Multi-goal Planning

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Lecture 05

B4M36UIR – Artificial Intelligence in Robotics

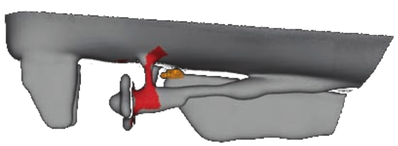
Overview of the Lecture

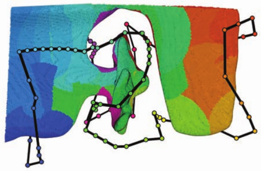
Part I

Part 1 – Multi-goal Planning

Robotic Information Gathering in Inspection of Vessel's Propeller

- The planning problem is to determine a shortest inspection path for an Autonomous Underwater Vehicle (AUV) to inspect the vessel's propeller.



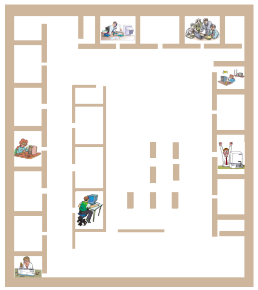


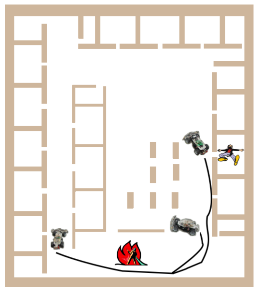
https://www.youtube.com/watch?v=8azP_9VnMtM

Englot, B., Hover, F.S.: *Three-dimensional coverage planning for an underwater inspection robot*, International Journal of Robotics Research, 32(9–10):1048–1073, 2013.

Example of Inspection Planning in Search Scenario

- Periodically visit particular locations of the environment and return to the starting locations.
- Use available floor plans to guide the search, e.g., finding victims in search-and-rescue scenario.

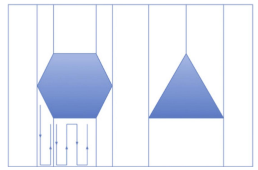




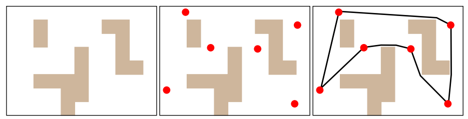
Inspection Planning

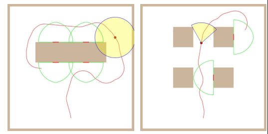
- **Inspection/coverage planning** stands to determine a plan (path) to inspect/cover the given areas or point of interest.
- We can directly find inspection/coverage plan using
  - predefined covering patterns such as *ox-plow* motion;
  - a "general" path satisfying coverage constraints.

    Galceran, E., Carreras, M.: *A survey on coverage path planning for robotics*, Robotics and Autonomous Systems, 61(12):1258–1276, 2013.
- **Decoupled** approach where locations to be visited are determined before path planning as the **sensor placement** problem.



Trapezoidal decomposition method



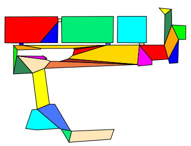


Kafka, Faigl, Váňa: ICRA 2016

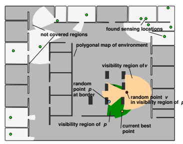
Inspection Planning – Decoupled Approach

1. Determine sensing locations such that the whole environment would be inspected (seen) by visiting them (**Sampling design problem**).

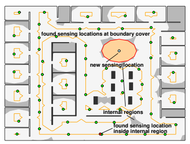
   In the geometrical-based approach, a solution of the Art Gallery Problem.



Convex Partitioning (Kazazakis and Argyros, 2002)



Randomized Dual Sampling (González-Baños et al., 1998)



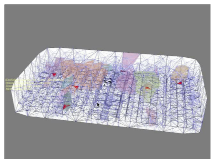
Boundary Placement (Faigl et al., 2006)

The problem is related to the sensor placement and sampling design.

2. Create a roadmap connecting the sensing location.

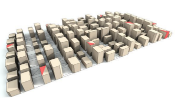
   E.g., using visibility graph or randomized sampling based approaches.
3. Find the inspection path visiting all the sensing locations as a solution of the multi-goal path planning (a solution of the robotic TSP).

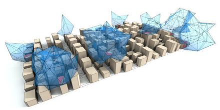
Planning to Capture Areas of Interest using UAV

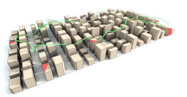


- Determine a cost-efficient path from which a given set of target regions is covered.
- For each target region a subspace $S \subset \mathbb{R}^3$ from which the target can be covered is determined. *S represents the neighborhood.*
- **We search for the best sequence of visits to the regions.** *Combinatorial optimization*
- The PRM is utilized to construct the planning roadmap (a graph).

  PRM – Probabilistic Roadmap Method – sampling-based motion planner, see lecture 8.
- The problem can be formulated as **the Traveling Salesman Problem with Neighborhoods**, as it is not necessary to visit exactly a single location to capture the area of interest.





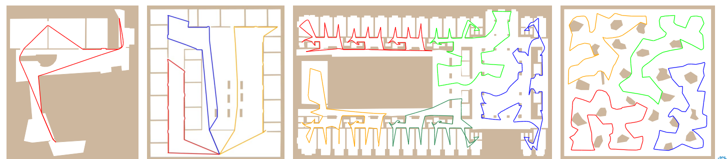


Janoušek and Faigl, ICRA 2013

Inspection Planning – "*Continuous Sensing*"

- If we do not prescribe a discrete set of sensing locations, we can formulate the problem as the **Watchman route problem**.

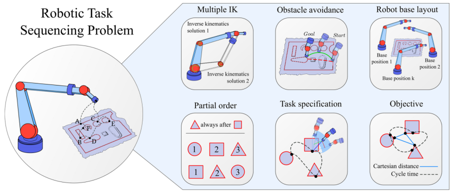
Given a map of the environment $\mathcal{W}$ determine the shortest, closed, and collision-free path, from which the whole environment is covered by an omnidirectional sensor with the radius ρ.



Faigl, J.: *Approximate Solution of the Multiple Watchman Routes Problem with Restricted Visibility Range*, IEEE Transactions on Neural Networks, 21(10):1668-1679, 2010.

Multi-Goal Planning

- Having a **set of locations** to be visited, determine the cost-efficient path to visit them.
  - Locations where a robotic arm or mobile robot performs some task. *The operation can be repeated–closed path.*
- The problem is called **robotic task sequencing problem** for robotic manipulators.



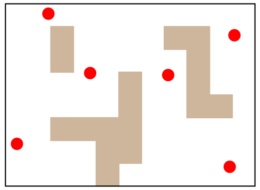
Alatartsev, S., Stellmacher, S., Ortmeier, F. (2015): Robotic Task Sequencing Problem: A Survey. Journal of Intelligent & Robotic Systems.

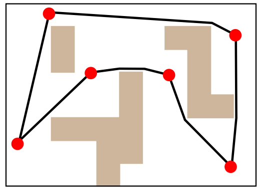
- The problem is also called **Multi-goal Path Planning** (MTP) problem or **Multi-goal Planning** (**MGP**).

Also studied in its Multi-goal Motion Planning (MGMP) variant.

Multi-Goal Path Planning (MTP)

- Multi-goal planning problem is a problem how to visit the given set of locations.
- It consists of **point-to-point path planning** on how to reach one location from another.
- The *challenge* is to determine the optimal sequence of the visits to the locations w.r.t. cost-efficient path to visit all the given locations.





- Determination the sequence of visits is a **combinatorial optimization problem** that can be formulated as the **Traveling Salesman Problem** (**TSP**).

Traveling Salesman Problem (TSP)

Given a set of cities and the distances between each pair of cities, what is the shortest possible route that visits each city exactly once and returns to the origin city.

- The TSP can be formulated for a graph $\boldsymbol{G}(V, E)$, where V denotes a set of locations (cities) and E represents edges connecting two cities with the associated travel cost c (distance), i.e., for each $v_i, v_j \in V$ there is an edge $e_{ij} \in E$, $e_{ij} = (v_i, v_j)$ with the cost c_{ij}.
- If the associated cost of the edge (v_i, v_j) is the Euclidean distance $c_{ij} = |(v_i, v_j)|$, the problem is called the **Euclidean TSP** (ETSP).
- It is known, the TSP is NP-hard (its decision variant) and several algorithms can be found in literature.

William J. Cook (2012) – In Pursuit of the Traveling Salesman: Mathematics at the Limits of Computation.

Traveling Salesman Problem (TSP)

- Let S be a set of n sensor locations $S = \{\boldsymbol{s}_1, \ldots, \boldsymbol{s}_n\}$, $\boldsymbol{s}_i \in \mathbb{R}^2$ and $c(\boldsymbol{s}_i, \boldsymbol{s}_j)$ is a cost of travel from $\boldsymbol{s}_i$ to $\boldsymbol{s}_j$
- **Traveling Salesman Problem** (**TSP**) is a problem to determine a closed tour visiting each $\boldsymbol{s} \in S$ such that the total tour length is minimal.
  - We are searching for the optimal **sequence of visits** $\Sigma = (\sigma_1, \ldots, \sigma_n)$ such that

$$\begin{aligned} &\text{minimize}_{\Sigma} \quad L = \left(\sum_{i=1}^{n-1} c(\boldsymbol{s}_{\sigma_i}, \boldsymbol{s}_{\sigma_{i+1}})\right) + c(\boldsymbol{s}_{\sigma_n}, \boldsymbol{s}_{\sigma_1}) \\ &\text{subject to} \quad \Sigma = (\sigma_1, \ldots, \sigma_n), 1 \le \sigma_i \le n, \sigma_i \ne \sigma_j \text{ for } i \ne j. \end{aligned} \tag{1}$$

- The TSP can be considered on a graph $G(V, E)$ where the set of vertices V represents sensor locations S and E are edges connecting the nodes with the cost $c(\boldsymbol{s}_i, \boldsymbol{s}_j)$.
- For simplicity we can consider $c(\boldsymbol{s}_i, \boldsymbol{s}_j)$ to be Euclidean distance; otherwise, we also need to address the path/motion planning problem. **Euclidean TSP**
- If $c(\boldsymbol{s}_i, \boldsymbol{s}_j) \ne c(\boldsymbol{s}_j, \boldsymbol{s}_i)$ it is the **Asymmetric TSP**.
- The TSP is known to be NP-hard unless P=NP.

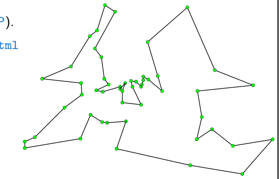
Traveling vs Travelling – `http://www.math.uwaterloo.ca/tsp/history/travelling.html`

Existing Approaches to the TSP

- Exact solutions
  - Branch&Bound, Branch&Cut, and Integer Linear Programming (ILP).

Concorde-`http://www.math.uwaterloo.ca/tsp/concorde.html`

- Approximation algorithms
  - Minimum Spanning Tree (MST) heuristic with $L \le 2L_{opt}$.
  - Christofides's algorithm with $L \le \frac{3/2}{L}{}_{opt}$.
- Heuristic algorithms
  - Constructive heuristic – Nearest Neighborhood (NN) algorithm
  - 2-Opt – local search algorithm proposed by Croes 1958
  - **LKH** – K. Helsgaun efficient implementation of the Lin-Kernighan heuristic (1998). `http://www.akira.ruc.dk/~keld/research/LKH/`
- Combinatorial meta-heuristics
  - **Variable Neighborhood Search** (**VNS**)
  - **Greedy Randomized Adaptive Search Procedure** (**GRASP**)
- Soft-computing techniques, evolutionary methods, and unsupervised learning

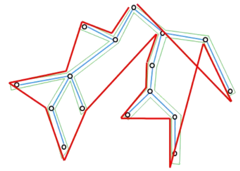


Problem Berlin52 from the TSPLIB

MST-based Approximation Algorithm to the TSP

- Minimum Spanning Tree heuristic

1. Compute the MST (denoted T) of the input graph G.
2. Construct a graph H by doubling every edge of T.
3. Shortcut repeated occurrences of a vertex in the tour.



- For the triangle inequality, the length of such a tour L is

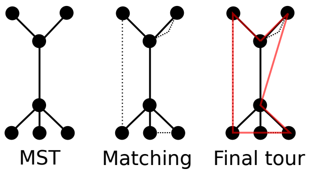
$$L \le 2L_{optimal},$$

where $L_{optimal}$ is the cost of the optimal solution of the TSP.

Christofides's Algorithm to the TSP

- Christofides's algorithm

1. Compute the MST of the input graph G.
2. Compute the minimal matching on the odd-degree vertices.
3. Shortcut a traversal of the resulting Eulerian graph.



- For the triangle inequality, the length of such a tour L is

$$L \le \frac{3}{2} L_{optimal},$$

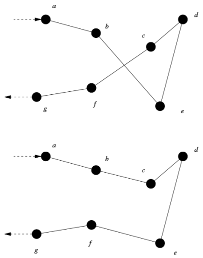
where $L_{optimal}$ is the cost of the optimal solution of the TSP.

Length of the MST is $\le L_{optimal}$

Sum of lengths of the edges in the matching $\le \frac{1}{2} L_{optimal}$

2-Opt Heuristic

1. Use a construction heuristic to create an initial route
   - NN algorithm, cheapest insertion, farther insertion
2. Repeat until no improvement is made
   2.1 Determine swapping that can shorten the tour (i, j) for $1 \le i \le n$ and $i + 1 \le j \le n$
   - route[0] to route[i-1];
   - route[i] to route[j] in reverse order;
   - route[j] to route[end];
   - Determine length of the route;
   - Update the current route if the length is shorter than the existing solution.



Croes, G.A.: *A method for solving traveling salesman problems*, Operations Research 6:791–812, 1958.

Overview of the Variable Neighborhood Search (VNS) for the TSP

- The **Variable Neighborhood Search** (**VNS**) is a metaheuristic for solving combinatorial optimization and global optimization problems by searching distant neighborhoods of the current **incumbent solution** using **shake** and **local search** procedures.

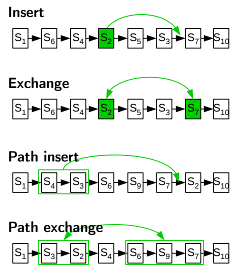
Mladenović and Hansen, 1997

1. **Shake** explores the neighborhood of the current solution to escape from a local minima using operators
   - **Insert** – moves one element;
   - **Exchange** – exchanges two elements.
2. **Local search** improves the solution by
   - **Path insert** – moves a subsequence;
   - **Path exchange** – exchanges two subsequences.

Algorithm 1: VNS-based Solver to the TSP

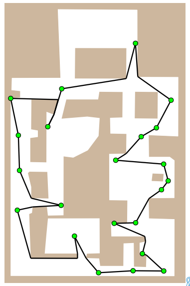
Input: S – Set of the target locations to be visited.
Output: Σ – Found sequence of visits to locations S.

$\Sigma^* \leftarrow$ Initial sequence found by cheapest insertion
while *terminal condition is not met* **do**
  $\Sigma' \leftarrow$ shake(Σ^*)
  for n^2*-times* **do**
    $\Sigma'' \leftarrow$ localSearch(Σ')
    if Σ'' *is "better" than* Σ' **then**
      $\Sigma' \leftarrow \Sigma''$ // Select Σ'' instead of Σ'
  if Σ' *is "better" than* Σ^* **then**
    $\Sigma^* \leftarrow \Sigma'$ // Replace the incumbent sequence.
return Σ^*



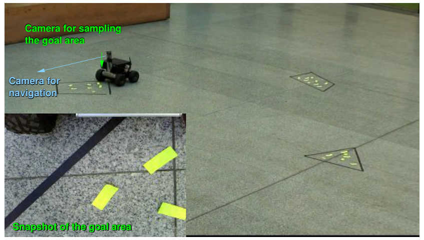
Multi-Goal Path Planning (MTP) Problem

- **MTP problem** is a **robotic variant of the TSP** with the edge costs as the length of the *shortest* path connecting the locations.
- Variants of the **robotic TSP** includes additional constraints arising from limitations of real robotic systems such as
  - obstacles, curvature-constraints, sensing range, location precision.
- For n locations, we need to compute up to n^2 shortest paths.
- Having a **roadmap** (graph) representing $\mathcal{C}_{free}$, the paths can be found in the graph (roadmap), from which the $G(V, E)$ for the TSP can be constructed. *Visibility graph as a roadmap for a point robot provides a straight forward solution, but such a shortest path may not be necessarily feasible for more complex robots.*
- We can determine the roadmap using randomized sampling-based motion planning techniques. See lecture 8.



Multi-goal Path Planning with Goal Regions

- It may be sufficient to visit a goal region instead of the particular point location.



Not only a ***sequence*** *of goals visit has to be determined, but also an* ***appropriate location*** *at each region has to be found.*

The problem with goal regions can be considered as a variant of the **Traveling Salesman Problem with Neighborhoods** (**TSPN**).

Traveling Salesman Problem with Neighborhoods

Given a set of n regions (neighbourhoods), what is the shortest closed path that visits each region.

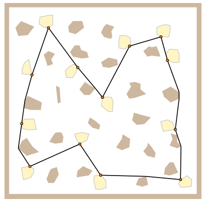
- The problem is NP-hard and APX-hard, it cannot be approximated to within factor $2 - \epsilon$, where $\epsilon > 0$. *Safra and Schwartz (2006) – Computational Complexity*
- Approximate algorithms exist for particular problem variants such as disjoint unit disk neighborhoods.
- **TSPN provides a suitable problem formulation for planning various inspection and data collection missions.**
- It enables to exploit non-zero sensing range, and thus find shortest (more cost-efficient) **data collection plans**.

Traveling Salesman Problem with Neighborhoods (TSPN)

- Instead visiting a particular location $\boldsymbol{s} \in S$, $\boldsymbol{s} \in \mathbb{R}^2$ as in the TSP, we request to visit a set of regions $R = \{r_1, \ldots, r_n\}$, $r_i \subset \mathbb{R}^2$ to save travel cost.
- The TSP becomes the **TSP with Neighborhoods** (**TSPN**) where, in addition to the determination of the **sequence** Σ, we determine a suitable locations of visits $P = \{\boldsymbol{p}_1, \ldots, \boldsymbol{p}_n\}$, $\boldsymbol{p}_i \in r_i$.
- The problem is a combination of combinatorial optimization to determine Σ with **continuous optimization** to determine P.

$$\text{minimize}_{\Sigma, P} \quad L = \left(\sum_{i=1}^{n-1} c(\boldsymbol{p}_{\sigma_i}, \boldsymbol{p}_{\sigma_{i+1}}) \right) + c(\boldsymbol{p}_{\sigma_n}, \boldsymbol{p}_{\sigma_1})$$

$$\begin{aligned} \text{subject to} \quad & R = \{r_1, \ldots, r_n\}, r_i \subset \mathbb{R}^2 \\ & P = \{\boldsymbol{p}_1, \ldots, \boldsymbol{p}_n\}, \boldsymbol{p}_i \in r_i \\ & \Sigma = (\sigma_1, \ldots, \sigma_n), 1 \le \sigma_i \le n, \\ & \sigma_i \ne \sigma_j \text{ for } i \ne j \\ & \text{Foreach } r_i \in R \text{ there is } \boldsymbol{p}_i \in r_i. \end{aligned}$$

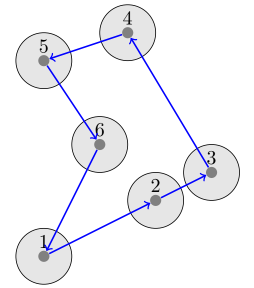


Approaches to the TSPN

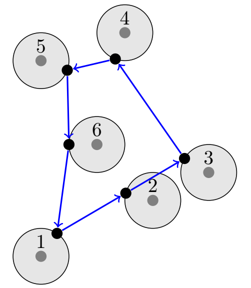
- A direct solution of the TSPN – approximation algorithms and heuristics
  E.g., using evolutionary techniques or unsupervised learning
- Euclidean TSPN with, disk-shaped δ neighborhoods is called **Closed Enough TSP** (**CETSP**).
  - Simplified variant with regions as disks with radius δ – remote sensing with the δ communication range.
- **Decoupled approach**
  1. Determine sequence of visits Σ independently on the locations P, e.g., as a solution of the TSP using centroids of the (convex) regions R.
  2. For the sequence Σ determine the locations P to minimize the total tour length using
     - Touring polygon problem (TPP);
     - Sampling possible locations and use a forward search for finding the best locations;
     - Continuous optimization such as hill-climbing.
       E.g., Local Iterative Optimization (LIO), Váňa & Faigl (IROS 2015)
- **Sampling-based** approaches
  - For each region, sample possible locations of visits into a discrete set of locations for each region.
  - The problem can be then formulated as the **Generalized Traveling Salesman Problem** (**GTSP**).

Close Enough Traveling Salesman Problem (CETSP)

- **Close Enough TSP** (**CETSP**) is a variant of the TSPN with disk shaped δ-neighborhoods.



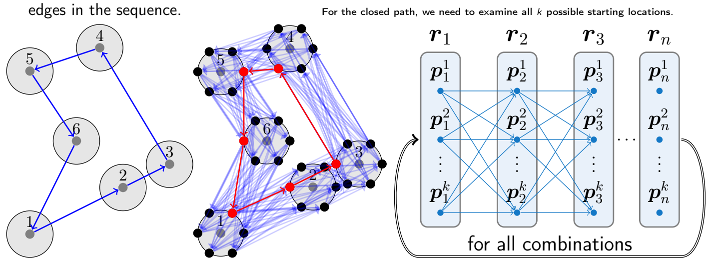
A solution of the TSP for the centers of the disks



A solution of the CETSP

Decoupled Sampling-based Solution of the TSPN / CETSP

- **Decoupled** – Determine sequence of visits as a solution of the Euclidean TSP for the representatives of the regions R, e.g., using centroids.
- Sample each region (neighborhood) with k samples, e.g., $k = 6$.
- Construct graph and find the shortest tour in by graph search in $\mathcal{O}(nk^3)$ for n regions and nk^2 edges in the sequence. *For the closed path, we need to examine all k possible starting locations.*

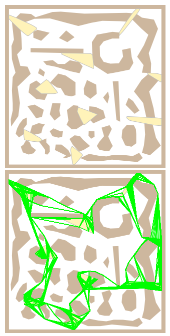


Iterative Refinement in the Multi-goal Planning Problem with Regions

- Let the sequence of n polygon regions be $R = (r_1, \ldots, r_n)$.
  Li, F., Klette, R.: Approximate algorithms for touring a sequence of polygons. 2008

1. Sampling regions into a discrete set of points and determine all shortest paths between each sampled points in the sequence of visits to the regions.
   E.g., using visibility graph
2. *Initialization:* Construct an initial touring polygons path using a sampled point of each region. Let the path be defined by $P = (\boldsymbol{p}_1, \boldsymbol{p}_2, \ldots, \boldsymbol{p}_n)$, where $\boldsymbol{p}_i \in r_i$ and $L(P)$ be the length of the shortest path induced by P.
3. *Refinement:* **For** $i = 1, 2, \ldots, n$:
   - Find $\boldsymbol{p}_i^* \in r_i$ minimizing the length of the path $d(\boldsymbol{p}_{i-1}, \boldsymbol{p}_i^*) + d(\boldsymbol{p}_i^*, \boldsymbol{p}_{i+1})$, where $d(\boldsymbol{p}_k, \boldsymbol{p}_l)$ is the path length from $\boldsymbol{p}_k$ to $\boldsymbol{p}_l$, $\boldsymbol{p}_0 = \boldsymbol{p}_n$, and $\boldsymbol{p}_{n+1} = \boldsymbol{p}_1$.
   - If the total length of the current path over point $\boldsymbol{p}_i^*$ is shorter than over $\boldsymbol{p}_i$, replace the point $\boldsymbol{p}_i$ by $\boldsymbol{p}_i^*$.
4. Compute the path length L_{new} using the refined points.
5. *Termination condition:* If $L_{new} - L < \epsilon$ Stop the refinement. Otherwise $L \leftarrow L_{new}$ and go to Step 3.
6. *Final path construction:* Use the last points and construct the path using the shortest paths among obstacles between two consecutive points.

On-line sampling during the iterations – Local Iterative Optimization (LIO), Váňa & Faigl (IROS 2015).



Part II

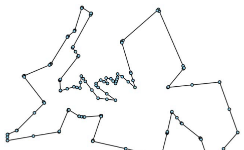
Part 2 – Unsupervised Learning for Multi-goal Planning

Unsupervised Learning based Solution of the TSP

- Iterative learning procedure where neurons (nodes) adapt to the target locations.
- Based on self-organizing map by T. Kohonen.

  Somhom, S., Modares, A., Enkawa, T. (1999)
- Deployed in robotic problems such as inspection and search-and-rescue planning. Faigl, J. et al. (2011)
  - Generalized to polygonal domain with (overlapping) regions.
- Evolved to **Growing Self-Organizing Array** (**GSOA**).

  A general heuristic for various routing problems with neighborhoods; including routing problems with profit aka the orienteering problem.



Unsupervised Learning based Solution of the TSP

Kohonen's type of **unsupervised** two-layered neural network (**Self-Organizing Map**)

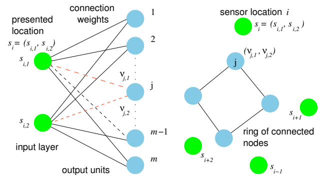
- Neurons' weights represent nodes $\mathcal{N} = \{\boldsymbol{\nu}_1, \ldots, \boldsymbol{\nu}_m\}$ in a plane (input space $\mathbb{R}^2$).
- Nodes are organized into a ring that evolved in the output space $\mathbb{R}^2$).
- Target locations $\boldsymbol{S} = \{\boldsymbol{s}_1, \ldots \boldsymbol{s}_n\}$ are presented to the network in a random order.
- Nodes compete to be winner according to their distance to the presented goal $\boldsymbol{s}$

$$\boldsymbol{\nu}^* = \text{argmin}_{\boldsymbol{\nu} \in \mathcal{N}} |\mathcal{D}(\{\boldsymbol{\nu}, \boldsymbol{s})|.$$

- The winner and its neighbouring nodes are adapted (moved) towards the target according to the neighbouring function $\boldsymbol{\nu}' \leftarrow \mu f(\sigma, d)(\boldsymbol{\nu} - \boldsymbol{s})$

$$f(\sigma, d) = \begin{cases} e^{-\frac{d^2}{\sigma^2}} & \text{for } d < m/n_f, \\ 0 & \text{otherwise,} \end{cases}$$

- Best matching unit $\boldsymbol{\nu}$ to the presented prototype $\mathbf{s}$ is determined according to the distance function $|\mathcal{D}(\boldsymbol{\nu}, s)|$.



- For the Euclidean TSP, $\mathcal{D}$ is the Euclidean distance
- However, for problems with obstacles, the multi-goal path planning, $\mathcal{D}$ should correspond to the length of the shortest, collision-free path.

Fort, J.C. (1988), Angéniol, B. et al. (1988), Somhom, S. et al. (1997), and further improvements.

Unsupervised Learning based Solution of the TSP - Detail

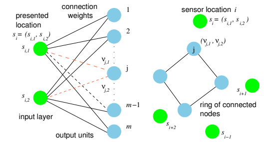
- Target (sensor) locations $S = \{\boldsymbol{s}_1, \ldots, \boldsymbol{s}_n\}$, $\boldsymbol{s}_i \in \mathbb{R}^2$; Neurons $\mathcal{N} = (\boldsymbol{\nu}_1, \ldots, \boldsymbol{\nu}_m)$, $\boldsymbol{\nu}_i \in \mathbb{R}^2$, $m = 2.5n$.
- Learning gain σ; epoch counter i; gain decreasing rate $\alpha = 0.1$; learning rate $\mu = 0.6$.

1. $\mathcal{N} \leftarrow$ init ring of neurons as a small ring around some $\boldsymbol{s}_i \in S$, e.g., a circle with radius 0.5.
2. $i \leftarrow 0$; $\sigma \leftarrow 12.41n + 0.06$;
3. $I \leftarrow \emptyset$ *//clear inhibited neurons*
4. **foreach** $\boldsymbol{s} \in \Pi(S)$ (a permutation of S)
   - 4.1 $\boldsymbol{\nu}^* \leftarrow \text{argmin}_{\boldsymbol{\nu} \in \mathcal{N} \setminus I} \|(\boldsymbol{\nu}, \boldsymbol{s})\|$
   - 4.2 **foreach** $\boldsymbol{\nu}$ in d neighborhood of $\boldsymbol{\nu}^*$

$$\boldsymbol{\nu} \leftarrow \boldsymbol{\nu} + \mu f(\sigma, d)(\boldsymbol{s} - \boldsymbol{\nu})$$

$$f(\sigma, d) = \begin{cases} e^{-\frac{d^2}{\sigma^2}} & \text{for } d < 0.2m, \\ 0 & \text{otherwise,} \end{cases}$$

   - 4.3 $I \leftarrow I \bigcup \{\boldsymbol{\nu}^*\}$ // inhibit the winner
5. $\sigma \leftarrow (1 - \alpha)\sigma$; $i \leftarrow i + 1$;
6. `If` (**termination condition** is not satisfied) `Goto` Step 3; `Otherwise` retrieve solution.

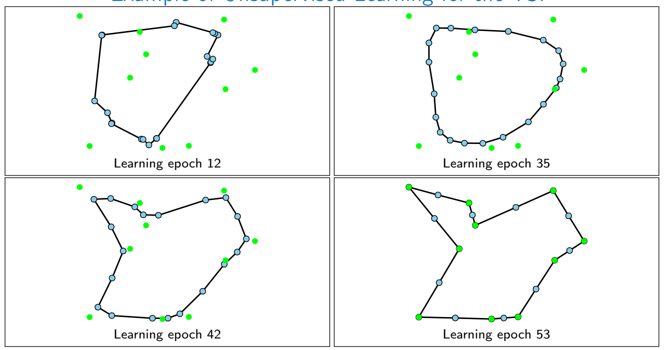


Termination condition can be

- Maximal number of learning epochs $i \leq i_{max}$, e.g., $i_{max} = 120$.
- Winner neurons are negligibly close to sensor locations, e.g., less than 0.001.

Somhom, S., Modares, A., Enkawa, T. (1999): Competition-based neural network for the multiple travelling salesmen problem with minmax objective. Computers & Operations Research.
Faigl, J. et al. (2011): An application of the self-organizing map in the non-Euclidean Traveling Salesman Problem. Neurocomputing.

Example of Unsupervised Learning for the TSP

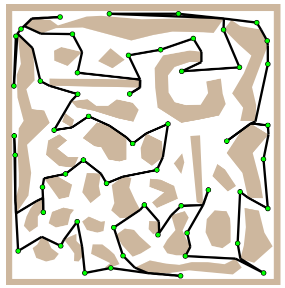


Unsupervised Learning for the Multi-Goal Path Planning

- Unsupervised learning procedure for the Multi-goal Path Planning (**MTP**) problem a robotic variant of the Traveling Salesman Problem (TSP).

Algorithm 2: SOM-based MTP solver

$\mathcal{N} \leftarrow$ initialization$(\nu_1, \ldots, \nu_m)$;
repeat
  error $\leftarrow 0$;
  foreach $g \in \Pi(\boldsymbol{S})$ **do**
    $\nu^* \leftarrow$
    selectWinner $\text{argmin}_{\nu \in \mathcal{N}} |S(g, \nu)|$;
    adapt$(S(g, \nu), \mu f(\sigma, l)|S(g, \nu)|)$;
    error $\leftarrow \max\{error, |S(g, \nu^\star)|\}$;
  $\sigma \leftarrow (1 - \alpha)\sigma$;
until *error* $\leq \delta$;



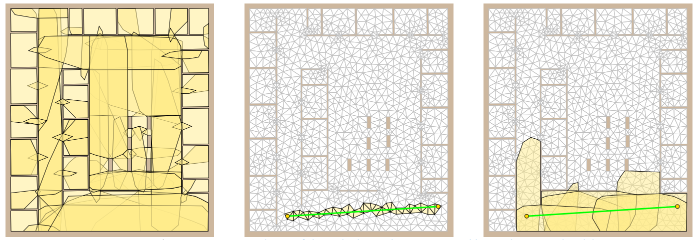
- For multi-goal path planning – the **selectWinner** and **adapt** procedures are based on the solution of the **path planning problem.**

Faigl, J., Kulich, M., Vonásek, V., Přeučil, L.: *An Application of Self-Organizing Map in the non-Euclidean Traveling Salesman Problem*, **Neurocomputing**, 74(5):671-679, 2011.

SOM for the TSP in the Watchman Route Problem – *Inspection Planning*

During the unsupervised learning, we can compute coverage of $\mathcal{W}$ from the current ring (solution represented by the neurons) and adapt the network towards uncovered parts of $\mathcal{W}$.

- Convex cover set of $\mathcal{W}$ created on top of a triangular mesh.
- Incident convex polygons with a straight line segment are found by walking in a triangular mesh.

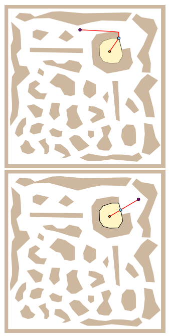


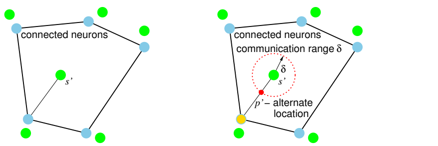
Faigl, J.: *Approximate solution of the multiple watchman routes problem with restricted visibility range*, **IEEE Transactions on Neural Networks**, 21(10):1668-1679, 2010.

Unsupervised Learning for the TSPN

- A suitable location of the region can be sampled during the winner selection.
- We can use the centroid of the region for the shortest path computation from ν to the region r presented to the network.
- Then, an intersection point of the path with the region can be used as an alternate location.

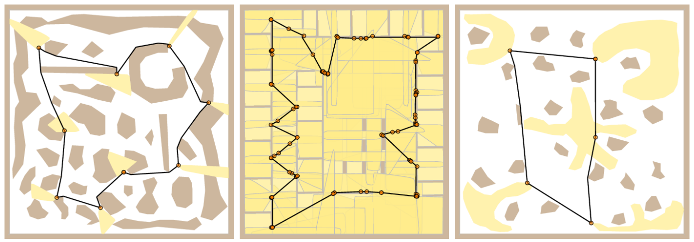
  Faigl, J. et al. (2013): Visiting convex regions in a polygonal map. Robotics and Autonomous Systems.
- For the Euclidean TSPN with disk-shaped δ neighborhoods, we can compute the alternate location directly from the Euclidean distance.





SOM for the Traveling Salesman Problem with Neighborhoods (TSPN)

- Unsupervised learning of the SOM for the TSP allows to generalize the adaptation procedure to the TSPN.
- It also provides solutions for non-convex regions, overlapping regions, and coverage problems.

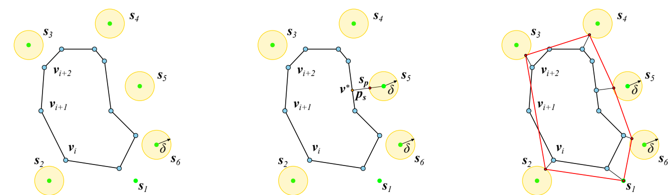


Polygonal Goals	Convex Cover Set	Non-Convex Goals
n=9, T= 0.32 s	n=106, T=5.1 s	n=5, T=0.1 s

Faigl, J., Vonásek, V., Přeučil, L.: *Visiting Convex Regions in a Polygonal Map*, **Robotics and Autonomous Systems**, 61(10):1070–1083, 2013.

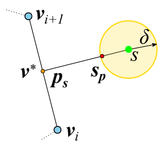
Growing Self-Organizing Array (GSOA)

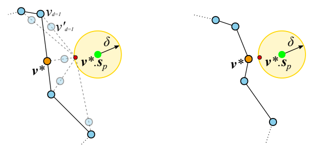
- **Growing Self-Organizing Array** (**GSOA**) is generalization of the unsupervised learning to routing problems motivated by data collection planning, i.e., routing with neighborhoods such as the **Close Enough TSP**.
- The GSOA is an array of nodes $\mathcal{N} = \{\nu_1, \ldots, \nu_M\}$ that evolves in the problem space using unsupervised learning.
- The array adapts to each $s \in S$ (in a random order) and for each s a **new winner node** $\boldsymbol{\nu}^*$ is determined together with the corresponding $\boldsymbol{s}_p$, such that $\|(\boldsymbol{s}_p, \boldsymbol{s})\| \leq \delta(s)$. It adaptively adjusts the number of nodes.
- The winner and its neighborhoods are adapted (moved) towards $\boldsymbol{s}_p$.
- After the adaptation to all $\boldsymbol{s} \in S$, each $\boldsymbol{s}$ has its $\boldsymbol{\nu}$ and $\boldsymbol{s}_p$, and the array defines the sequence Σ and the requested waypoints P.



GSOA – Winner Selection and Its Adaptation

- Selecting winner node ν^* for s and its waypoint s_p
- Winner adaptation

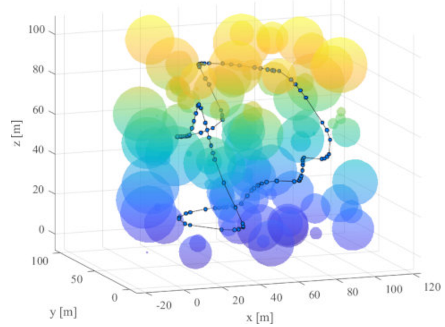




- For each $s \in S$, we create new node ν^*, and therefore, all not winning nodes are removed after processing all locations in S (one learning epoch) to balance the number of nodes in the GSOA.
- After **each learning epoch**, the **GSOA encodes a feasible solution of the CETSP**.
- The power of adaptation is decreasing using a cooling schedule after each learning epoch.
- The GSOA converges to a stable solution in tens of epochs. Number of epochs can be set.

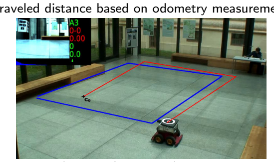
Faigl, J. (2018): GSOA: Growing Self-Organizing Array - Unsupervised learning for the Close-Enough Traveling Salesman Problem and other routing problems. Neurocomputing 312: 120-134 (2018).

GSOA Evolution in solving the 3D CETSP



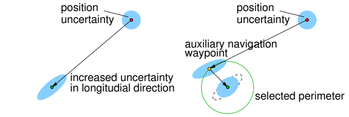
Example – TSPN for Planning with Localization Uncertainty

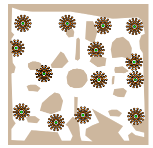
- Teach-and-repeat autonomous navigation using vision-based bearing corrections that are more precise than estimation of the traveled distance based on odometry measurements.

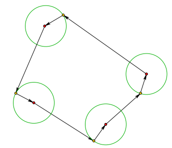


Krajník, T., Faigl, J., Vonásek, V., Košnar, K., Kulich, M., and Přeučil, L.: *Simple yet stable bearing-only navigation*, Journal of Field Robotics, 27(5):511-533, 2010.

- The localization uncertainty can be decreased by visiting auxiliary navigation waypoints prior the target locations.
- It can be formulated as a variant of the TSPN with auxiliary navigation waypoints.





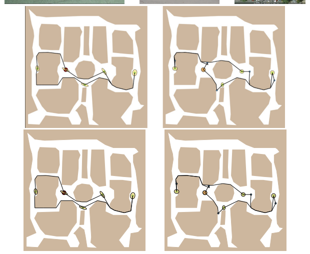


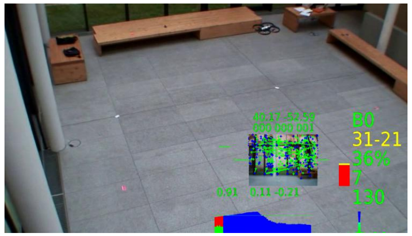
- The adaptation procedure is modified to select the auxiliary navigation waypoint to decrease the expected localization error at the target locations.

Faigl, J., Krajník, T., Vonásek, V., and Přeučil, L.: *On localization uncertainty in an autonomous inspection*, IEEE International Conference on Robotics and Automation (ICRA), 2012, pp. 1119-1124.

Example – Results on the TSPN for Planning with Localization Uncertainty

- Deployment of the method in indoor and outdoor environment with ground mobile robots and aerial vehicle in indoor environment.
- In the indoor with the small MMP5 robot, the error decreased from 16.6 cm → 12.8 cm.
- In the outdoor with the P3AT robot, the real overall error at the goals decreased from 0.89 m → 0.58 m (about 35%).
- Deployment with a small aerial vehicle the Parrot AR.Drone, the success of the locations' visits improved from 83% to 95%.





TSP: L=184 m, E_{avg}=0.57 m TSPN: L=202 m, E_{avg}=0.35 m

Summary of the Lecture

Topics Discussed

- Robotic information gathering in inspection missions
- Inspection planning and multi-goal path planning - coverage planning
- **Multi-goal path planning (MTP)**
  - Robotic Traveling Salesman Problem (TSP)
  - Traveling Salesman Problem with Neighborhoods (TSPN) and Close Enough Traveling Salesman Problem (CETSP)
    - **Decoupled** and **Sampling-based** approaches
    - TSP can be solved by efficient heuristics such as **LKH**
    - Optimal, approximation, and heuristics solutions
    - **Generalized TSP** (**GTSP**)
- Next: Data collection planning