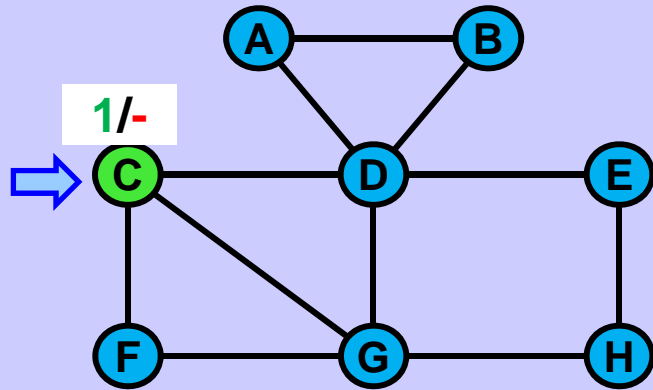
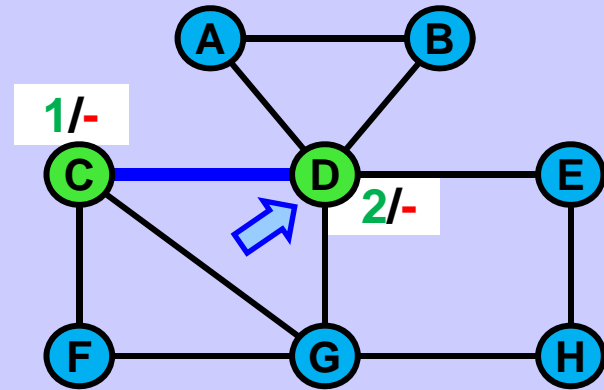


# Depth-first search (DFS) in a graph



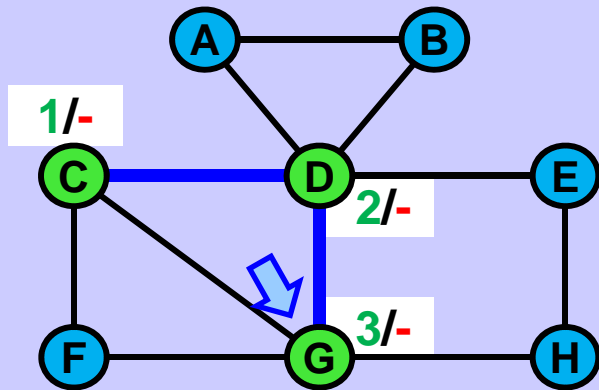
Stack C

Output C



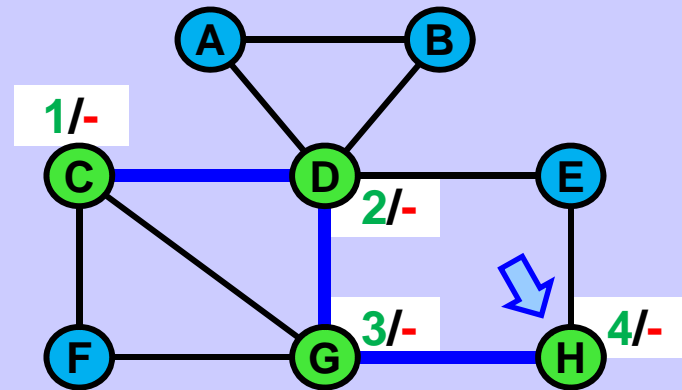
Stack C D

Output C D



Stack C D G

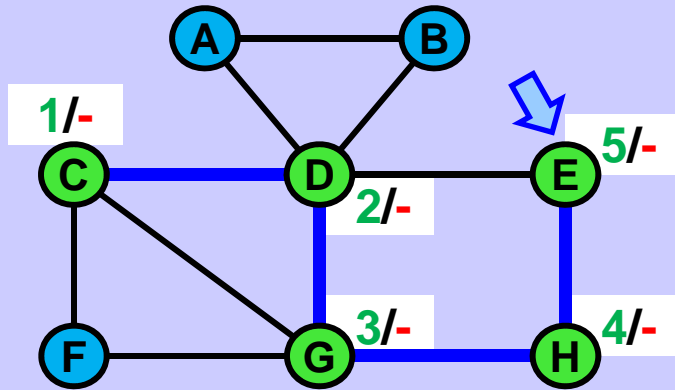
Output C D G



Stack C D G H

Output C D G H

# Depth-first search (DFS) in a graph

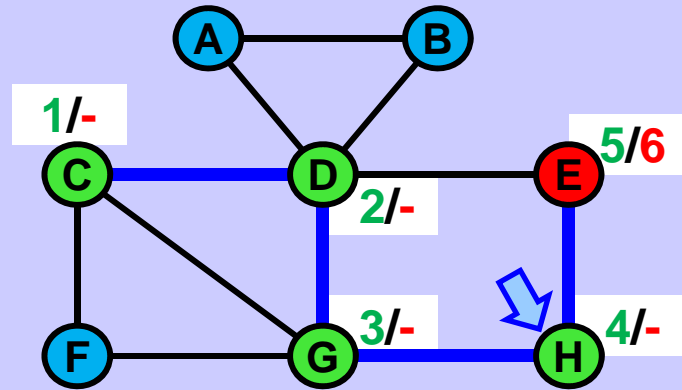


Stack

C D G H E

Output

C D G H E

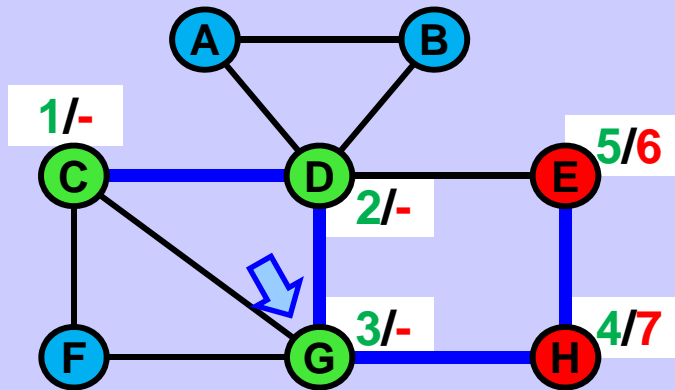


Stack

C D G H

Output

C D G H E

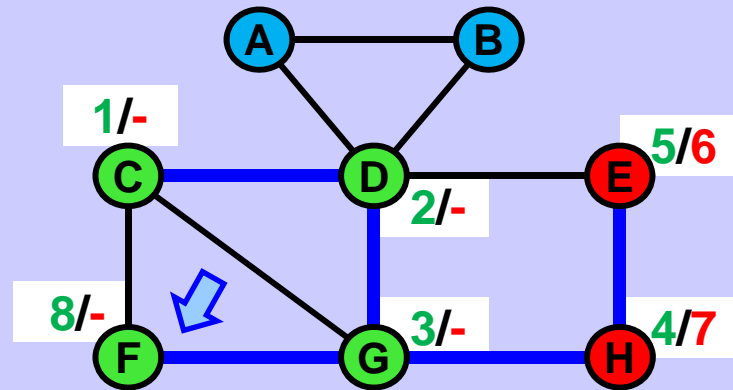


Stack

C D G

Output

C D G H E



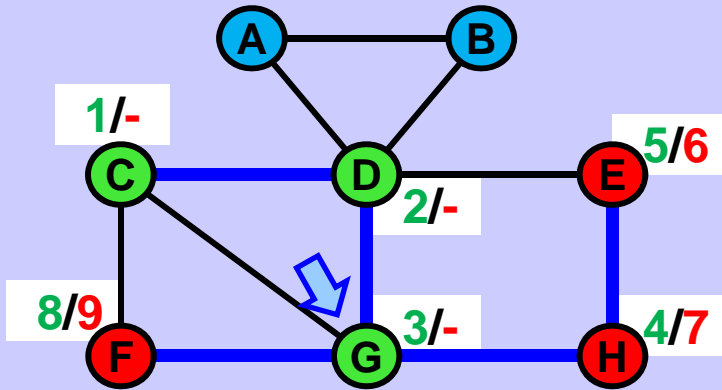
Stack

C D G F

Output

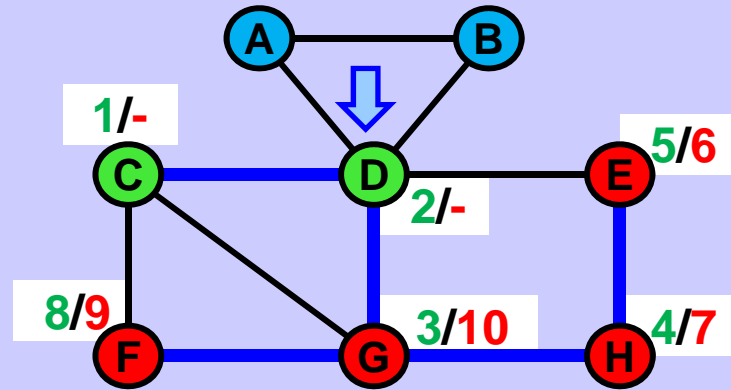
C D G H E F

# Depth-first search (DFS) in a graph



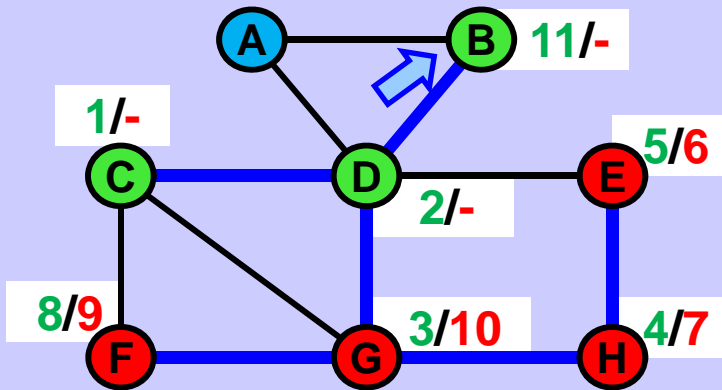
Stack: C D G

Output: C D G H E F



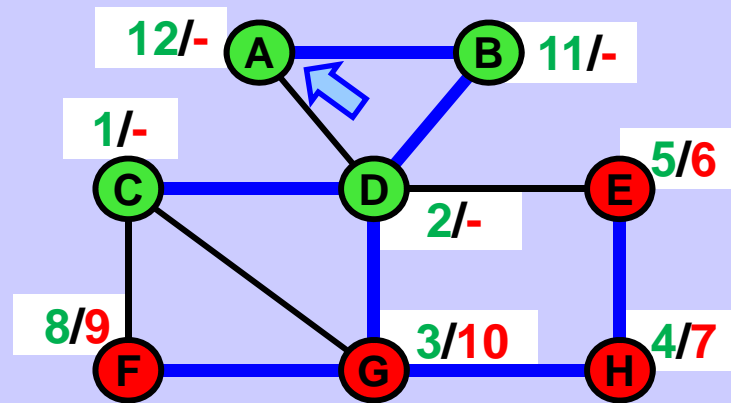
Stack: C D

Output: C D G H E F



Stack: C D B

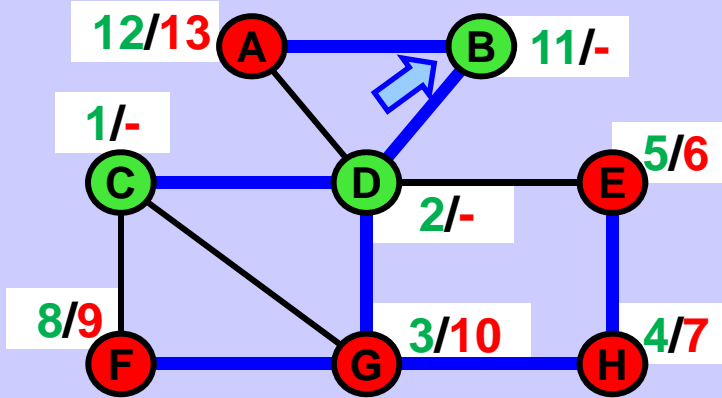
Output: C D G H E F B



Stack: C D B A

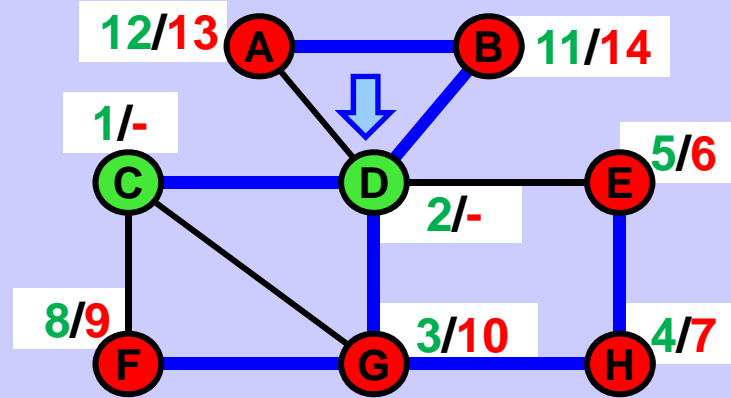
Output: C D G H E F B A

# Depth-first search (DFS) in a graph



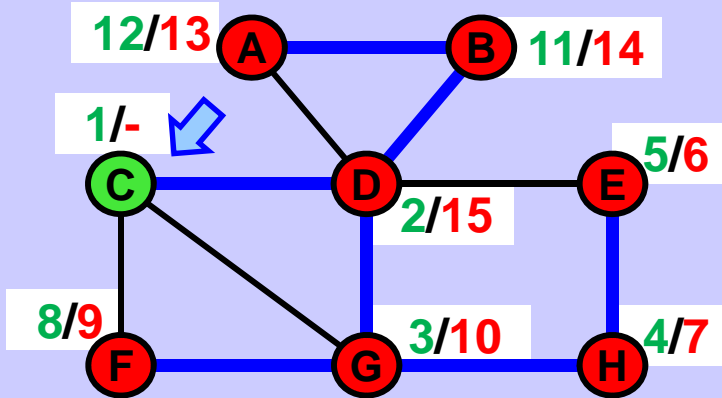
Stack: C D B

Output: C D G H E F B A



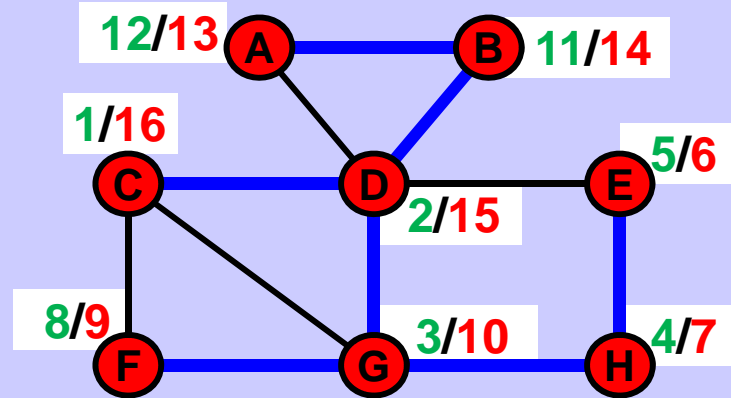
Stack: C D

Output: C D G H E F B A



Stack: C

Output: C D G H E F B A



Stack:

Output: C D G H E F B A

## Depth-first search (DFS) in a graph

### Life cycle of a node during the DFS

Fresh - open - closed

#### Fresh

Fresh nodes are those nodes which have not been visited yet.

Before the search starts, all nodes are fresh.

A fresh node becomes open when it is visited for the first time.

The set of fresh nodes shrinks or remains the same during the search.

#### Open

Open nodes are those nodes which have been already visited but were not closed yet.

The set of open nodes may grow and shrink during the search.

#### Closed

Closed nodes are those nodes which will not be visited any more.

When each neighbour of a current node in the search is either open or closed the current node becomes closed.

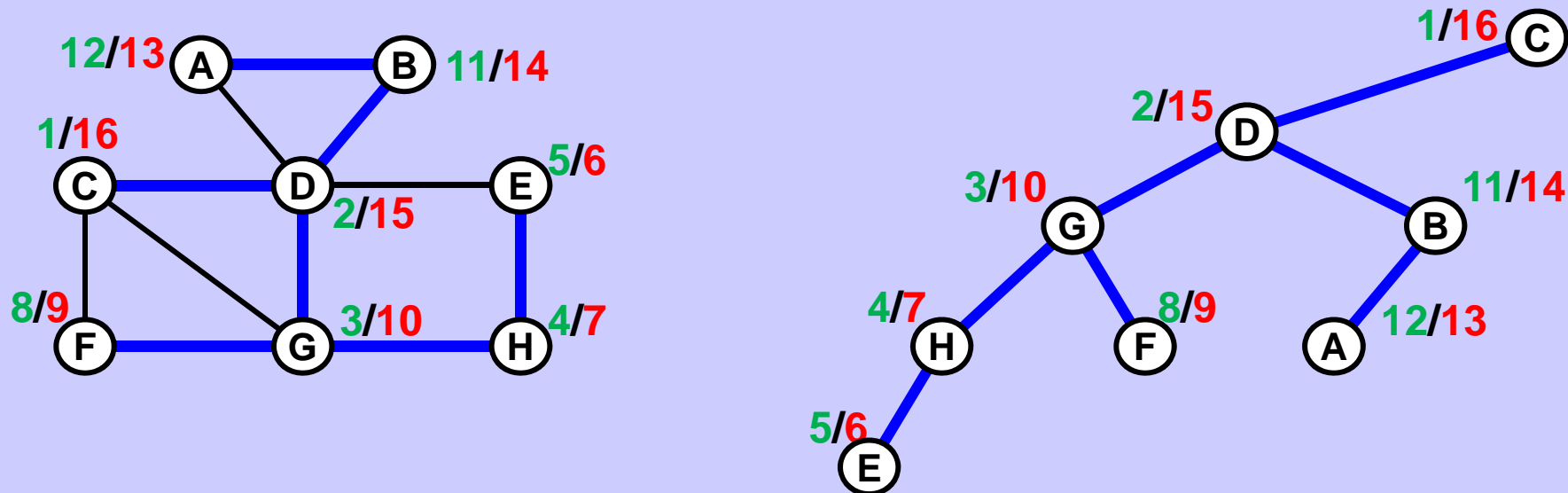
The set of closed does only grow during the search.

When the search terminates all nodes are closed.



## Depth-first search (DFS) in a graph

DFS-tree  
with open and close times of the nodes



Note, that in a subtree with a root  $X$  it holds for each node  $Y \neq X$ :  
 $\text{OpenTime}(X) < \text{OpenTime}(Y) < \text{CloseTime}(Y) < \text{CloseTime}(X)$ .  
 On the other hand, when  $Y$  is not a part of the subtree rooted in  $X$  then  
 $\text{CloseTime}(Y) < \text{OpenTime}(X)$  or  $\text{CloseTime}(X) < \text{OpenTime}(Y)$

The number of nodes in the subtree rooted in  $X$  is always  
 $(\text{CloseTime}(X) + 1 - \text{OpenTime}(X)) / 2$ .

## Depth-first search (DFS) in a graph

### Asymptotic complexity

Each single operation on the stack and each single operation on additional data structures and nodes/edges is of constant time (and memory) complexity.

Each node enters the stack only once and it leaves the stack only once.

The state of the node (fresh/open/closed) is tested more times.

The number of these tests is equal to the degree of the node (the search tries to access the node from its neighbours).

The sum of all node degrees is equal to twice the number of edges, in any graph.

In total

$$\Theta( |V| + |E| ).$$