Question 1. (2 points)

Consider a version-space agent whose initial hypothesis class $\mathcal{H}_1$ contains all non-contradictory conjunctions on 3 propositional variables.

1. Determine $|\mathcal{H}_1|$.
2. Give an upper bound on $|\mathcal{H}_2|$ given that $r_2 = -1$.

Question 2. (2 points)

Let $r_{\leq K}$ be a reward sequence of a standard agent and $h_{\leq K}$ be its sequence of hypotheses. Denote $M = \sum_{k=1}^{K} |r_k|$. Show that there is a hypothesis h retained for at least $\frac{K}{M+1}$ consecutive steps in $h_{\leq K}$, i.e.

$$h_{\leq K} = h_1, h_2, \ldots \underbrace{h, h, \ldots h}_{\text{at least } \frac{K}{M+1} \text{ times}}, \ldots h_K$$

Question 3. (5 points)

Let an agent PAC-learn $\mathcal{C}$ from X. Show that for any target concept from $\mathcal{C}$ on X, an arbitrary distribution $P(x)$ on X and arbitrary numbers $0 < \epsilon, \delta < 1$ and $K \in \mathbb{N}$, the condition $\mathsf{err}(h_K) \leq \epsilon$ with probability at least $1 - \delta$ implies that h_K is consistent with all observations in $x_{<K}$.

Question 4. (5 points)

Let X contain all real numbers from $[0; 1]$ which can be represented using 256 bits. Let $\mathcal{H} = X$, and the decision policy given by a $h \in \mathcal{H}$ is

$$h(x) = 1 \text{ iff } x > h$$

Determine a k such that with probability at least 0.9, $\mathsf{err}(h) < 0.1$, where h is an arbitrary hypothesis from $\mathcal{H}$ consistent with k i.i.d. examples from X. Estimate it using:

1. $\ln |\mathcal{H}|$
2. $\mathsf{VC}(\mathcal{H})$