

Curvature-Constrained Data Collection Planning
Dubins Traveling Salesman Problem with Neighborhoods (DTSPN)
and
Dubins Orienteering Problem with Neighborhoods (DOPN)

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Lecture 07

B4M36UIR – Artificial Intelligence in Robotics



Overview of the Lecture

- Part 1 – Curvature-Constrained Data Collection Planning
 - Dubins Vehicle and Dubins Planning
 - Dubins Touring Problem (DTP)
 - Dubins Traveling Salesman Problem
 - Dubins Traveling Salesman Problem with Neighborhoods
 - Dubins Orienteering Problem
 - Dubins Orienteering Problem with Neighborhoods
 - Planning in 3D – Examples and Motivations

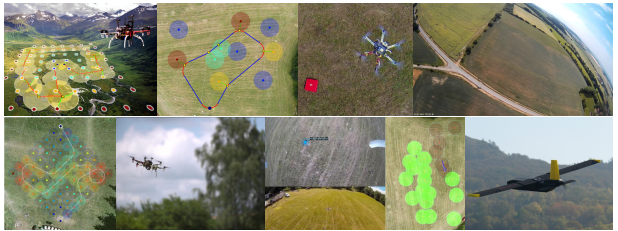


Part I
Part 1 – Curvature-Constrained Data Collection Planning



Motivation – Surveillance Missions with Aerial Vehicles

- Provide **curvature-constrained** path to collect the most valuable measurements with shortest possible path/time or under limited travel budget.



- Formulated as routing problems with Dubins vehicle
 - Dubins Traveling Salesman Problem with Neighborhoods
 - Dubins Orienteering Problem with Neighborhoods



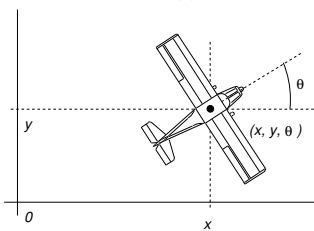
Dubins Vehicle

- Non-holonomic vehicle such as car-like or aircraft can be modeled as the Dubins vehicle:
 - Constant forward velocity;
 - Limited minimal turning radius ρ ;
 - Vehicle state is represented by a triplet $q = (x, y, \theta)$, where
 - Position is $(x, y) \in \mathbb{R}^2$, vehicle heading is $\theta \in \mathbb{S}^1$, and thus $q \in SE(2)$.

The vehicle motion can be described by the equation

$$\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\theta} \end{bmatrix} = v \begin{bmatrix} \cos \theta \\ \sin \theta \\ \frac{u}{\rho} \end{bmatrix}, \quad |u| \leq 1,$$

where u is the control input.



Optimal Maneuvers for Dubins Vehicle

- For two states $q_1 \in SE(2)$ and $q_2 \in SE(2)$ in the environment **without obstacles** $\mathcal{W} = \mathbb{R}^2$, the optimal path connecting q_1 with q_2 can be characterized as one of two main types
 - CCC type: LRL, RLR;
 - CSC type: LSL, LSR, RSL, RSR;
 where S – straight line arc, C – circular arc oriented to left (L) or right (R).
L. E. Dubins (1957) – American Journal of Mathematics
- The optimal paths are called **Dubins maneuvers**.
 - Constant velocity: $v(t) = v$ and turning radius ρ .
 - Six types of trajectories connecting any configuration in $SE(2)$. *(Without obstacles)*
 - The control u is according to C and S type one of three possible values $u \in \{-1, 0, 1\}$.



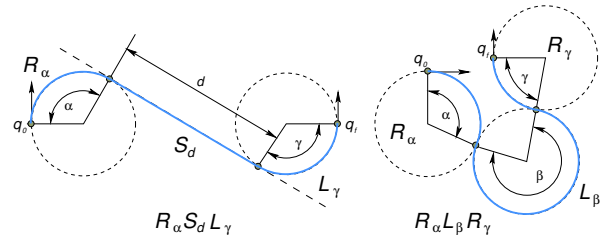
Parametrization of Dubins Maneuvers

- Parametrization of each trajectory phase:

$$\{L_\alpha R_\beta L_\gamma, R_\alpha L_\beta R_\gamma, L_\alpha S_d L_\gamma, L_\alpha S_d R_\gamma, R_\alpha S_d L_\gamma, R_\alpha S_d R_\gamma\}$$

for $\alpha \in [0, 2\pi)$, $\beta \in (\pi, 2\pi)$, $d \geq 0$.

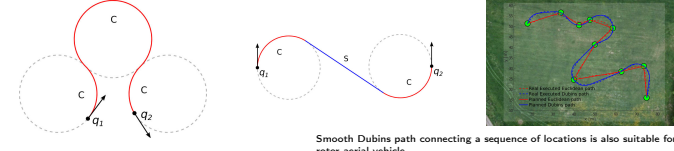
Notice the prescribed orientation at q_0 and q_f .



Multi-goal Dubins Path

- Minimal turning radius ρ and constant forward velocity v .
- State of the Dubins vehicle is $q = (x, y, \theta)$, $q \in SE(2)$, $(x, y) \in \mathbb{R}^2$ and $\theta \in \mathbb{S}^1$.

$$\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\theta} \end{bmatrix} = v \begin{bmatrix} \cos \theta \\ \sin \theta \\ \frac{u}{\rho} \end{bmatrix}$$

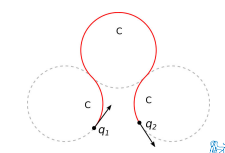
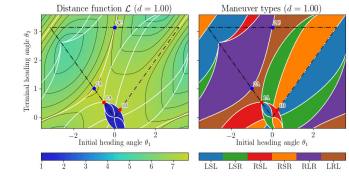
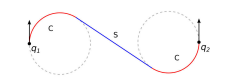
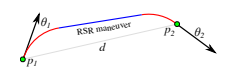


- Optimal path connecting $q_1 \in SE(2)$ and $q_2 \in SE(2)$ consists only of straight line arcs and arcs with the maximal curvature, i.e., two types of maneuvers CCC and CSC and the solution can be found analytically. *(Dubins, 1957)*
- In **multi-goal Dubins path planning**, we need to solve the underlying TSP.



Difficulty of Dubins Vehicle in the Solution of the TSP

- For the minimal turning radius ρ , the **optimal path** connecting $q_1 \in SE(2)$ and $q_2 \in SE(2)$ can be found analytically. *L. E. Dubins (1957) – American Journal of Mathematics*
- Two types of optimal Dubins maneuvers: CSC and CCC.
- The length of the optimal maneuver \mathcal{L} has a closed-form solution.
 - It is **piecewise-continuous function**; *Can be computed in less than 0.5 μ s*
 - (continuous for $\|(\mathbf{p}_1, \mathbf{p}_2)\| > 4\rho$).



Dubins Vehicle and Dubins Planning DTP DTSP DTSPN DOP DOPN Planning in 3D

Dubins Traveling Salesman Problem (DTSP)

- Determine (closed) shortest Dubins path visiting each $p_i \in \mathbb{R}^2$ of the given set of n locations $P = \{p_1, \dots, p_n\}$.
- Permutation $\Sigma = (\sigma_1, \dots, \sigma_n)$ of visits (sequencing). *Combinatorial optimization*
- Headings $\Theta = \{\theta_{\sigma_1}, \theta_{\sigma_2}, \dots, \theta_{\sigma_n}\}$, $\theta_i \in [0, 2\pi)$, for $p_{\sigma_i} \in P$. *Continuous optimization*
- DTSP is an optimization problem over all possible sequences Σ and headings Θ at the states $(q_{\sigma_1}, q_{\sigma_2}, \dots, q_{\sigma_n})$ such that $q_{\sigma_i} = (p_{\sigma_i}, \theta_{\sigma_i})$, $p_{\sigma_i} \in P$

$$\text{minimize}_{\Sigma, \Theta} \sum_{i=1}^{n-1} \mathcal{L}(q_{\sigma_i}, q_{\sigma_{i+1}}) + \mathcal{L}(q_{\sigma_n}, q_{\sigma_1})$$

where

$$\text{subject to } q_i = (p_i, \theta_i) \quad i = 1, \dots, n,$$

$\mathcal{L}(q_i, q_j)$ is the length of Dubins path between q_i and q_j .

The continuous domain of the heading angles is similar to the regions in the TSPN-like problem formulations.

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Challenges of the Dubins Traveling Salesman Problem

- The key difficulty of the DTSP is that the path length mutually depends on
 - Order of the visits to the locations;
 - Headings at the target locations.
- We need the sequence to determine headings, but headings may influence the sequence.
- The Dubins TSP is **sequence dependent problem**.
- Two fundamental approaches can be found in literature.

- Decoupled** approach based on a given sequence of the locations, e.g., found by a solution of the Euclidean TSP.
- Sampling-based** approach with sampling of the headings at the locations into discrete sets of values and considering the problem as the variant of the **Generalized TSP**.

Besides, further approaches are

- Genetic and memetic techniques (evolutionary algorithms);
- Unsupervised learning based approaches.

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Existing Approaches to the DTSP(N)

- Heuristic (decoupled & evolutionary) approaches
 - Savla et al., 2005
 - Ma and Castanon, 2006
 - Macharet et al., 2011
 - Macharet et al., 2012
 - Ny et al., 2012
 - Yu and Hang, 2012
 - Macharet et al., 2013
 - Zhant et al., 2014
 - Macharet and Campost, 2014
 - Vaňa and Faigl, 2015
 - Isaiah and Shima, 2015
 - ...
- Sampling-based approaches
 - Obermeyer, 2009
 - Oberlin et al., 2010
 - Macharet et al., 2016
- Convex optimization
 - (Only if the locations are far enough)
 - Goac et al., 2013
- Lower bound for the DTSP
 - Dubins Interval Problem (DIP)
 - Manyam et al., 2016
 - DIP-based inform sampling
 - Vaňa and Faigl, 2017
- Lower bound for the DTSPN
 - Using Generalized DIP (GDIP)
 - Vaňa and Faigl, 2018, 2020

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Planning with Dubins Vehicle – Summary

- The optimal path connecting two configurations can be found analytically. *E.g., for UAVs that usually operates in environment without obstacles.*
- The Dubins maneuvers can also be used in randomized-sampling based motion planners, such as RRT, in the control based sampling.
- Dubins vehicle model can be considered in the multi-goal path planning such as surveillance, inspection or monitoring missions to periodically visits given target locations (areas).
- Dubins Touring Problem (DTP)**
 - Given a sequence of locations, what is the shortest path visting the locations, i.e., what are the headings of the vehicle at the locations.
- Dubins Traveling Salesman Problem (DTSP)**
 - Given a set of locations, what is the shortest Dubins path that visits each location exactly once and returns to the origin location.
- Dubins Orienteering Problem (DOP)**
 - Given a set of locations, each with associated reward, what is the Dubins path visting the most rewarding locations and not exceeding the given travel budget.

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Dubins Touring Problem – DTP

- For a sequence of the n waypoint locations $P = (p_1, \dots, p_n)$, $p_i \in \mathbb{R}^2$, the **Dubins Touring Problem (DTP)** stands to determine the **optimal headings** $T = \{\theta_1, \dots, \theta_n\}$ at the waypoints q_i such that

$$\text{minimize } \tau \quad \mathcal{L}(T, P) = \sum_{i=1}^{n-1} \mathcal{L}(q_i, q_{i+1}) + \mathcal{L}(q_n, q_1)$$

$$\text{subject to } q_i = (p_i, \theta_i), \quad \theta_i \in [0, 2\pi), \quad p_i \in P,$$

where $\mathcal{L}(q_i, q_j)$ is the length of the Dubins maneuver connecting q_i with q_j .

- The DTP is a **continuous optimization problem**.
- The term $\mathcal{L}(q_n, q_1)$ is for possibly closed tour that can be for example requested in the TSP with Dubins vehicle, a.k.a. DTSP.
- On the other, the DTP can also be utilized for open paths such as solutions of the OP with Dubins vehicle.
- In some cases, it may be suitable to relax the heading at the first/last locations in finding closed tours (i.e., solving DTSP).

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Sampling-based Solution of the DTP

- For a closed sequence of the waypoint locations $P = (p_1, \dots, p_n)$.
- We can sample possible heading values at each location i into a discrete set of k headings, i.e., $\Theta^i = \{\theta_1^i, \dots, \theta_k^i\}$ and create a graph of all possible Dubins maneuvers.

For open sequence we do not need to evaluate all possible initial headings, and the complexity is $O(nk^2)$.

- The problem is to determine the most suitable heading samples.

- For a set of heading samples, the optimal solution can be found by a forward search of the graph in $O(nk^3)$.

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Example of Heading Sampling – Uniform vs. Informed

Uniform sampling

$N = 224$, $T_{CPU} = 128$ ms
 $\mathcal{L} = 19.8$, $\mathcal{L}_U = 13.8$

Informed sampling

$N = 128$, $T_{CPU} = 76$ ms
 $\mathcal{L} = 14.4$, $\mathcal{L}_U = 14.2$

- N is the total number of samples, i.e., 32 samples per waypoint for uniform sampling.
- \mathcal{L} is the length of the tour (blue) and \mathcal{L}_U is the lower bound (red) determined as a solution of the **Dubins Interval Problem (DIP)**.

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Dubins Interval Problem (DIP)

- Dubins Interval Problem (DIP)** is a generalization of Dubins maneuvers to the shortest path connecting two points p_i and p_j .
- In the DIP, the leaving interval Θ_i at p_i and the arrival interval Θ_j at p_j are consider (not a single heading value).
- The optimal solution can be found analytically. *Manyam et al. (2015)*

Notice, for $\Theta_i = \Theta_j = (0, 2\pi)$ the optimal maneuver for DIP is a straight line segment.

- Solution of the DIP is a tight lower bound for the DTP.
- Solution of the DIP is not a feasible solution of the DTP.

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Lower Bound of the DTP

- For a discrete set of heading intervals $\mathcal{H} = \{H_1, \dots, H_n\}$, where $H_i = \{\Theta_1^i, \Theta_2^i, \dots, \Theta_n^i\}$, a similar graph as for the DTP can be constructed with the edge cost determined by the solution of the associated DIP.

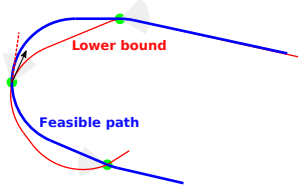
- The forward search of the graph with dense samples provides a **tight lower bound of the DTP**. *Manyam and Rathinam, 2015*

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Lower Bound and Feasible Solution of the DTP

- The arrival and departure angles may not be the same.

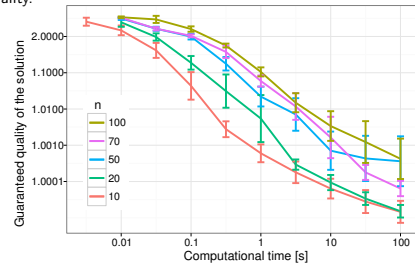
The lower bound solution is not a feasible solution of the DTP.



- DTP solution – use any particular heading of each interval in the lower bound solution.

The DIP-based Sampling of Headings in the DTP

- Using heading intervals for a sequence of waypoints and a solution of the DIP, we can determine **lower bound** of the DTP using the forward search graph as for the DTP.
- The ratio between the lower bound and feasible solution of the DTP provides an estimation of the solution quality.



Iteratively-Refined Informed Sampling (IRIS) of Headings in the Solution of the DTP

- Iterative refinement of the heading intervals \mathcal{H} up to the angular resolution ϵ_{req} .
- The angular resolution is gradually decreased for the most promising intervals.
- refineDTP – divide the intervals of the lower bound solution.
- solveDTP – solve DTP using the heading from the refined intervals.

```

Algorithm 1: Iterative Informed Sampling-based DTP Algorithm
Input: P – Target locations to be visited
Input:  $\epsilon_{req}$  – Requested angular resolution
Input:  $\alpha_{req}$  – Requested quality of the solution
Output: T – A tour visiting the targets
 $\epsilon \leftarrow 2\pi$  // initial angular resolution;
 $\mathcal{H} \leftarrow \text{createIntervals}(P, \epsilon)$  // initial intervals;
 $\mathcal{L}_L \leftarrow 0$  // init lower bound;
 $\mathcal{L}_U \leftarrow \infty$  // init upper bound;
while  $\epsilon > \epsilon_{req}$  and  $\mathcal{L}_U/\mathcal{L}_L > \alpha_{req}$  do
     $\epsilon \leftarrow \epsilon/2$ ;
     $(\mathcal{H}, \mathcal{L}_L) \leftarrow \text{refineDTP}(P, \epsilon, \mathcal{H})$ ;
     $(T, \mathcal{L}_U) \leftarrow \text{solveDTP}(P, \mathcal{H})$ ;
end
return T;
    
```

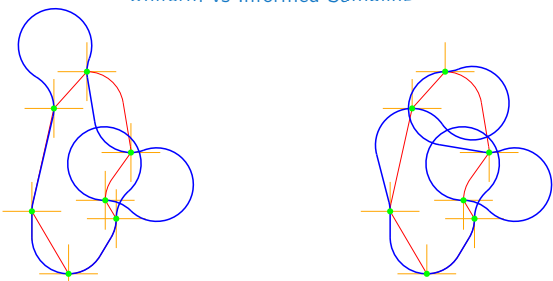
- It simultaneously provides **feasible and lower bound** solutions of the DTP.

The lower bound provides a tight estimation of the solution quality.

- The first solution is provided very quickly – **any-time algorithm**.

Faigl, J., Váňa, P., Saska, M., Báča, T., and Spurný, V.: On solution of the Dubins touring problem, ECMR, 2017.

Uniform vs Informed Sampling



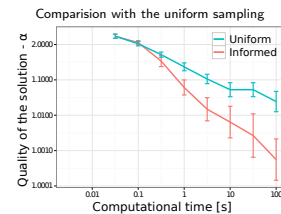
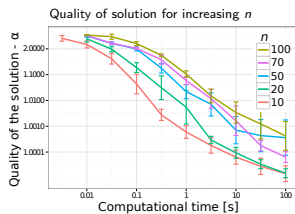
$\epsilon = 2\pi/4, N = 28, T_{CPU} = 8$ ms
 $\mathcal{L} = 27.9, \mathcal{L}_U = 13.2$

$\epsilon = 2\pi/4, N = 21, T_{CPU} = 8$ ms
 $\mathcal{L} = 29.9, \mathcal{L}_U = 13.2$

Results and Comparison with Uniform Sampling

- Random instances of the DTSP with a sequence of visits to the targets determined as a solution of the Euclidean TSP.
- The waypoints placed in a squared bounding box with the side $s = (\rho\sqrt{n})/d$ for the $\rho = 1$ and density $d = 0.5$.

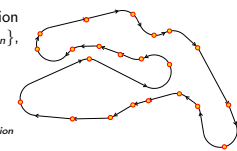
It matters on the density of targets!



- The informed sampling-based approach provides solutions up to 0.01% from the optima.
- A solution of the DTP is a fundamental building block for **routing problems with Dubins vehicle**.

Dubins Traveling Salesman Problem (DTSP)

- Determine a closed shortest Dubins path visiting each location $p_i \in P$ of the given set of n locations $P = \{p_1, \dots, p_n\}$, $p_i \in \mathbb{R}^2$.
- Permutation $\Sigma = (\sigma_1, \dots, \sigma_n)$ of visits. *Sequencing part of the problem*
- Headings $\Theta = \{\theta_{\sigma_1}, \theta_{\sigma_2}, \dots, \theta_{\sigma_n}\}$ for $p_{\sigma_i} \in P$. *Continuous optimization*



- DTSP is an optimization problem over all possible **permutations** Σ and **headings** Θ in the states $(q_{\sigma_1}, q_{\sigma_2}, \dots, q_{\sigma_n})$ such that $q_{\sigma_i} = (p_{\sigma_i}, \theta_{\sigma_i})$

$$\begin{aligned} & \text{minimize}_{\Sigma, \Theta} \sum_{i=1}^{n-1} \mathcal{L}(q_{\sigma_i}, q_{\sigma_{i+1}}) + \mathcal{L}(q_{\sigma_n}, q_{\sigma_1}) \\ & \text{subject to } q_i = (p_i, \theta_i) \quad i = 1, \dots, n, \end{aligned}$$

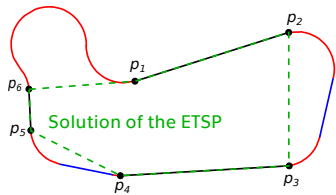
where $\mathcal{L}(q_{\sigma_i}, q_{\sigma_j})$ is the length of Dubins path between q_{σ_i} and q_{σ_j} .

Decoupled Solution of the DTSP – Alternating Algorithm

Alternating Algorithm (AA) provides a solution of the DTSP for an **even** number of targets n .

Savla, K., Frazzoli, E., Bullo, F.: On the point-to-point and traveling salesman problems for Dubins' vehicle, IEEE American Control Conference, 2005.

- Solve the related Euclidean TSP. *Relaxed motion constraints*
- Establish headings for even edges using straight line segments.
- Determine optimal maneuvers for odd edges using the analytical form for Dubins maneuvers. *Headings are known.*

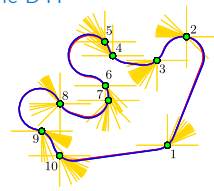
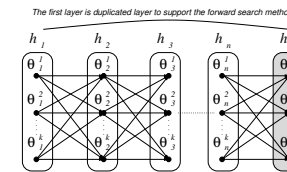


Courtesy of P. Váňa

DTSP with the Given Sequence of the Visits to the Targets

- If the sequence of the visits Σ to the target locations is given.
- the problem is to determine the optimal heading at each location.
- and the problem becomes the **Dubins Touring Problem (DTP)**.
- Let for each location $g_i \in G$ sample possible heading to k values, i.e., for each g_i the set of headings be $h_i = \{\theta_1^i, \dots, \theta_k^i\}$.
- Since Σ is given, we can construct a graph connecting two consecutive locations in the sequence by all possible headings.
- For such a graph and particular headings $\{h_1, \dots, h_n\}$, we can find an optimal headings and thus, **the optimal solution of the DTP**.

DTSP as a Solution of the DTP



- The edge cost corresponds to the length of Dubins maneuver.
- Better solution of the DTP can be found for more samples, which will also improve the DTSP but only for the given sequence.

Two questions arise for a practical solution of the DTP.

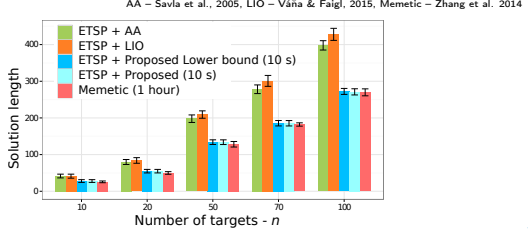
- How to sample the headings? More samples makes finding solution more demanding.
- What is the solution quality? Is there a tight lower bound?

We need to sample the headings in a "smart" way, i.e. guided sampling using lower bound of the DTP?

Yes, the lower bound can be computed as a solution of the Dubins Interval Problem (DIP).

DTP Solver in Solution of the DTSP

- The solution of the DTP can be used to solve DTSP for the given sequence of the waypoints. *E.g., determined as a solution of the Euclidean TSP as in the Alternating Algorithm.*
- Comparison with the Alternating Algorithm (AA), Local Iterative Optimization (LIO), and Memetic algorithm.



Dubins Traveling Salesman Problem with Neighborhoods

- In surveillance planning, it may be required to visit a set of target regions $G = \{R_1, \dots, R_n\}$ by the Dubins vehicle.
- Then, for each target region R_i , we have to determine a particular point of the visit $p_i \in R_i$ and DTSP becomes the **Dubins Traveling Salesman Problem with Neighborhoods (DTSPN)**. *In addition to Σ and headings Θ , waypoint locations P have to be determined.*
- DTSPN is an optimization problem over all permutations Σ , headings $\Theta = \{\theta_{\sigma_1}, \dots, \theta_{\sigma_n}\}$ and points $P = (p_{\sigma_1}, \dots, p_{\sigma_n})$ for the states $(q_{\sigma_1}, \dots, q_{\sigma_n})$ such that $q_{\sigma_i} = (p_{\sigma_i}, \theta_{\sigma_i})$ and $p_{\sigma_i} \in R_{\sigma_i}$:

$$\begin{aligned} & \text{minimize}_{\Sigma, \Theta, P} \sum_{i=1}^{n-1} \mathcal{L}(q_{\sigma_i}, q_{\sigma_{i+1}}) + \mathcal{L}(q_{\sigma_n}, q_{\sigma_1}) \\ & \text{subject to } q_i = (p_i, \theta_i), p_i \in R_i, i = 1, \dots, n. \end{aligned}$$

- $\mathcal{L}(q_{\sigma_i}, q_{\sigma_j})$ is the length of the shortest possible Dubins maneuver connecting the states q_{σ_i} and q_{σ_j} .

DTSPN – Local Iterative Optimization (LIO)

- Instead of sampling into a discrete set of waypoint locations each with sampled possible headings, we can perform local optimization, e.g., hill-climbing technique.
- At each waypoint location p_i , the heading can be $\theta_i \in [0, 2\pi)$.
- A waypoint location p_i can be parametrized as a point on the boundary of the respective region R_i , i.e., as a parameter $\alpha_i \in [0, 1)$ measuring a normalized distance on the boundary of R_i .
- The multi-variable optimization is treated independently for each particular variable θ_i and α_i iteratively.

```

Algorithm 2: Local Iterative Optimization (LIO) for the DTSPN
Data: Input sequence of the goal regions
       $G = (R_{\sigma_1}, \dots, R_{\sigma_n})$ , for the permutation  $\Sigma$ 
Result: Waypoints  $(q_{\sigma_1}, \dots, q_{\sigma_n})$ ,  $q_i = (p_i, \theta_i)$ ,  $p_i \in \delta R_i$ 
initialization() // random assignment of  $q_i \in \delta R_i$ ;
while global solution is improving do
  for every  $R_i \in G$  do
     $\theta_i := \text{optimizeHeadingLocally}(\theta_i)$ ;
     $\alpha_i := \text{optimizePositionLocally}(\alpha_i)$ ;
     $q_i := \text{checkLocalMinima}(\alpha_i, \theta_i)$ ;
  end
end
    
```

Vána, P. and Faigl, J.: *On the Dubins Traveling Salesman Problem with Neighborhoods*, IROS, 2015, pp. 4029–4034.

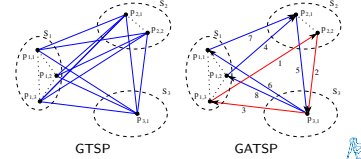
DTSP – Sampling-based Approach

- Sampled heading values can be directly utilized to find the sequence as a solution of the **Generalized Traveling Salesman Problem (GTSP)**. *Notice For Dubins vehicle, it is the Generalized Asymmetric TSP (GATSP).*

The problem is to determine a shortest tour in a graph that visits all specified subsets of the graph's vertices.

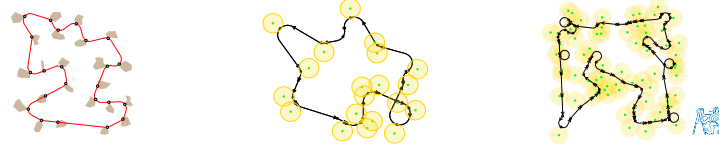
The TSP is a special case of the GTSP when each subset to be visited consists just a single vertex.

- GTSP \rightarrow ATSP; *Noon and Bean (1991)*
- ATSP can be solved by LKH;
- ATSP \rightarrow TSP, which can be solved optimally, e.g., by Concorde.



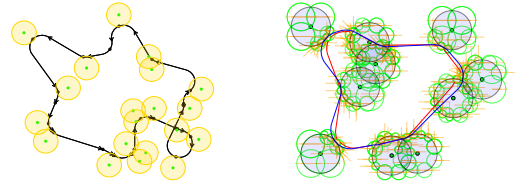
DTSPN – Approaches and Examples of Solution

- Similarly to the DTSP, also the DTSPN can be addressed by
 - Decoupled approaches** for which a sequence of visits to the regions can be found as a solution of the ETSP(N);
 - Sampling-based approaches** and formulation as the GATSP.
 - Clusters of sampled waypoint locations each with sampled possible heading values.
 - Soft-computing** techniques such as memetic algorithms.
 - Unsupervised learning** techniques.
- Vána and Faigl (IROS 2015), Faigl and Vána (ICANN 2016, IJCNN 2017)
- Similarly to the lower bound of the DTSP based on the **Dubins Interval Problem (DIP)** a lower bound for the DTSPN can be computed using the **Generalized DIP (GDIP)**.



Lower Bound for the DTSP with Neighborhoods Generalized Dubins Interval Problem

- In the DTSPN, we need to determine the **headings** and also the **waypoint locations**.
- The **Dubins Interval Problem (DIP)** is not sufficient to provide tight lower-bound.

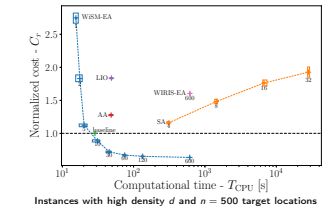
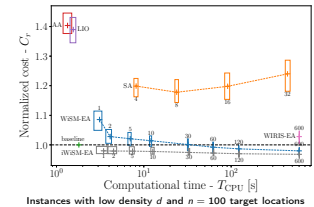


- Generalized Dubins Interval Problem (GDIP)** can be utilized for the DTSPN similarly as the DIP for the DTSP.

Vána and Faigl: *Optimal Solution of the Generalized Dubins Interval Problem*, RSS 2018, best student paper finalist.

DTSP – Evolutionary Approach with Surrogate Model

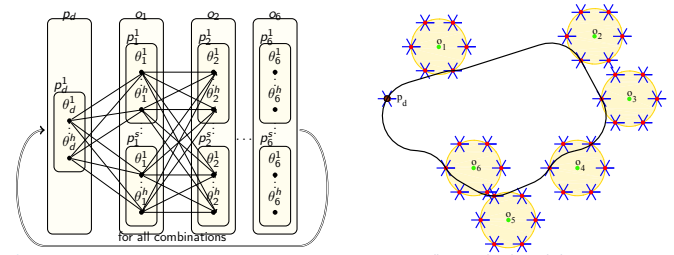
- Use standard genetic operators with tournament selection and OX1 crossover method.
- The population is evaluated using learned surrogate model based on multi-layer perceptron.
- The surrogate model estimates solution cost of candidate sequences (instances of the DTP).
- Massive speedup of the evaluation yields improved solutions and scalability.



Drchal, J., Vána, P., and Faigl, J.: *WISM: Windowing Surrogate Model for Evaluation of Curvature-Constrained Tours with Dubins vehicle*, IEEE Transactions on Cybernetics, 2020.

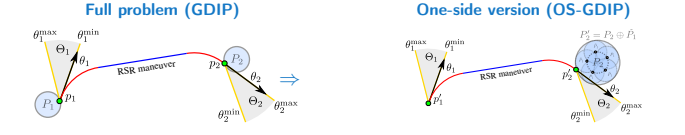
DTSPN – Decoupled Approach

- Determine a sequence of visits to the n target regions as the solution of the ETSP.
- Sample possible waypoint locations and for each such a location sample possible heading values, e.g., s locations per each region and h heading per each location.
- Construct a search graph and determine a solution in $O(n(sh)^3)$.
- An example of the search graph for $n = 6$, $s = 6$, and $h = 6$.



Generalized Dubins Interval Problem (GDIP)

- Determine the shortest Dubins maneuver connecting P_i and P_j given the angle intervals $\theta_i \in [\theta_i^{min}, \theta_i^{max}]$ and $\theta_j \in [\theta_j^{min}, \theta_j^{max}]$.



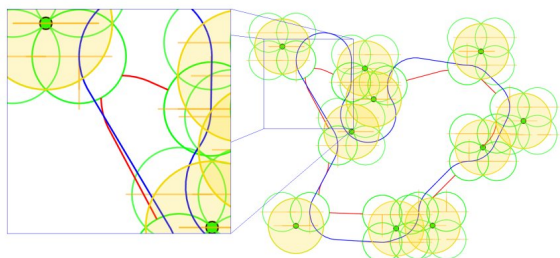
- Optimal solution** – Closed-form expressions for (1–6) and convex optimization (7).
- | Average computational time | | |
|----------------------------|-----------------|-------|
| Problem | Time [μ s] | Ratio |
| Dubins maneuver | 0.4 | 1.0 |
| DIP | 1.1 | 3.0 |
| GDIP | 5.4 | 14.5 |
- <https://github.com/consob/gdip>

Vána, P. and Faigl, J.: *Optimal Solution of the Generalized Dubins Interval Problem Finding the Shortest Curvature-constrained Path Through a Set of Regions*, Autonomous Robots, 44(7):1359–1376, 2020.

GDIP-based Informed Sampling for the DTSPN

- Iterative refinement of the neighborhood samples and heading samples.

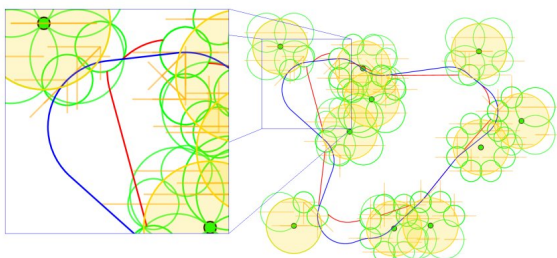
Resolution: 4 Gap: 69.3 % Time: 0.079 s



GDIP-based Informed Sampling for the DTSPN

- Iterative refinement of the neighborhood samples and heading samples.

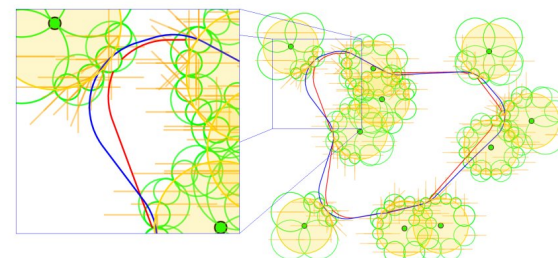
Resolution: 8 Gap: 39.4 % Time: 0.211 s



GDIP-based Informed Sampling for the DTSPN

- Iterative refinement of the neighborhood samples and heading samples.

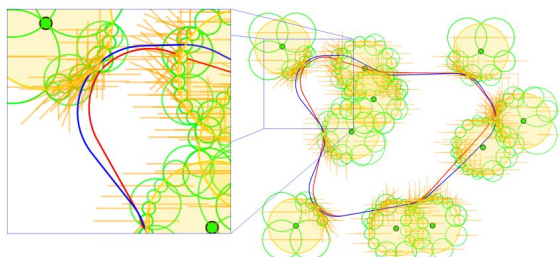
Resolution: 16 Gap: 19.9 % Time: 0.552 s



GDIP-based Informed Sampling for the DTSPN

- Iterative refinement of the neighborhood samples and heading samples.

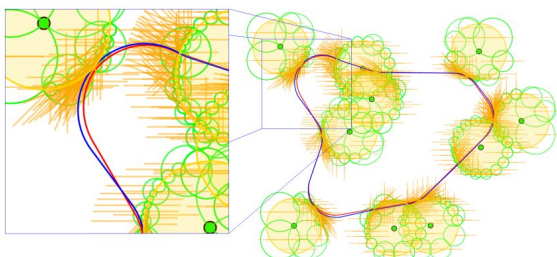
Resolution: 32 Gap: 10.7 % Time: 1.292 s



GDIP-based Informed Sampling for the DTSPN

- Iterative refinement of the neighborhood samples and heading samples.

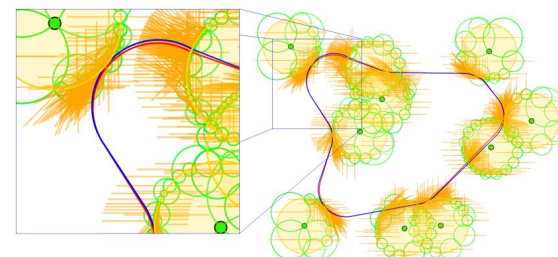
Resolution: 64 Gap: 5.3 % Time: 3.183 s



GDIP-based Informed Sampling for the DTSPN

- Iterative refinement of the neighborhood samples and heading samples.

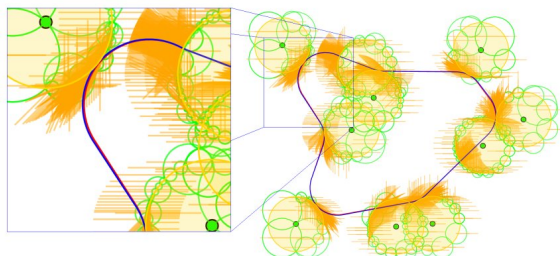
Resolution: 128 Gap: 2.6 % Time: 8.994 s



GDIP-based Informed Sampling for the DTSPN

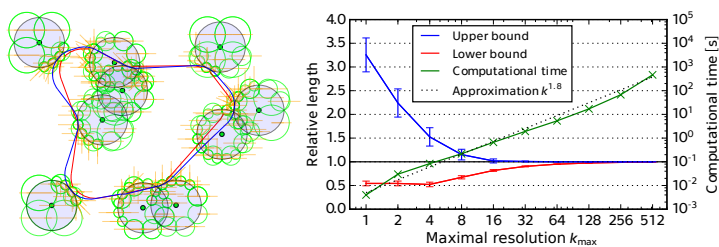
- Iterative refinement of the neighborhood samples and heading samples.

Resolution: 256 Gap: 1.3 % Time: 33.474 s



DTSPN – Convergence to the Optimal Solution

- For a given sequence of visits to the target regions (locations).



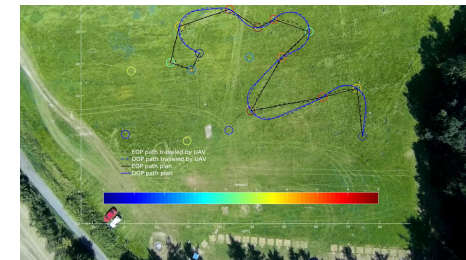
- The algorithm scales linearly with the number of locations.
- Complexity of the algorithm is approximately $O(nk^{1.8})$.

<https://github.com/comrob/gdip>

Data Collection / Surveillance Planning with Travel Budget

- Visit the most important targets because of limited travel budget.
- The problem can be formulated as the **Orienteering Problem** with Dubins vehicle, a.k.a. **Dubins Orienteering Problem (DOP)**.

Robert Pěnička, Jan Faigl, Petr Váňa and Martin Saska, RA-L 2017



<http://mrs.felk.cvut.cz/icra17dop>

Dubins Vehicle and Dubins Planning DTP DTSP DTSPN DOP DOPN Planning in 3D

Dubins Orienteering Problem

- Curvature-constrained data collection path respecting Dubins vehicle model with the minimal turning radius ρ and constant forward velocity v .
- The path is a sequence of waypoints $q_i \in SE(2)$, $q_i = (s, \theta)$.
- In addition to S_k, k, Σ (OP) determine headings $\Theta = (\theta_{\sigma_1}, \dots, \theta_{\sigma_k})$ such that

$$\text{maximize}_{k, S_k, \Sigma} R = \sum_{i=1}^k r_{\sigma_i}$$

$$\text{subject to} \sum_{i=2}^k \mathcal{L}(q_{\sigma_{i-1}}, q_{\sigma_i}) \leq T_{\max},$$

$$q_{\sigma_i} = (s_{\sigma_i}, \theta_{\sigma_i}), s_{\sigma_i} \in S, \theta_{\sigma_i} \in \mathbb{S}^1$$

$$s_{\sigma_1} = s_1, s_{\sigma_k} = s_n$$

The problem combines discrete combinatorial optimization (OP) with the continuous optimization for determining the vehicle headings.

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Dubins Vehicle and Dubins Planning DTP DTSP DTSPN DOP DOPN Planning in 3D

Variable Neighborhood Search (VNS)

- Variable Neighborhood Search (VNS)** is a general metaheuristic for combinatorial optimization (routing problems).
- The VNS is based on **shake** and **local search** procedures.
 - Shake** procedure aims to escape from local optima by changing the solution within the neighborhoods $N_{1, \dots, k_{\max}}$.
 - Local search** procedure searches fully specific neighborhoods of the solution using l_{\max} predefined operators.

Hansen, P. and Mladenović, N. (2001): *Variable neighborhood search: Principles and applications*. European Journal of Operational Research.

The neighborhoods are particular operators.

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Dubins Vehicle and Dubins Planning DTP DTSP DTSPN DOP DOPN Planning in 3D

Variable Neighborhood Search (VNS) for the DOP

- The solution is the first k locations of the sequence of all target locations satisfying T_{\max} .
- It is an improving heuristics, i.e., an initial solution has to be provided.
- A set of predefined neighborhoods are explored to find a better solution.
- Shake** – explores the configuration space and escape from a local minima using
 - Insert** – moves one random element;
 - Exchange** – exchanges two random elements.
- Local Search** – optimizes the solution using
 - Path insert** – moves a random sub-sequence;
 - Path exchange** – exchanges two random sub-sequences.
- Randomized VNS** – examines only n^2 changes in the *Local Search* procedure in each iteration.

Sevkil Z., Sevilgen F.E.: *Variable Neighborhood Search for the Orienteering Problem*, SCIS, 2006.

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Evolution of the VNS Solution to the DOP

Initial solution

$T_{CPU} = 10.9$ s,
 $\mathcal{L} = 79.6, R = 960$

4710th iteration
(4th improvement)

$T_{CPU} = 144.8$ s,
 $\mathcal{L} = 79.7, R = 990$

4790th iteration
(12th improvement)

$T_{CPU} = 147.3$ s,
 $\mathcal{L} = 79.3, R = 1008$

5560th iteration
(16th improvement)

$T_{CPU} = 170.0$ s,
 $\mathcal{L} = 79.1, R = 1050$

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Dubins Vehicle and Dubins Planning DTP DTSP DTSPN DOP DOPN Planning in 3D

Possible Solutions of the Dubins Orienteering Problem

- Solve the Euclidean OP (EOP) and then determine Dubins path.

The final path may exceed the budget and the vehicle can miss the locations because of motion control.
- Directly solve the **Dubins Orienteering Problem (DOP)** such as
 - Sample possible heading values and use Variable Neighborhood Search (VNS); Penička, R., Faigl, J., Vaňa, P., and Saska, M.: *Dubins Orienteering Problem*, IEEE Robotics and Automation Letters, 2(2):1210–1217, 2017.
 - Unsupervised learning based on Self-Organizing Maps (SOM); Faigl, J.: *Self-organizing map for orienteering problem with dubins vehicle*, Advances in Self-Organizing Maps, Learning Vector Quantization, Clustering and Data Visualization, 2017, pp. 125–132.

The VNS-based approach provides better solutions than the SOM-based solution, but it tends to be more demanding.

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Dubins Vehicle and Dubins Planning DTP DTSP DTSPN DOP DOPN Planning in 3D

Dubins Orienteering Problem with Neighborhoods

- Curvature-constrained path respecting Dubins vehicle model.
- Each waypoint consists of location $p \in \mathbb{R}^2$ and the heading $\theta \in \mathbb{S}^1$.
- In addition to S_k, k, Σ determine locations $P_k = (p_{\sigma_1}, \dots, p_{\sigma_k})$ and headings $\Theta = (\theta_{\sigma_1}, \dots, \theta_{\sigma_k})$ such that

$$\text{maximize}_{k, S_k, \Sigma} R = \sum_{i=1}^k r_{\sigma_i}$$

$$\text{subject to} \sum_{i=2}^k \mathcal{L}(q_{\sigma_{i-1}}, q_{\sigma_i}) \leq T_{\max},$$

$$q_{\sigma_i} = (p_{\sigma_i}, \theta_{\sigma_i}), p_{\sigma_i} \in \mathbb{R}^2, \theta_{\sigma_i} \in \mathbb{S}^1$$

$$\|p_{\sigma_i}, s_{\sigma_i}\| \leq \delta, s_{\sigma_i} \in S_k$$

$$p_{\sigma_1} = s_1, p_{\sigma_k} = s_n$$

We need to solve the continuous optimization for determining the vehicle heading at each waypoint and the waypoint locations $P_k = \{p_{\sigma_1}, \dots, p_{\sigma_k}\}$, $p_{\sigma_i} \in \mathbb{R}^2$.

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Dubins Vehicle and Dubins Planning DTP DTSP DTSPN DOP DOPN Planning in 3D

Variable Neighborhood Search (VNS) for the DOPN

```

Algorithm 3: VNS based method for the DOPN
Input : S – Set of the target locations
Input : T_max – Maximal allowed budget
Input : phi – Initial number of position waypoints for each target
Input : m – Initial number of heading values for each waypoints
Input : r_i – Local waypoint improvement ratio
Input : l_max – Maximal neighborhood number
Output : P – Found data collecting path
S_i ← getReachableLocations(S, T_max)
P ← createInitialPath(S, T_max)
while Stopping condition is not met do
    l ← phi
    while l ≤ l_max do
        P' ← shake(P, l)
        P'' ← localSearch(P', l, r_i)
        if L(P'') ≤ T_max and
           [(R(P'') > R(P)) or (R(P'') == (P) and L_d(P'') < L_d(P, L_d(P')))]
        then
            P ← P''
        else
            l ← l + 1
        end
    end
end
    
```

The particular l for the individual operators of the **shake** procedure are:

- Waypoint Shake** ($l = 1$);
- Path Move** ($l = 2$);
- Path Exchange** ($l = 3$).

The **local search** procedure consists of three operators and the particular l for the individual operators of the **local search** procedure are:

- Waypoint Improvement** ($l = 1$);
- One Point Move** ($l = 2$);
- One Point Exchange** ($l = 3$).

Penička, R., Faigl, J., Saska, M., and Vaňa, P.: *Data collection planning with non-zero sensing distance for a budget and curvature constrained unmanned aerial vehicle*, Autonomous Robots 43(3):1937–1956, 2019.

Penička, R., Faigl, J., Vaňa, P., and Saska, M.: *Dubins Orienteering Problem with Neighborhoods*, International Conference on Unmanned Aircraft Systems (ICUAS), 2017, pp. 1555–1562.

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Dubins Vehicle and Dubins Planning DTP DTSP DTSPN DOP DOPN Planning in 3D

VNS for DOPN – Example of the Shake Operators

Path Move

Path Exchange

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Comparison of the DOPN Solvers

- VNS-based DOPN solver with $s = 16$ sampled waypoint locations per sensor and $h = 16$ heading samples per waypoint location.
- SOM-based DOPN solver with $h = 3$.
- Aggregate results using average relative percentage error (ARPE) and relative percentage error (RPE) to the reference (best found) solution.

Problem set	VNS-based			SOM-based ($h = 3$)		
	ARPE	T_{cpu} [s]	RPE	ARPE	T_{cpu} [s]	RPE
Set 3, $\delta = 0.0$	1.0	1,178.9	3.6	7.4	7.0	
Set 3, $\delta = 0.5$	0.9	13,273.3	6.6	10.6	7.9	
Set 3, $\delta = 1.0$	0.5	13,304.4	5.5	9.2	8.3	
Set 64, $\delta = 0.0$	1.9	5,272.2	17.4	23.8	17.9	
Set 64, $\delta = 0.5$	2.8	13,595.6	18.7	24.2	20.2	
Set 64, $\delta = 1.0$	1.3	13,792.3	9.9	15.2	22.2	
Set 66, $\delta = 0.0$	1.5	6,546.6	3.6	9.1	22.9	
Set 66, $\delta = 0.5$	1.4	13,650.1	6.7	11.8	25.5	
Set 66, $\delta = 1.0$	3.2	13,824.5	16.1	21.3	26.7	

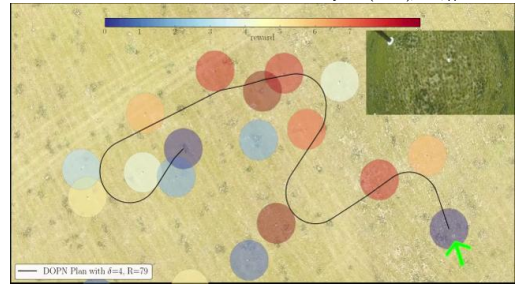
*The results have been obtained with a grid Xeon CPUs running at 2.2 GHz to 3.4 GHz due to computational requirements.

Penička, Faigl, et al. (ICUAS 2017)
Faigl, Penička (IROS 2017)

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DOPN – Example of Solution and Practical Deployment

- VNS-based solution of the DOPN.
- Peňička, R., Faigl, J., Váňa, P., and Saska, M.: *Dubins Orienteering Problem with Neighborhoods*, International Conference on Unmanned Aircraft Systems (ICUAS), 2017, pp. 1555–1562.



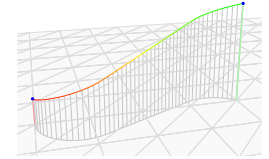
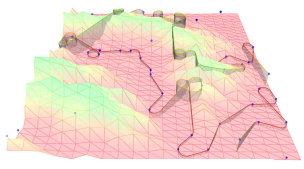
3D Data Collection Planning with Dubins Airplane Model

- Dubins Airplane model describes the vehicle state $q = (p, \theta, \psi)$, $p \in \mathbb{R}^3$ and $\theta, \psi \in \mathbb{S}^1$ as

$$\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{z} \\ \dot{\theta} \\ \dot{\psi} \end{bmatrix} = v \begin{bmatrix} \cos \theta \cdot \cos \psi \\ \sin \theta \cdot \cos \psi \\ \sin \psi \\ u_\theta \cdot \rho^{-1} \end{bmatrix}$$

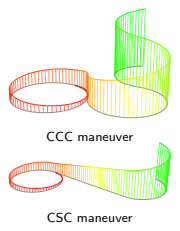
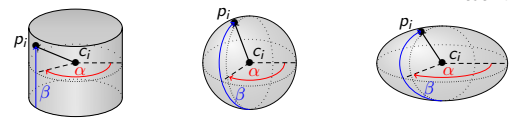
H. Chitsaz and S. M. LaValle: *Time-optimal paths for a Dubins airplane*, IEEE Conference on Decision and Control, 2007, pp. 2379–2384.

- Constant forward velocity v , the minimal turning radius ρ , and limited pitch angle, i.e., $\psi \in [\psi_{min}, \psi_{max}]$.
- u_θ controls the vehicle heading, $|u_\theta| \leq 1$, and v is the forward velocity.
- Generation of the 3D trajectory is based on the 2D Dubins maneuver.
- If altitude changes are too high, additional helix segments are inserted.



The DTSPN in 3D

- Using the same principles as for the DTSPN in 2D, we can generalize the approaches for 3D planning using the Dubins Airplane model instead of simple Dubins vehicle.
- The regions can be generalized to 3D and the problem can be addressed by decoupled or sampling-based approaches, i.e., using GATSP formulation.
- In the case of LIO, we need a parametrization of the possible waypoint location, such as point on the object boundary.

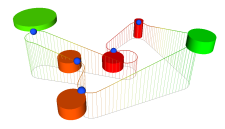
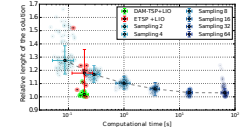


Solutions of the 3D-DTSPN

```

Algorithm 4: LIO-based Solver for 3D-DTSPN
Data: Regions R
Result: Solution represented by Q and Σ
Σ ← getInitialSequence(R, Σ)
Q ← getInitialSolution(R, Σ);
while terminal condition do
  Q ← optimizeHeadings(Q, R, Σ);
  Q ← optimizeAlpha(Q, R, Σ);
  Q ← optimizeBeta(Q, R, Σ);
end
return Q, Σ;
    
```

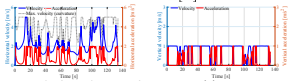
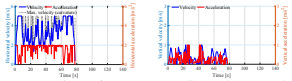
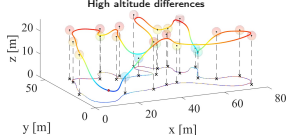
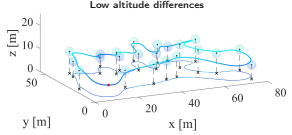
- Solutions based on LIO (ETSP+LIO), TSP with the travel cost according to Dubins Airplane Model (DAM-TSP+LIO), and sampling-based approach with transformation of the GTSP to the ATSP solved by LKH.



Váňa, P., Faigl, J., Šlamma, J., and Peňička, R.: *Data collection planning with Dubins airplane model and limited travel budget* European Conference on Mobile Robots (ECMR), 2017.

3D Surveillance Planning

- Parametrization of smooth 3D multi-goal trajectory as a sequence of Bézier curves.
- Unsupervised learning for the TSPN can be generalized for such trajectories.
- During the solution of the sequencing part of the problem, we can determine a velocity profile along the curve and compute the so-called *Travel Time Estimation* (TTE).
- Bézier curves better fit the limits of the multi-rotor UAVs that are limited by the maximal accelerations and velocities rather than minimal turning radius as for Dubins vehicle.

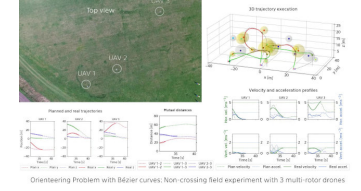
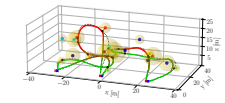


Faigl, J. and Váňa, P.: *Surveillance Planning With Bézier Curves*, IEEE Robotics and Automation Letters, 3(2):750–757, 2018.

- Low altitude differences saturate horizontal velocity while high altitudes changes saturate vertical velocity.

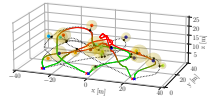
Multi-Vehicle Multi-Goal Planning with Limited Travel Budget – Curvature-Constrained Team Orienteering Problem (with Neighborhoods)

- Operational time of multi-rotor aerial vehicles is limited and only a subset of locations can be visited.
- Planning multi-goal trajectories as a sequence of Bézier curves.



Orienteering Problem with Bézier curves, Non-crossing field experiment with 3 multi-rotor drones

- Targets are missed in a case of colliding trajectories, because of local collision avoidance and optimal trajectory following.
- There is a practical need to include coordination in multi-vehicle multi-goal trajectory planning.



Faigl, J., Váňa, P., and Peňička, R.: *Multi-Vehicle Close Enough Orienteering Problem with Bézier Curves for Multi-Rotor Aerial Vehicles*, ICRA 2019, pp. 3039–3044.

Summary of the Lecture

- Data collection planning with curvature-constrained paths/trajectories
 - The **Traveling Salesman Problem (TSP)** and **Orienteering Problem (OP)** with Dubins Vehicle, i.e., **DTSP** and **DOP**.
 - It is a combination of the combinatorial and continuous (determining optimal headings) optimization.
 - The continuous part can be solved using **Dubins Touring Problem (DTP)**.
 - Using a solution of the **Dubins Interval Problem (DIP)** we can establish **tight lower bound** of the DTP and DTSP with a particular sequence of visits.
 - The problems can be further extended to **DTSP with Neighborhoods (DTSPN)** and **OP with Neighborhoods (DOPN)**, and its **Close Enough** variants.
- The key ideas of the presented problems and approaches are as follows.
 - Consider proper assumptions that fits the original problem being solved.
 - Suitability of the vehicle model, requirements on the solution quality, and benefit of optimal or computationally demanding solutions.
 - Employing lower bound based on "a bit different problem" such as the **DIP** and **GDIP**, to find high quality solutions, even using decoupled approaches.
 - Challenging problems with continuous optimization can be addressed by decoupled and sampling-based approaches.
 - Be aware that the optimal solutions found for discretized problems, e.g., using ILP or combinatorial solvers, are not optimal solutions of the original (continuous) problem!

Summary

- Dubins vehicles and planning – Dubins maneuvers
- Dubins Interval Problem (DIP)** (lower bound estimation to the DTP, DTSP)
- Dubins Touring Problem (DTP)**
- Dubins Traveling Salesman Problem (DTSP) and Dubins Traveling Salesman with Neighborhoods (DTSPN)
 - Decoupled approaches – Alternating Algorithm
 - Sampling-based approaches – GATSP
- Generalized Dubins Interval Problem (GDIP)** (lower bound estimation to the DTSPN)
- Dubins Orienteering Problem (OP) and Dubins Orienteering Problem with Neighborhoods (DOPN)
- Data collection and surveillance planning in 3D
- Next: Sampling-based motion planning**