

A0M33EOA:
Differential Evolution.
Other Types of Metaheuristics.

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Contents

Differential evolution (DE):

- Another successful heuristic for optimization in R^D .

Swarm intelligence:

- Particle Swarm Optimization (PSO, optimization in R^D).
- Ant Colony Optimization (ACO, optimization on graphs).

Differential Evolution**Differential Evolution**

Developed by Storn and Price [SP97].

- Simple algorithm, easy to implement.
- Unusual breeding pipeline.

Algorithm 1: DE Breeding Pipeline

Input: Population X with fitness in f .

Output: Offspring population X_N .

```

1 begin
2    $X_N \leftarrow \emptyset$ 
3   foreach  $x \in X$  do
4      $(x_1, x_2, x_3) \leftarrow \text{Select}(X, f, x)$ 
5      $u \leftarrow \text{Mutate}(x, x_1, x_2)$ 
6      $x_N \leftarrow \text{Recombine}(u, x_3)$ 
7      $X_N \leftarrow X_N \cup \text{BetterOf}(x, x_N)$ 
8   return  $X_N$ 

```

- Vectors x, x_1, x_2, x_3 shall all be different, x_1, x_2, x_3 chosen uniformly.
- For each population member x , an offspring x_N is created.
- x_N replaces x in population if it is better.

[SP97] Rainer Storn and Kenneth Price. Differential evolution – a simple and efficient heuristic for global optimization over continuous spaces. *Journal of Global Optimization*, 11(4):341–359, December 1997.

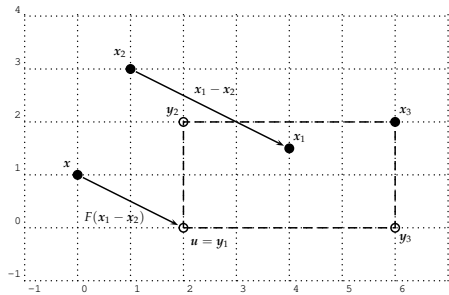
DE Mutation and Recombination

- Mutation and recombination:

$$u \leftarrow x_1 + F(x_2 - x_3), \quad F \in (0, 2)$$

$$x_N, d \leftarrow \begin{cases} u_d & \text{iff } \text{rand}_d \leq CR \text{ or } d = I_{\text{rand}} \\ x_{4,d} & \text{iff } \text{rand}_d > CR \text{ and } d \neq I_{\text{rand}} \end{cases}$$

- $\text{rand}_d \sim \mathcal{U}(0, 1)$, different for each dimension
- I_{rand} is a random index of the dimension that is always copied from u
- $2^D - 1$ possible candidate points y



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DE Variants

Small variations of the base algorithm:

- DE/rand vs DE/best: the “best” variant uses the best of 4 parent vectors in place of x when generating the offspring.
- DE/./n: n is the number of difference vectors taken into account during mutation.
- DE/././bin vs DE/././exp: binomial recombination (described above), exponential recombination (not described here)

Many adaptive variants: SaDE, JADE, ...

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Swarm Algorithms

Swarm intelligence:

- In nature: swarm (cz: roj, hejno) of small simple 'units' is able to create very complex behavioral patterns via cooperation.
- **Emergence**: non-linear interactions of simple rules → complex behavior of the whole system.
- Analogy to the behavior of bees, wasps, ants, fish, birds, ...

An engineering view:

- Is it possible to model these systems *in silico* and use that model to solve a practical task?
- How to design the simple units and their interactions such that a practically useful system emerges?

Particle Swarm Optimization

Particle Swarm Optimization

Particle Swarm Optimization (PSO): an optimization algorithm inspired by the behavior of birds.

Inspiration:

- Birds fly over the landscape and land on the highest hill.
- Birds are modeled by particles in a multidimensional vector space.
- The particles have their *position* and *speed* (and momentum).
- They remember their own best position (i.e., the highest place of the landscape they flew over), but also
- they communicate and use the best position of their neighboring particles to update their own position and speed.
- The communication is usually of 2 types:
 1. **Globally best position** is known to all particles and is updated as soon as any particle finds an improvement.
 2. **Best position in neighborhood** is shared among a group of neighboring particles.

PSO Algorithm

Algorithm 2: Canonical PSO

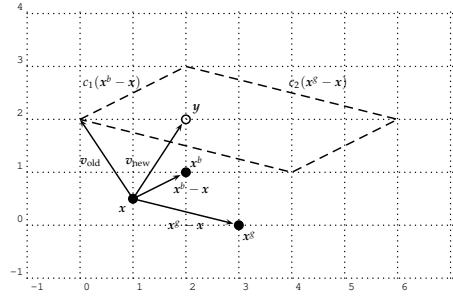
```

1 begin
2   Initialize positions  $x_i$  and velocities  $v_i$ .
3   Initialize personal best positions  $x_i^b \leftarrow x_i$ .
4   Initialize globally best position  $x^g \leftarrow x_k, \forall i : f(x_k) \leq f(x_i)$ 
5   for  $i = 1, \dots, N$  do
6      $v_i \leftarrow \omega v_i + c_1 r_1 \circ (x_i^b - x_i) + c_2 r_2 \circ (x^g - x_i)$ 
7      $x_i \leftarrow x_i + v_i$ 
8     If  $f(x_i) < f(x_i^b)$ ,  $x_i^b \leftarrow x_i$ .
9     If  $f(x_i) < f(x^g)$ ,  $x^g \leftarrow x_i$ .
10  If termination condition not satisfied, go to 5.

```

Meaning of symbols:

f objective function (landscape) $f : \mathcal{R}^D \rightarrow \mathcal{R}$
 N the number of particles
 x_i particle positions, $x_i \in \mathcal{R}^D$
 v_i particle velocities, $v_i \in \mathcal{R}^D$
 x_i^b personal best position

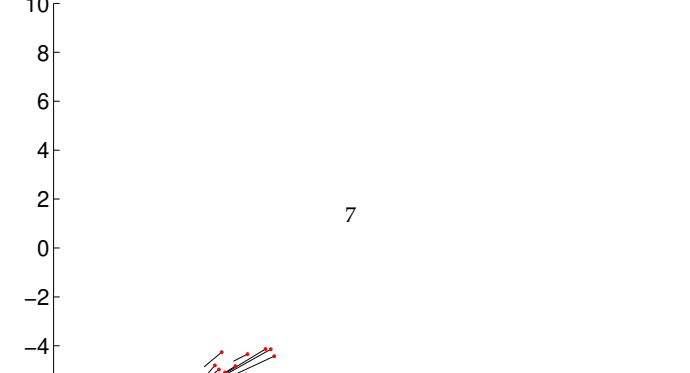
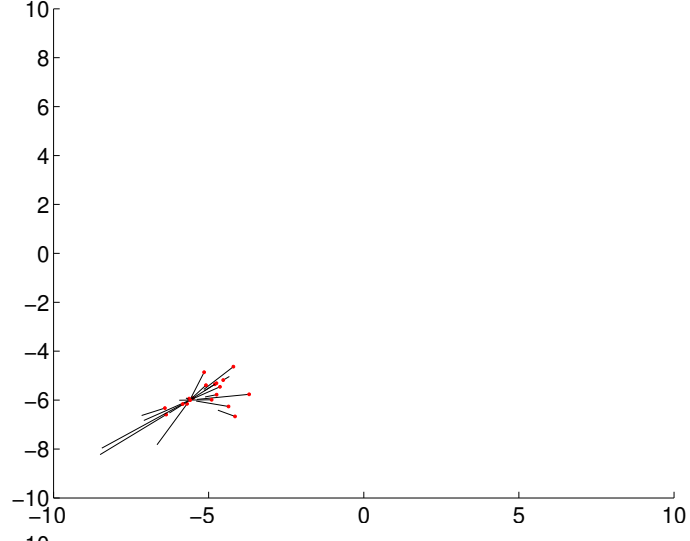
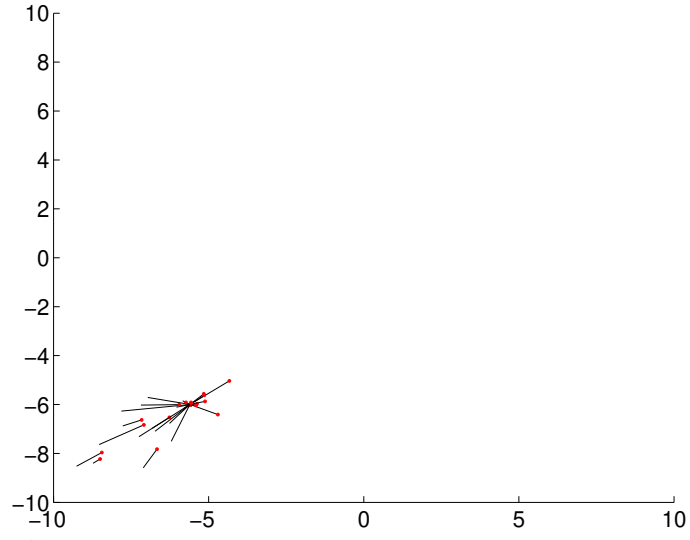
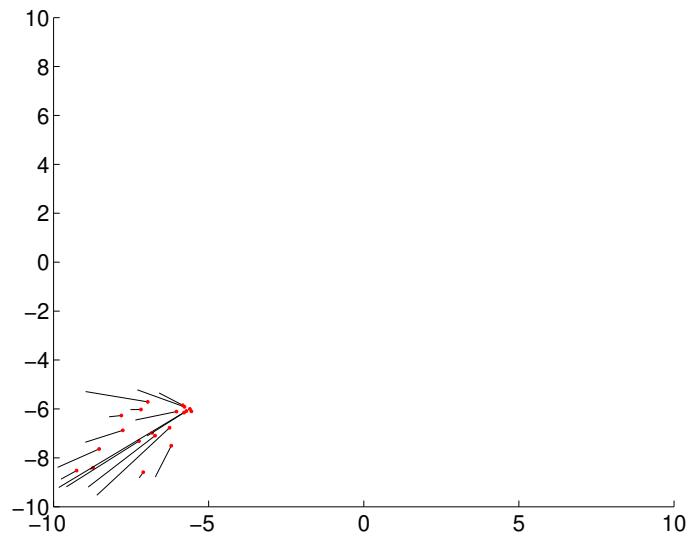


x^g globally best position
 ω particle momentum, suitable value is e.g. 0.9, sometimes it decreases during simulation e.g. to 0.4.
 c_1, c_2 attraction constants, 'cognitive' and 'social' components, suitable values between 1 and 2
 r_1, r_2 random vectors from $U(0,1)^D$
 \circ vector multiplication by items

[KE02] J. Kennedy and R. Eberhart. Particle swarm optimization. In *Neural Networks, 1995. Proceedings., IEEE International Conference on*, volume 4, pages 1942–1948, August 2002.

PSO: Demo

PSO on 2D Sphere function:



Ant colonies

Ants:

- Social insects
- Ant colonies exhibit an intelligent behavior:
 - labor division, work coordination
 - complex nests
 - ability to find 'low-energy' path between the nest and a food source
- They communicate by
 1. physical contact (they touch with their antennas)
 2. interaction with the environment (pheromone trails)

"In nature, ants first search their environment randomly, until they find a source of food. Then, they return to the nest and lay a pheromone trail behind. Other ants are able to sense this pheromone trail and are able to follow it, and thus make it stronger. The pheromone evaporates; after the food source is exhausted, the random foraging reemerges."

Ant Colony Optimization

Ant Colony Optimization (ACO) is a class of stochastic optimization algorithms for solving combinatorial problems.

Similarities with the real ants:

- a colony of cooperating individuals
- pheromone trail
- indirect communication via pheromone (stigmergy)
- probabilistic decision making, local strategies

Differences from the real ants:

- (usually) discrete world (a graph)
- inner state, memory
- the amount of pheromone train can depend on the solution quality
- may use several types of pheromones

Algorithm 3: ACO

```

1 begin
2   Initialize the pheromone trails on graph edges:  $\tau_{ij}(0) = \tau_0$ .
3   Set the initial position of ants in the graph.
4   while not termination condition do
5     foreach ant do
6       Build a solution.
7       Apply local search. // Optional, but used very often.
8     Update pheromone trails.
```

Algorithm parts

Ant k constructs a solution:

- Probability the ant will move from the current node i to neighboring node j is

$$p_{ij}^k(t) = \frac{(\tau_{ij}(t))^\alpha (\eta_{ij})^\beta}{\sum_{l \in \mathcal{N}_i^k} (\tau_{il}(t))^\alpha (\eta_{il})^\beta}, \text{ kde } j \in \mathcal{N}_i^k,$$

where τ_{ij} the amount of pheromone on edge $i \rightarrow j$,
 $\eta_{ij} = \frac{1}{d_{ij}}$ known heuristic information,
 α, β the influence of pheromone and heuristic information, respectively,
 \mathcal{N}_i^k a set of graph nodes accessible to ant k from node i .

- If $\alpha = 0$, only the heuristic information has an effect, and the solution construction reduces to greedy algorithm (nearest neighbor heuristic).
- If $\beta = 0$, only the pheromone trail has an effect. The paths found in the first iteration have a big influence. Moreover, if $\alpha > 1$, stagnation occurs very fast, i.e. all ants use the same (not optimal) path.
- Suggested values of parameters:

$$\alpha = 1 \qquad \beta = 2 \text{ až } 5 \qquad \rho = 0.5 \qquad m = n \text{ (TSP)} \qquad \tau_0 = m / C^m \text{ (TSP)}$$

m is the number of ants, n is the number of cities, C^m is the length of the path constructed by the nearest neighbor heuristic.

Algorithm parts (cont.)

Pheromone update on all edges

- Done after all ants find their solution.
- Pheromone evaporation: $\tau_{ij} \leftarrow (1 - \rho)\tau_{ij}$.
 ρ is the evaporation rate, allows to 'forget' bad paths.
- Pheromone deposition from all ants: $\tau_{ij} \leftarrow \tau_{ij} + \sum_{k=1}^m \Delta\tau_{ij}^k$, where

$$\Delta\tau_{ij}^k = \begin{cases} 1/C^k & \text{if ant } k \text{ used edge } i \rightarrow j \\ 0 & \text{otherwise,} \end{cases}$$

C^k is the length of the path of ant k .

Other options:

- The best path is reinforced the most.
- The amount of deposited pheromone is proportional to the ant rank according to the path lengths (i.e., not directly proportional to path lengths).
- Update of pheromone trails as soon as an ant uses and edge.
- More types of pheromones can be used:
 - Ants can start from both the nest and the food source.
 - We can have more types of ants.
- ...

Applications

ACO was able to find good solutions in the following tasks:

- Traveling salesperson problem
- Network routing, vehicle routing
- Scheduling
- Quadratic assignment problem
- Shortest common supersequence
- Classification rule learning
- ...

Advantages:

- The graph topology can change in time (e.g. in routing problems)

Demo: ant foraging

Conclusions

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Summary

- There are plenty of nature-inspired techniques, other than EAs.
- Swarm intelligence takes advantage of the emergent swarm behavior which is a result of simple interactions among individual swarm members.
- Particle swarm optimization primarily aims at real-parameter optimization, but there are also variants suitable for discrete spaces.
- Ant colonies are used to solve problems which can be reduced to search for the shortest path in a graph (combinatorial problems). Again, variants for real-parameter optimization exist (but are somewhat 'unnatural').