Question 1. (3 points)
Give a real-life meaning to binary random variables $A, B, C$, for which the following Bayes graph is appropriate

1. (A) $\rightarrow$ (B) $\rightarrow$ (C)
2. (A) $\rightarrow$ (B) $\leftarrow$ (C)
3. $(A) \leftarrow(B) \rightarrow(C)$

For each case, decide if the graph implies $A \Perp_{P} C \mid B$.

## Question 2. (2 points)

How many parameters are needed in the Bayesian network below to fully specify a joint distribution on the random variables in vertices, which are binary?


Question 3. (5 points)
Given the Bayesian network below (conditional probability tables not shown),


1. (1 point) list all variables that 'John calls' is independent of, given that 'Alarm' is observed.
2. (4 points) express the probability that neither Mary nor John calls given that both burglary and earthquake happens, using only those (conditional) probabilities which are encoded in the conditional probability tables appropriate for this network. (Produce a formula referring to the random events by symbols such as $A$ and their outcomes by symbols such as $b, \neg m$, etc.).

Question 4. (5 points)
Let $X_{1}, X_{2}, \ldots \Perp_{P} Y_{1}, Y_{2}, \ldots \mid \mathcal{E}$ denote that $\forall i, j \in\{1,2, \ldots\}: X_{i} \Perp_{P} Y_{j} \mid \mathcal{E}$. The empty set is denoted as $\emptyset$. Decide (true/false) for each of the statements below whether it is implied by the Bayesian graph for $P$


1. $Q \Perp_{P} X, Y, Z, P \mid W$,
2. $Z, Y, P \Perp_{P} W, Q \mid \emptyset$,
3. $Z \Perp_{P} X, W, Q \mid \emptyset$,
4. $Z \Perp_{P} X, W, Q \mid P$,
5. $Z, Y, P \Perp_{P} W, Q \mid X$,

Question 5. (15 points)
Consider the Bayes Network

$$
\begin{array}{llll}
P(a)=0.5 & P(b \mid a)=0.5 & P(c \mid b)=0.5 & P(d \mid c)=0.5 \\
& P(b \mid \neg a)=0 & P(c \mid \neg b)=0 & P(d \mid \neg c)=0.1
\end{array}
$$


$P(e \mid \neg b)=0$

1. Calculate $P(a \mid d)$ and $P(\neg a \mid d)$ by the factor method, i.e., using factor multiplication and variable elimination.
2. Determine $\arg \max _{A, B} P(A, B \mid d)$ by the factor method.
