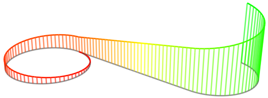
- Constant forward velocity $v$, the minimal turning radius $\rho$, and limited pitch angle, i.e., $\psi \in [\psi_{min}, \psi_{max}]$.
- $u_\theta$ controls the vehicle heading, $|u_\theta| \leq 1$, and $v$ is the forward velocity.
- Generation of the 3D trajectory is based on the 2D Dubins maneuver.
- If altitude changes are too high, additional helix segments are inserted.

# The DTSPN in 3D

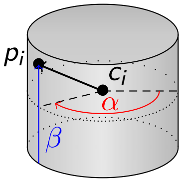
- Using the same principles as for the DTSPN in 2D, we can generalize the approaches for 3D planning using the Dubins Airplane model instead of simple Dubins vehicle.
- The regions can be generalized to 3D and the problem can be addressed by decoupled or sampling-based approaches, i.e., using GATSP formulation.
- In the case of LIO, we need a parametrization of the possible waypoint location, such as point on the object boundary.

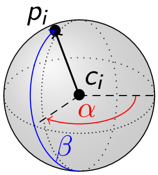


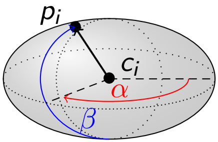
CCC maneuver



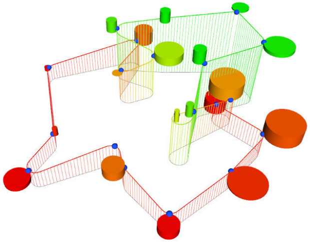
CSC maneuver



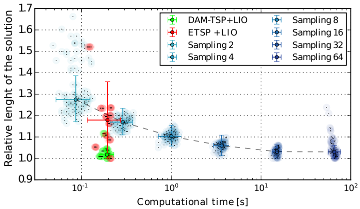
# Solutions of the 3D-DTSPN

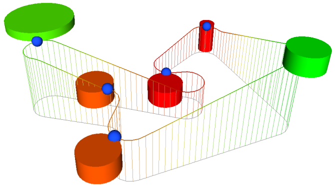


**Algorithm 4:** LIO-based Solver for 3D-DTSPN

**Data**: Regions $\mathcal{R}$
**Result**: Solution represented by $\mathcal{Q}$ and $\Sigma$
$\Sigma \leftarrow$ getInitialSequence($\mathcal{R}$);
$\mathcal{Q} \leftarrow$ getInitialSolution($\mathcal{R}, \Sigma$);
**while** *terminal condition* **do**
  $\mathcal{Q} \leftarrow$ optimizeHeadings($\mathcal{Q}, \mathcal{R}, \Sigma$);
  $\mathcal{Q} \leftarrow$ optimizeAlpha($\mathcal{Q}, \mathcal{R}, \Sigma$);
  $\mathcal{Q} \leftarrow$ optimizeBeta($\mathcal{Q}, \mathcal{R}, \Sigma$);
**end**
**return** $\mathcal{Q}, \Sigma$;

- Solutions based on LIO (ETSP+LIO), TSP with the travel cost according to Dubins Airplane Model (DAM-TSP+LIO), and sampling-based approach with transformation of the GTSP to the ATSP solved by LKH.



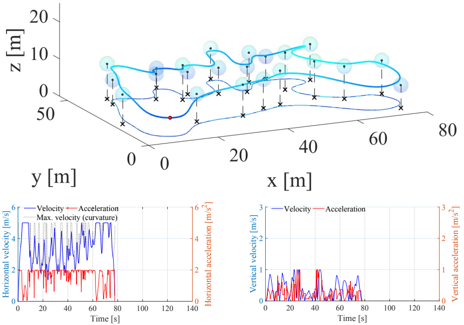



Váňa, P., Faigl, J., Sláma, J., and Pěnička, R.: *Data collection planning with Dubins airplane model and limited travel budget* **European Conference on Mobile Robots (ECMR), 2017.**

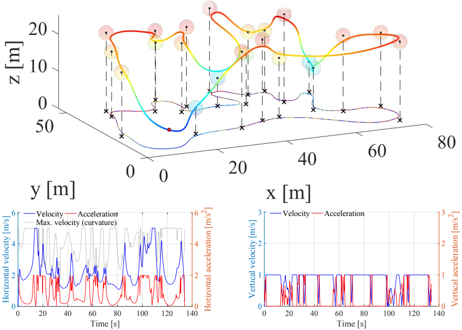
# 3D Surveillance Planning

- Parametrization of smooth 3D multi-goal trajectory as a sequence of Bézier curves.
- Unsupervised learning for the TSPN can be generalized for such trajectories.
- During the solution of the sequencing part of the problem, we can determine a velocity profile along the curve and compute the so-called *Travel Time Estimation* (TTE).
- Bézier curves better fit the limits of the multi-rotor UAVs that are limited by the maximal accelerations and velocities rather than minimal turning radius as for Dubins vehicle.

**Low altitude differences**




**High altitude differences**

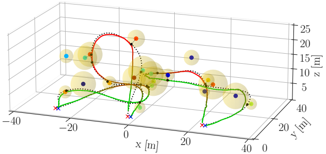


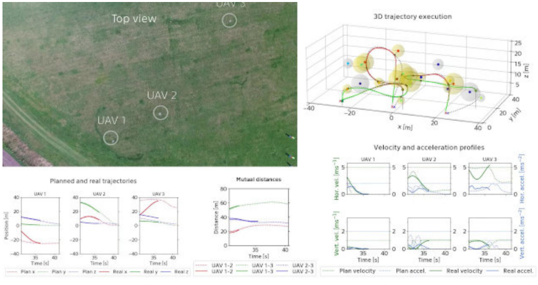

Faigl, J. and Váňa, P.: *Surveillance Planning With Bézier Curves*, **IEEE Robotics and Automation Letters**, 3(2):750–757, 2018.

- Low altitude differences saturate horizontal velocity while high altitudes changes saturate vertical velocity.

# Multi-Vehicle Multi-Goal Planning with Limited Travel Budget – Curvature-Constrained Team Orienteering Problem (with Neighborhoods)

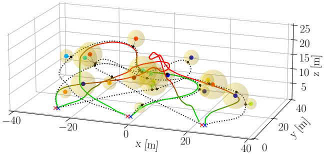
- Operational time of multi-rotor aerial vehicles is limited and only a subset of locations can be visited.
- Planning multi-goal trajectories as a sequence of Bézier curves.




Orienteering Problem with Bézier curves: Non-crossing field experiment with 3 multi-rotor drones

- Targets are missed in a case of colliding trajectories, because of local collision avoidance and optimal trajectory following.
- **There is a practical need to include coordination in multi-vehicle multi-goal trajectory planning**.




Faigl, J., Váňa, P., and Pěnička, R.: *Multi-Vehicle Close Enough Orienteering Problem with Bézier Curves for Multi-Rotor Aerial Vehicles*. **ICRA** 2019, pp. 3039–3044.

# Summary of the Lecture

# Summary

- Data collection planning with curvature-constrained paths/trajectories
  - The **Traveling Salesman Problem** (**TSP**) and **Orienteering Problem** (**OP**) with Dubins Vehicle, i.e., **DTSP** and **DOP**.
  - It is a combination of the combinatorial and continuous (determining optimal headings) optimization.
  - The continuous part can be solved using **Dubins Touring Problem** (**DTP**).
  - Using a solution of the **Dubins Interval Problem** (**DIP**) we can establish **tight lower bound** of the DTP and DTSP with a particular sequence of visits.
  - The problems can be further extended to **DTSP with Neighborhoods** (**DTSPN**) and **OP with Neighborhoods** (**DOPN**), and its **Close Enough** variants.
- The key ideas of the presented problems and approaches are as follows.
  - Consider proper assumptions that fits the original problem being solved.
    - Suitability of the vehicle model, requirements on the solution quality, and benefit of optimal or computationally demanding solutions.
  - Employing lower bound based on "a bit different problem" such as the **DIP** and **GDIP**, to find high quality solutions, even using decoupled approaches.
  - Challenging problems with continuous optimization can be addressed by decoupled and sampling-based approaches.
    - Be aware that the optimal solutions found for discretized problems, e.g., using ILP or combinatorial solvers, are not optimal solutions of the original (continuous) problem!

# Topics Discussed

- Dubins vehicles and planning – Dubins maneuvers
- **Dubins Interval Problem (DIP)** (lower bound estimation to the DTP, DTSP)
- **Dubins Touring Problem (DTP)**
- Dubins Traveling Salesman Problem (DTSP) and Dubins Traveling Salesman with Neighborhoods (DTSPN)
  - Decoupled approaches – Alternating Algorithm
  - Sampling-based approaches – GATSP
- **Generalized Dubins Interval Problem (GDIP)** (lower bound estimation to the DTSPN)
- Dubins Orienteering Problem (OP) and Dubins Orienteering Problem with Neighborhoods (DOPN)
- Data collection and surveillance planning in 3D

- Next: Sampling-based motion planning