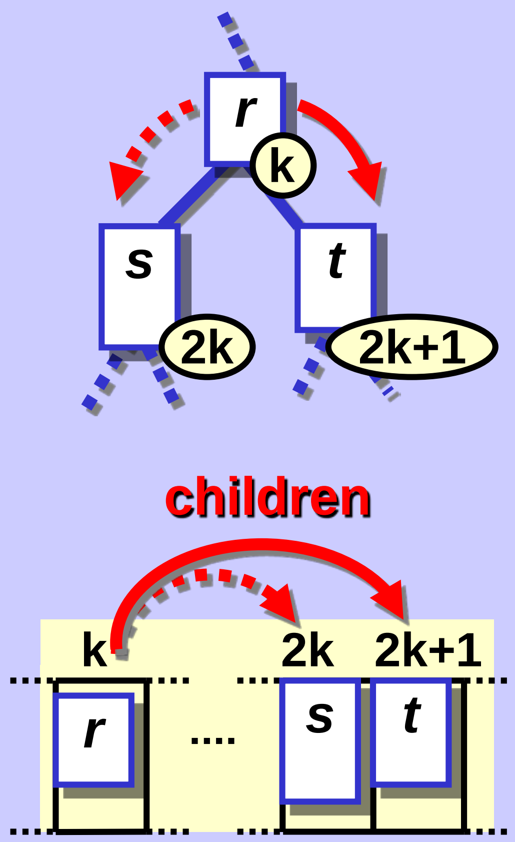
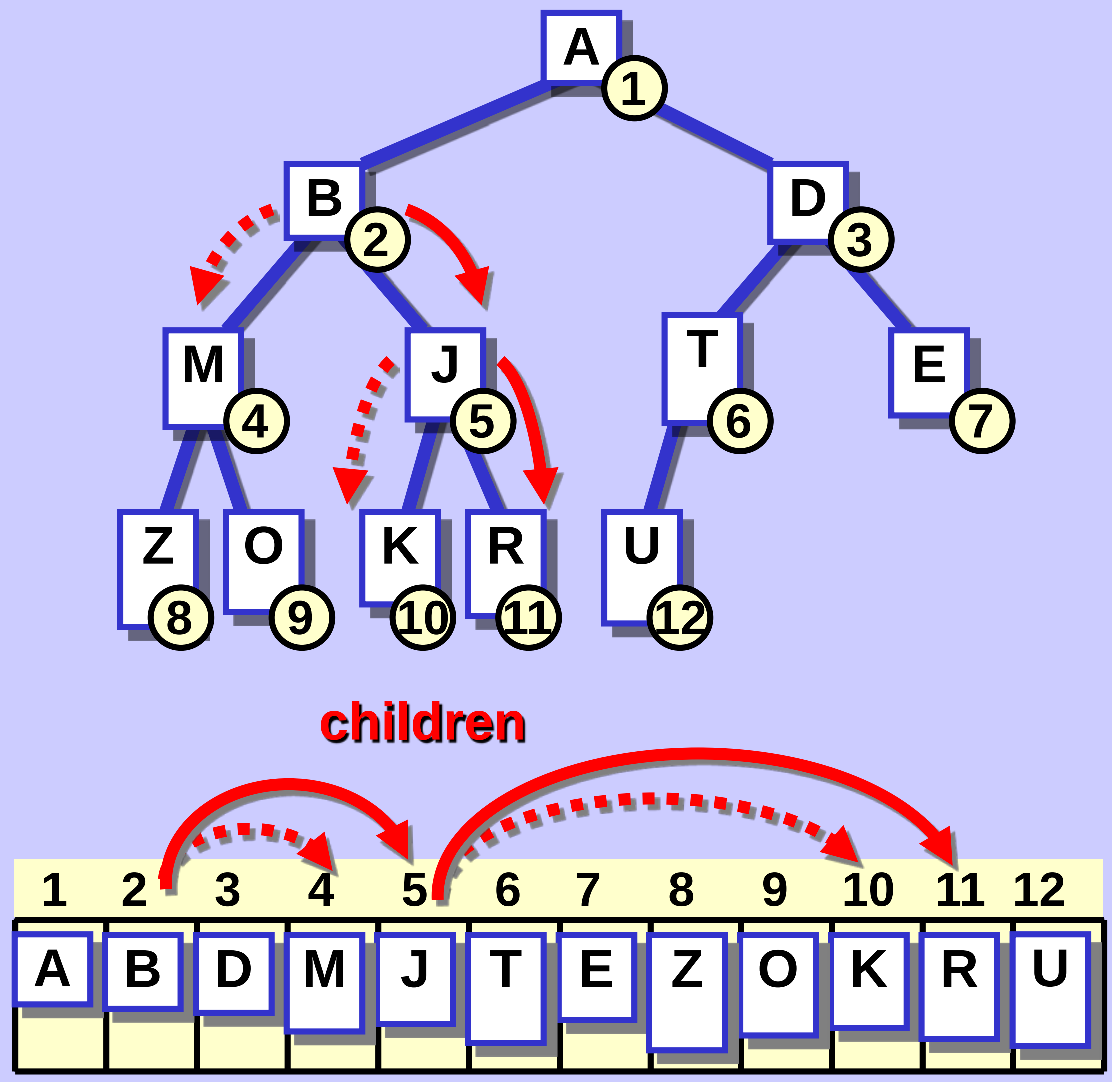
Heap in an array

Heap stored in an array

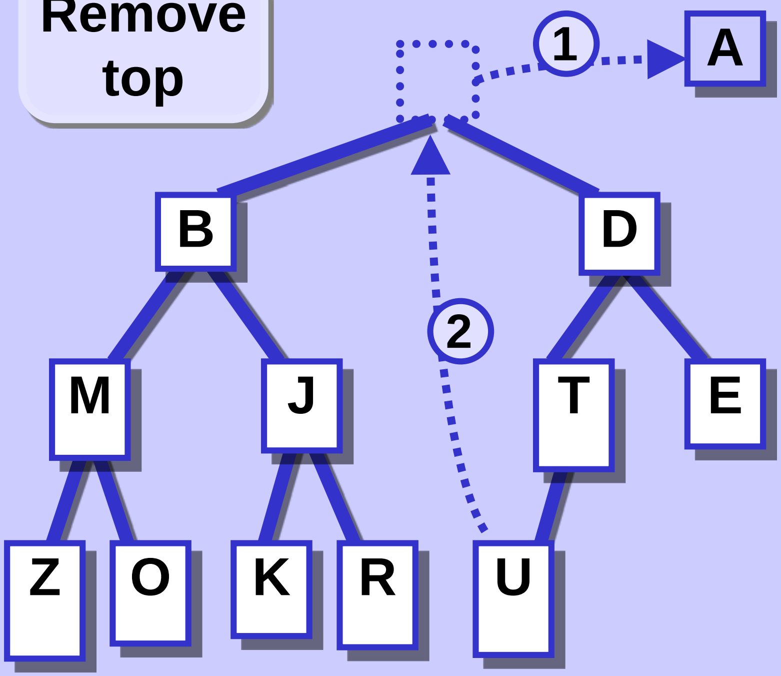




Heap repair

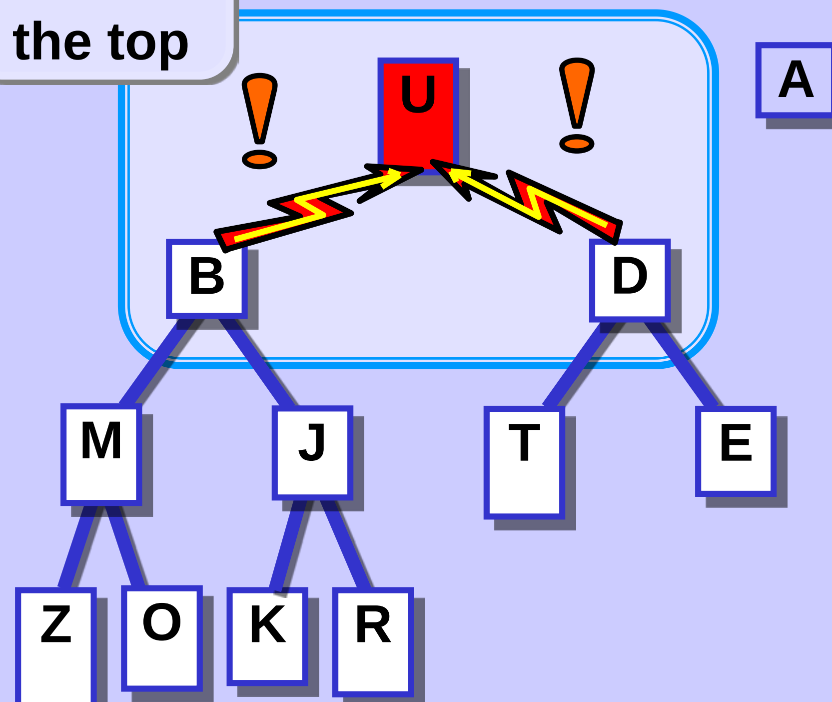
Top removed (1)

1 Remove top



2 last $\longrightarrow$ first

3 Put at the top



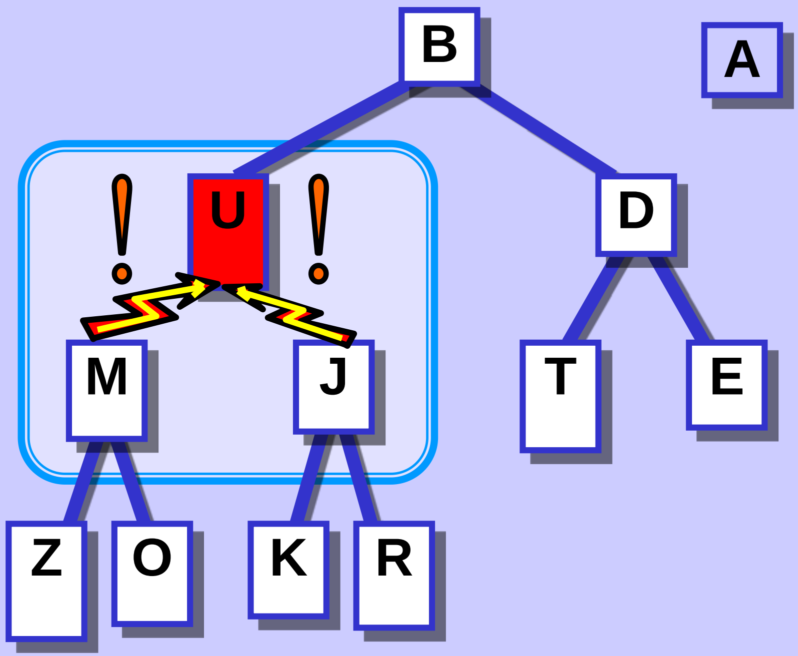
U > B, U > D, <u>B < D</u>

$\Rightarrow$ swap B $\longleftrightarrow$ U

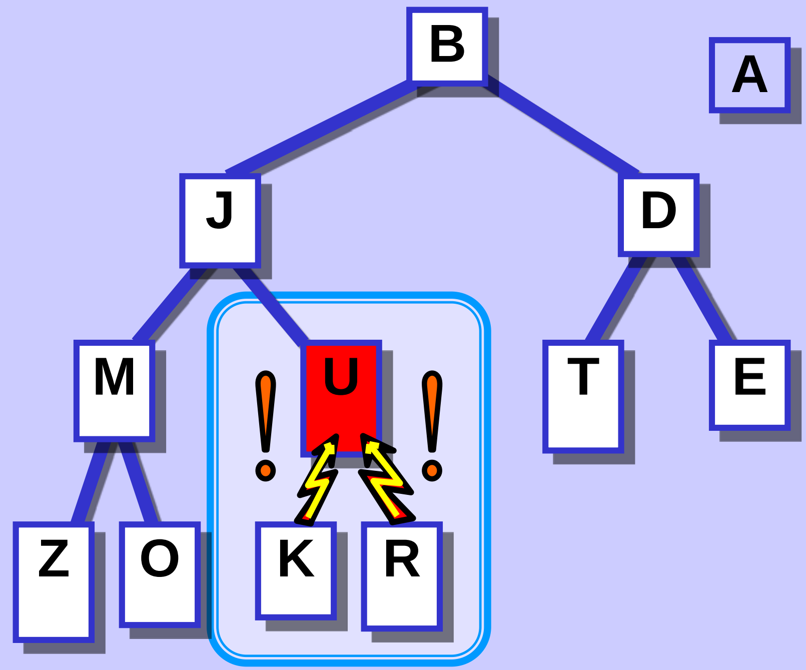
Heap repair

Top removed (2)

③ Put at the top - cont...



U > M, U > J, <u>J < M</u>
⇒ swap J ⟷ U

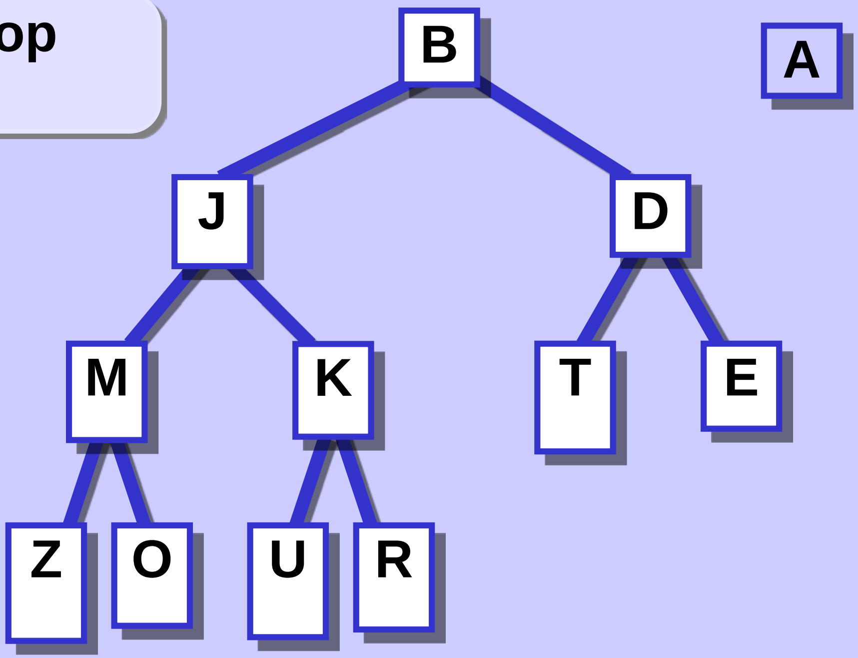


U > K, U > R, <u>K < R</u>
⇒ swap K ⟷ U

Heap repair

Top removed (3)

3 Put at the top
- done.



New heap

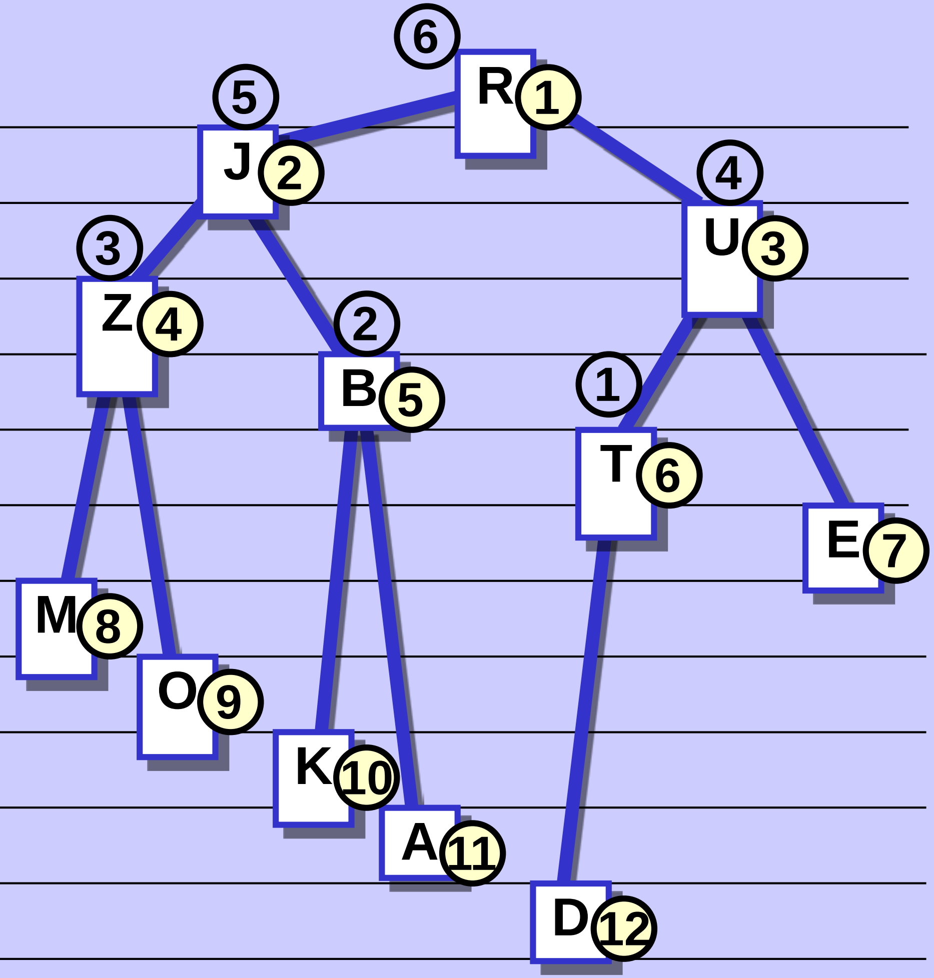
Make a heap -- Heapify

Array

6	1	R
5	2	J
4	3	U
3	4	Z
2	5	B
1	6	T
	7	E
	8	M
	9	O
	10	K
	11	A
	12	D

Array elements ordered randomly

Not a heap



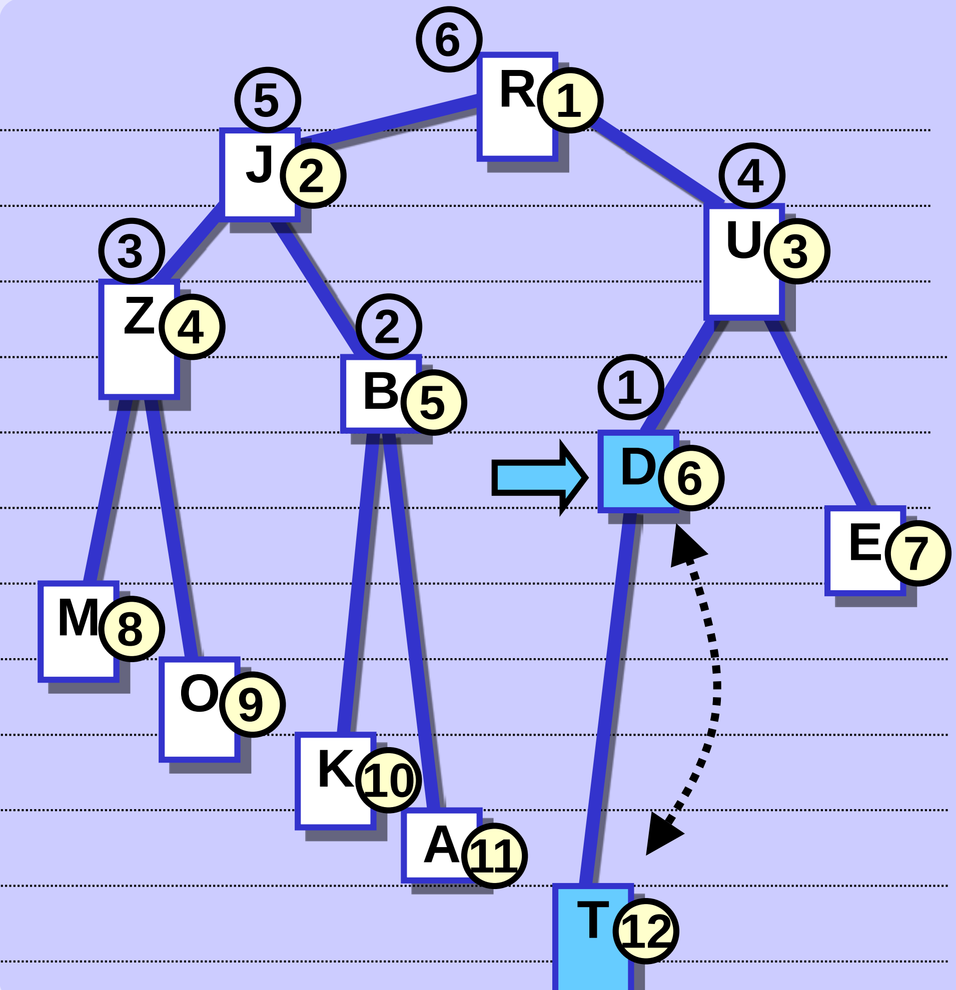
Make a heap -- Heapify

Moves

Array

Currently created heap

6	1	R
5	2	J
4	3	U
3	4	Z
2	5	B
1	6	D
	7	E
	8	M
	9	O
	10	K
	11	A
	12	T



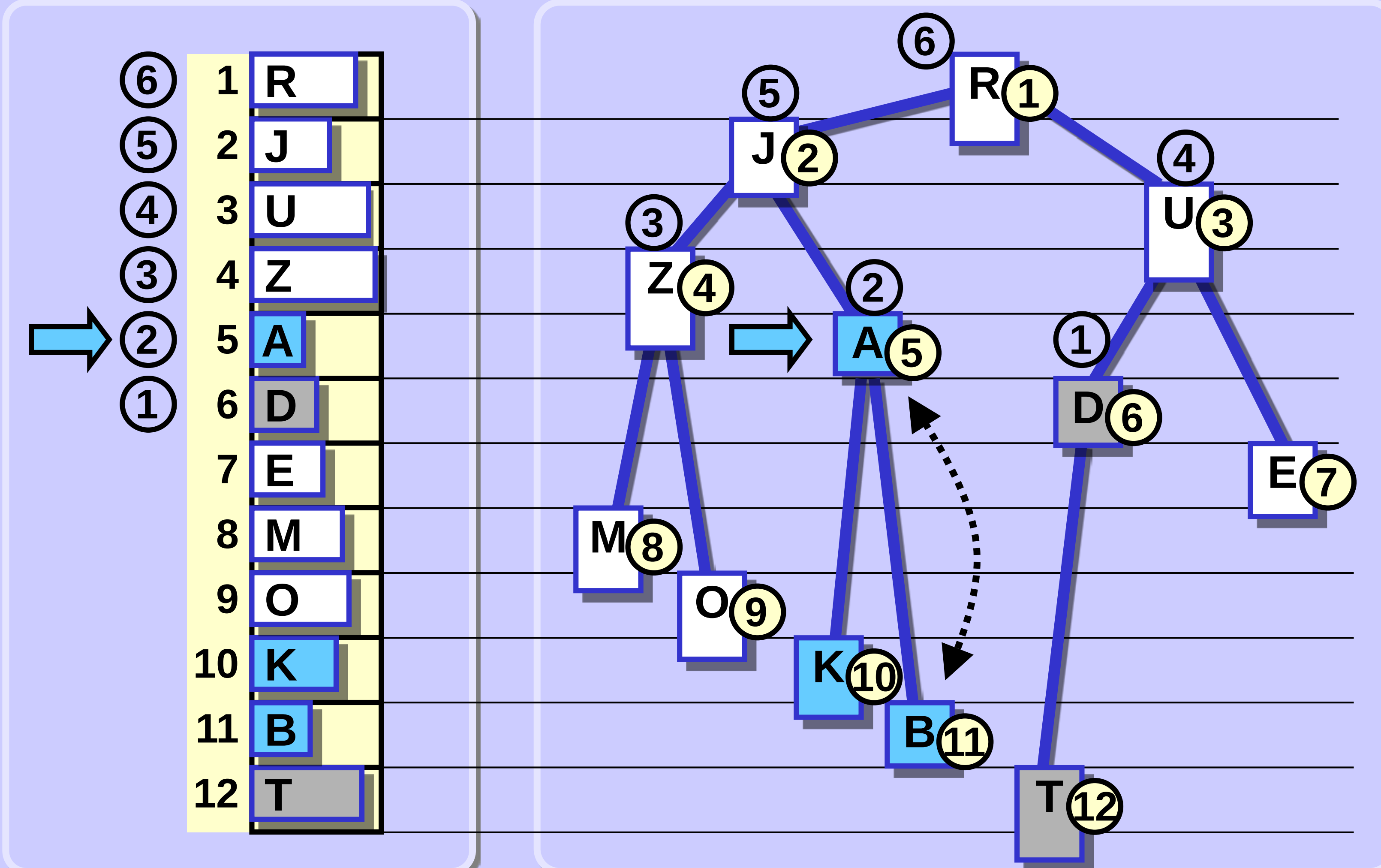
Make a heap -- Heapify

Earlier heap(s)

Moves

Currently created heap

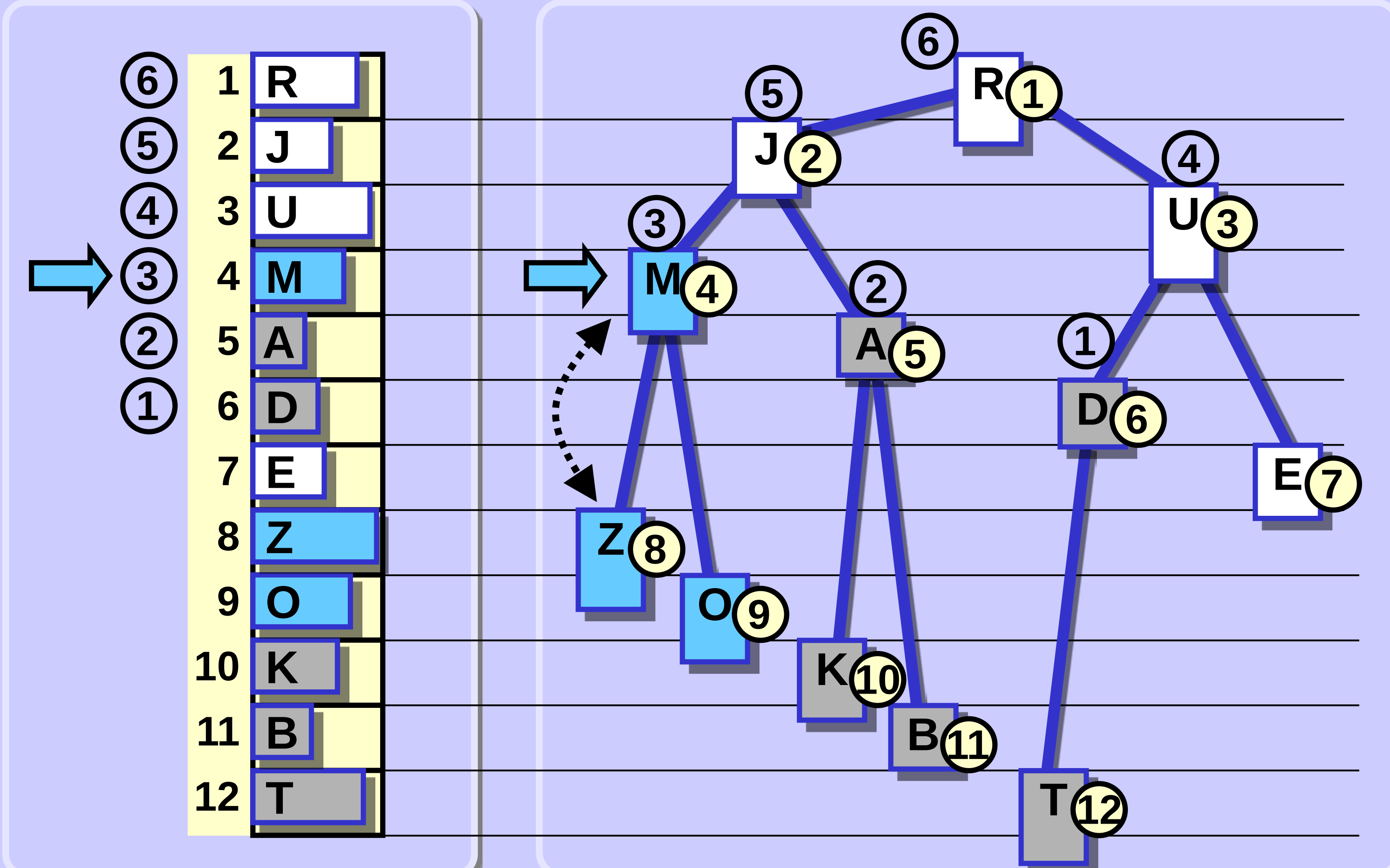
Array



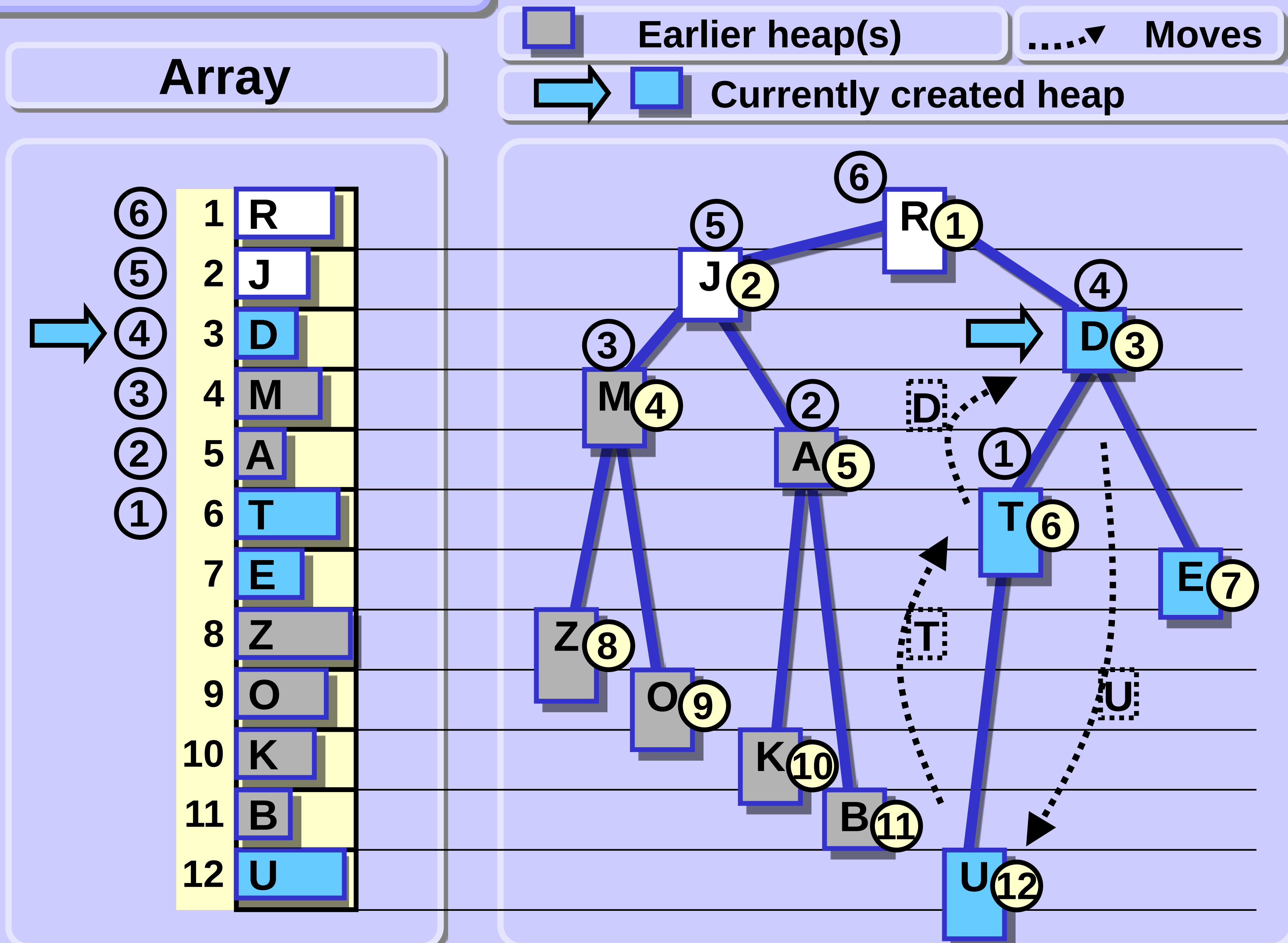
Make a heap -- Heapify

Earlier heap(s) | Moves

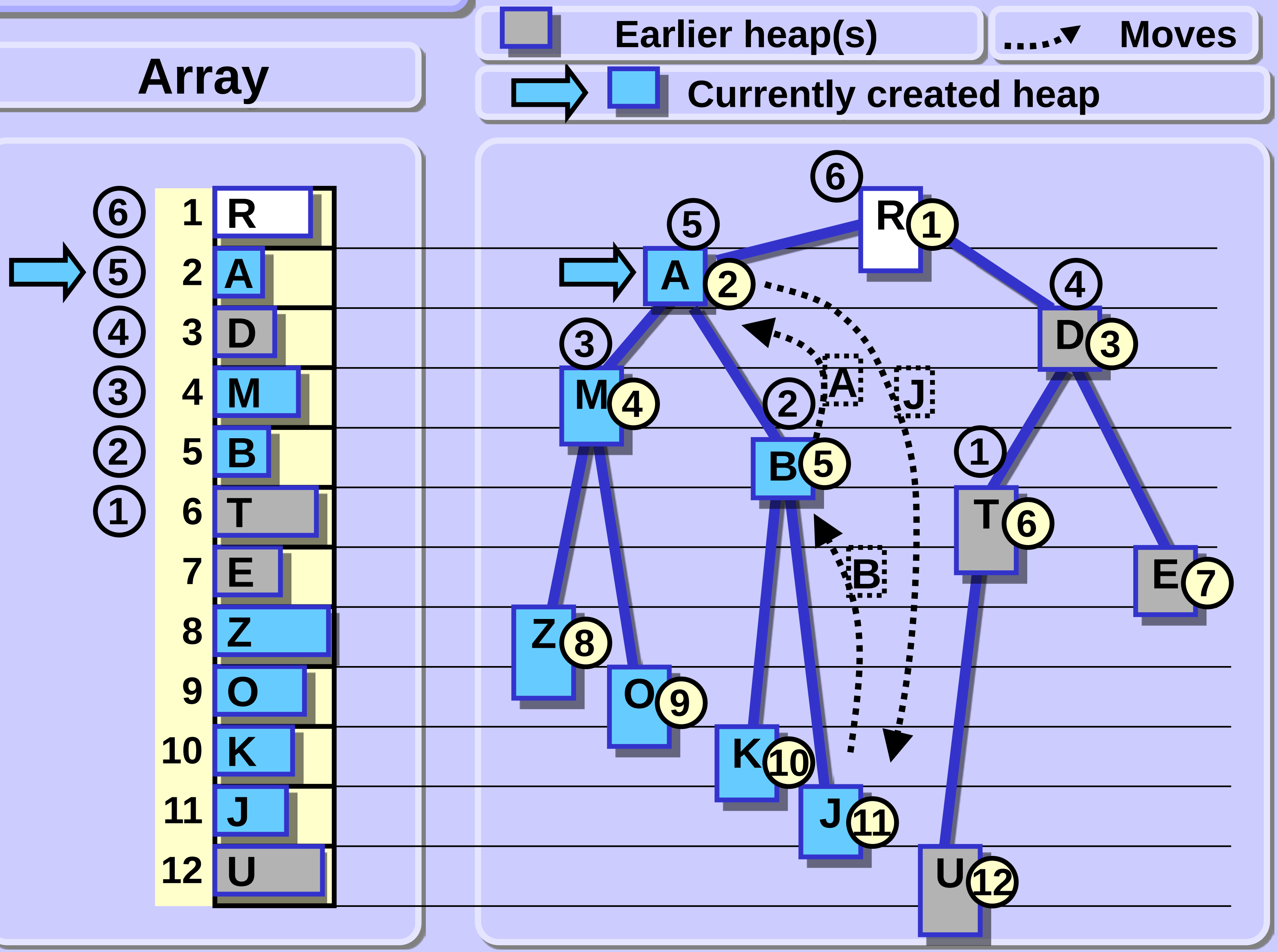
Currently created heap



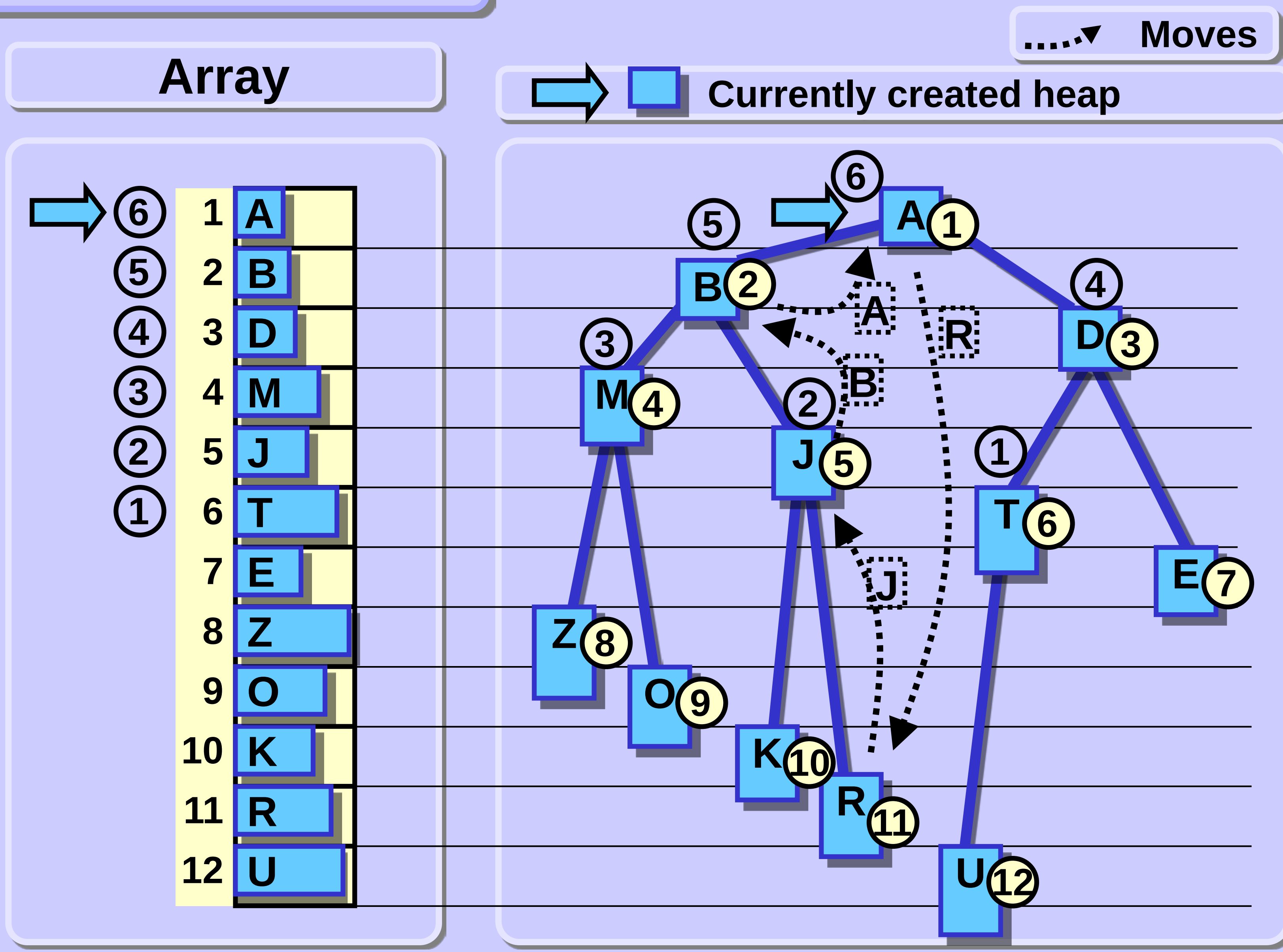
Make a heap -- Heapify



Make a heap -- Heapify



Make a heap -- Heapify

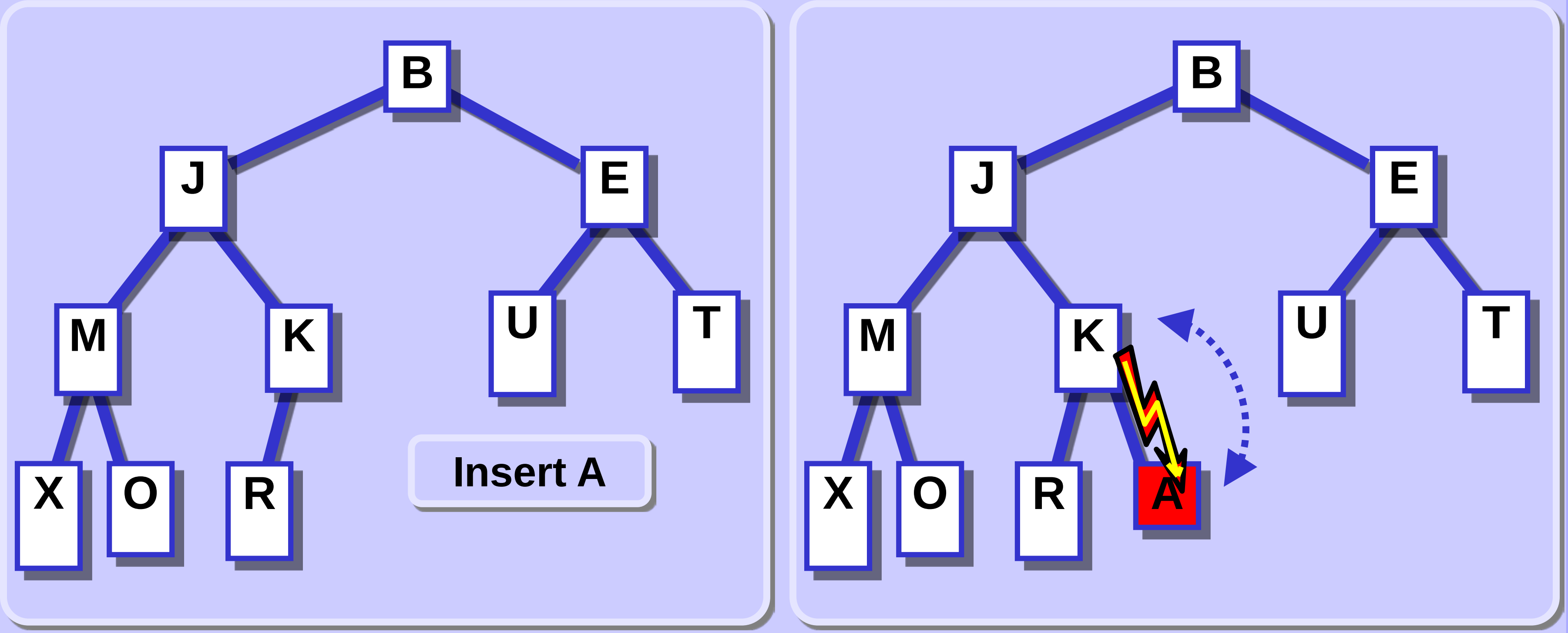


Heapify

```
def repairTop (arr, top, bottom):
    i = top     # arr[2*i] and arr[2*i+1]
    j = i*2     # are successors of arr[i]
    topVal = arr[top]
    # try to find a successor < topVal
    if j < bottom and arr[j] > arr[j+1]: j += 1
    # while successors < topVal move successors up
    while j <= bottom and topVal > arr[j]:
        arr[i] = arr[j]
        i = j; j = j*2     # move to next suceessor
        if j < bottom and arr[j] > arr[j+1]: j += 1
    # put topVal to its correct place
    arr[i] = topVal
```

```
def heapify (arr):
    n = len(arr)-1
    for i in range(n//2, 0, -1): # progress backwards!
        repairTop(arr, i, n)
```

Priority queue implemented with binary heap -- Insert

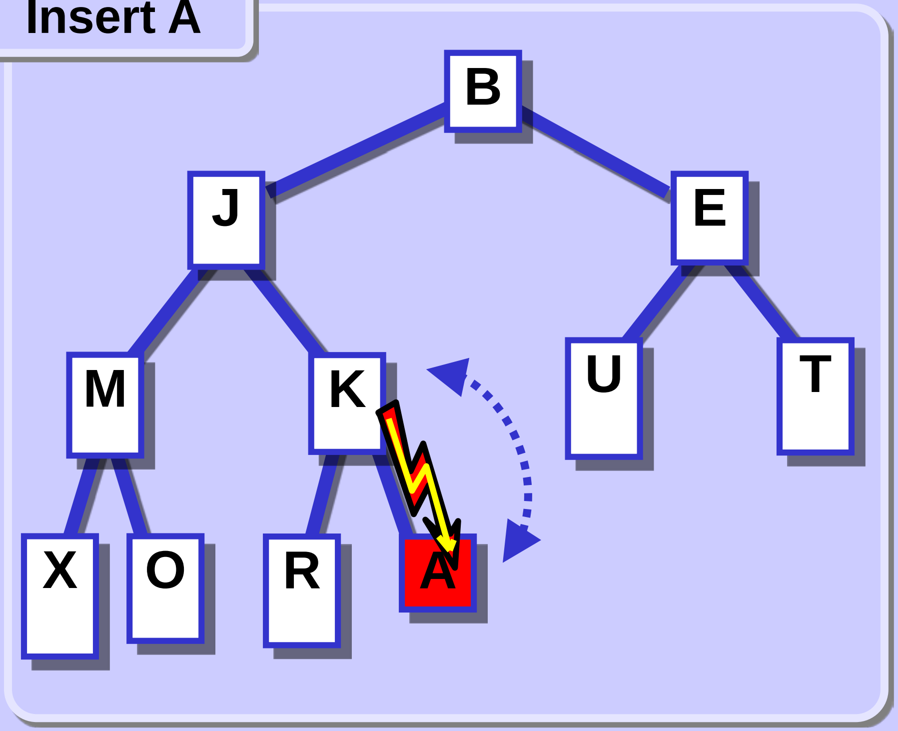


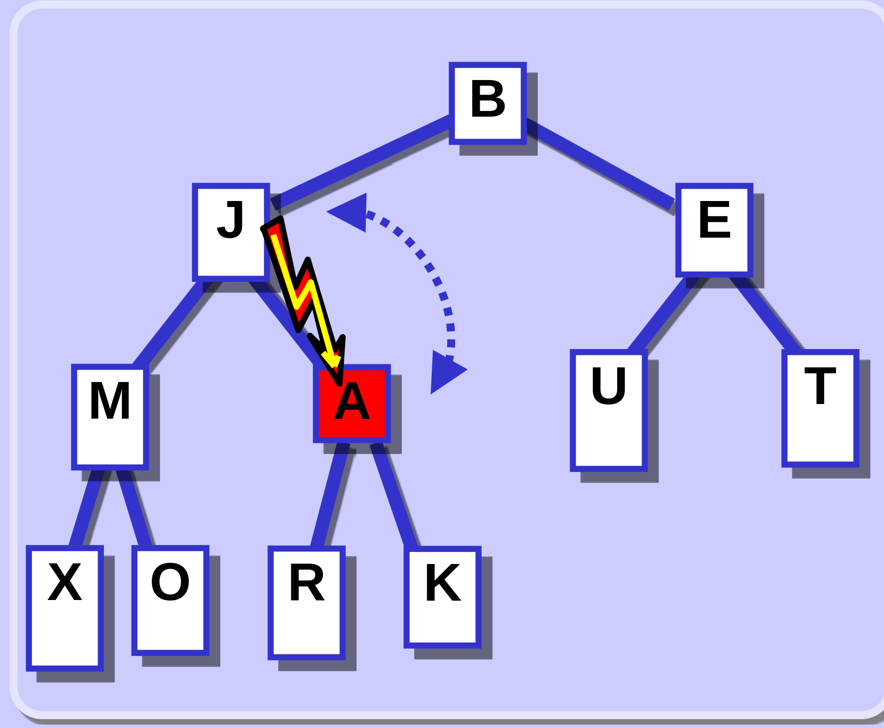
Insert the element at the end of the queue (end of the heap).

In most cases, this violates the heap property
and the heap has to be repaired.

Priority queue implemented with binary heap -- Insert

Insert A



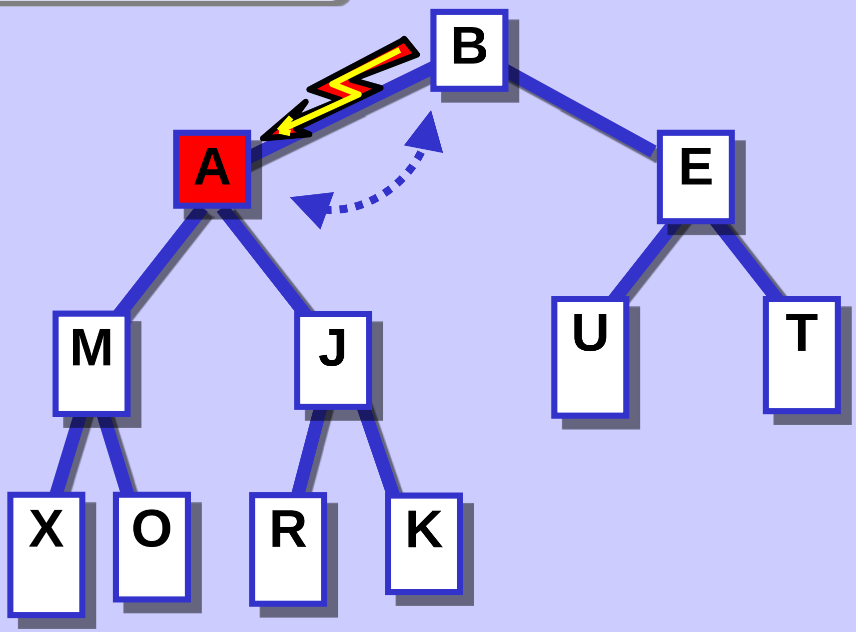


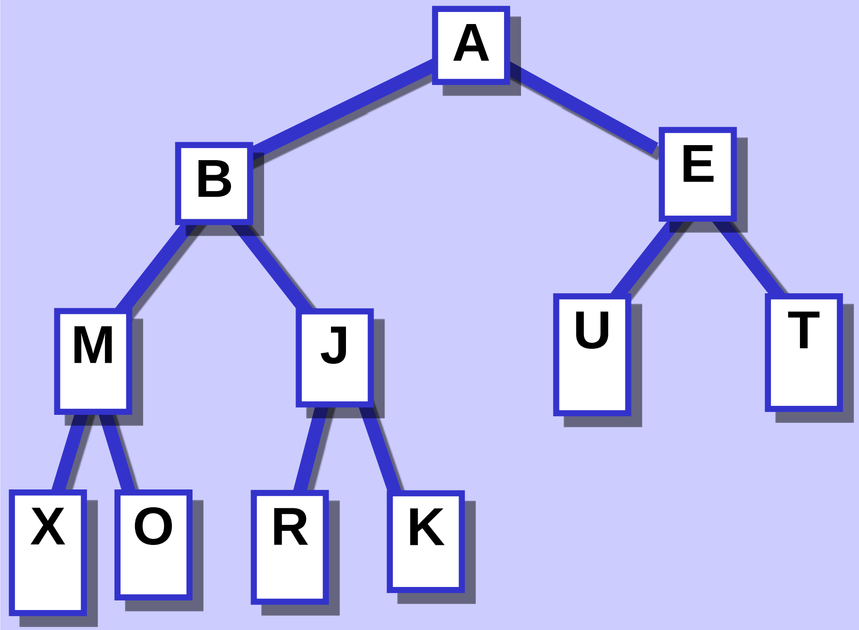
Heap property is violated,
swap the element with its parent.

Heap property is still violated,
swap the element with its parent.

Priority queue implemented with binary heap -- Insert

Inserting A



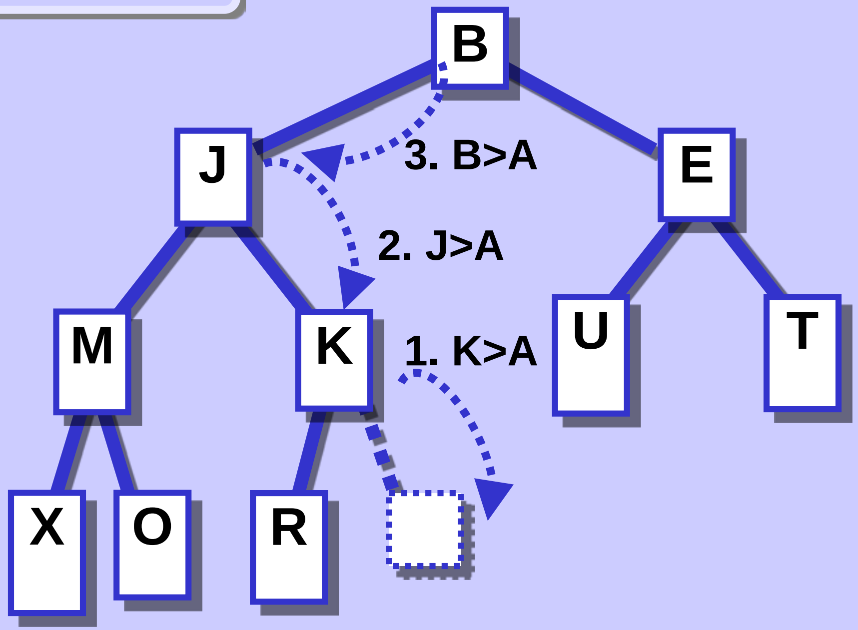


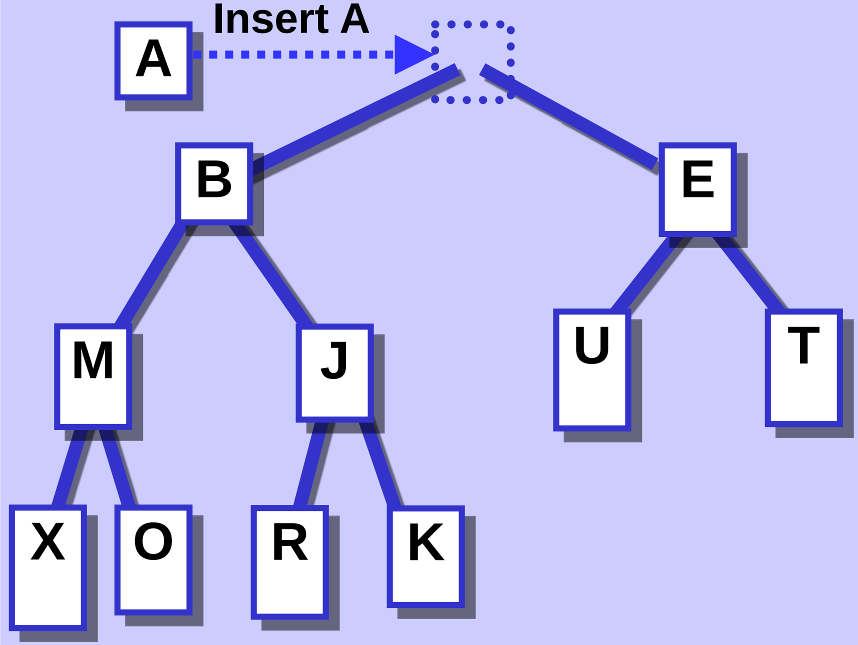
Heap property is still violated,
swap the element with its parent.

Heap property is respected,
the inserted element has found
its place in the queue (heap).

Binary heap -- Insert element more effectively

Insert A





Do not insert the element at the end of the queue.
First, find its place and while searching move down other elements encountered in the search.

Finally, store the inserted element at its correct position.

Binary heap – Insert

```
# beware!  array is arr[1] ... arr[n]
# bottom == ndx of last elem
def heapInsert(arr, x, bottom):
    bottom += 1   # expand the heap space
    j = bottom
    i = j/2       # parent index

    while i > 0 and arr[i] > x:
        arr[j] = arr[i]        # move elem down the heap
        j = i; i /= 2          # move indices up the heap

    arr[i] = x                # put inserted elem to its place
    return bottom
```

Insert -- Complexity

Inserting represents a traversal in a binary tree from a leaf to the root in the worst case. Therefore, the Insert complexity is $O(\log_2(n))$.