# ADVANCED ROBOTICS


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2017



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Head of Applied Algebra and Geometry Group

**Faculty of Electrical Engineering**
Associate Professor

**National Institute of Informatics Tokyo**
Visiting Associate Professor


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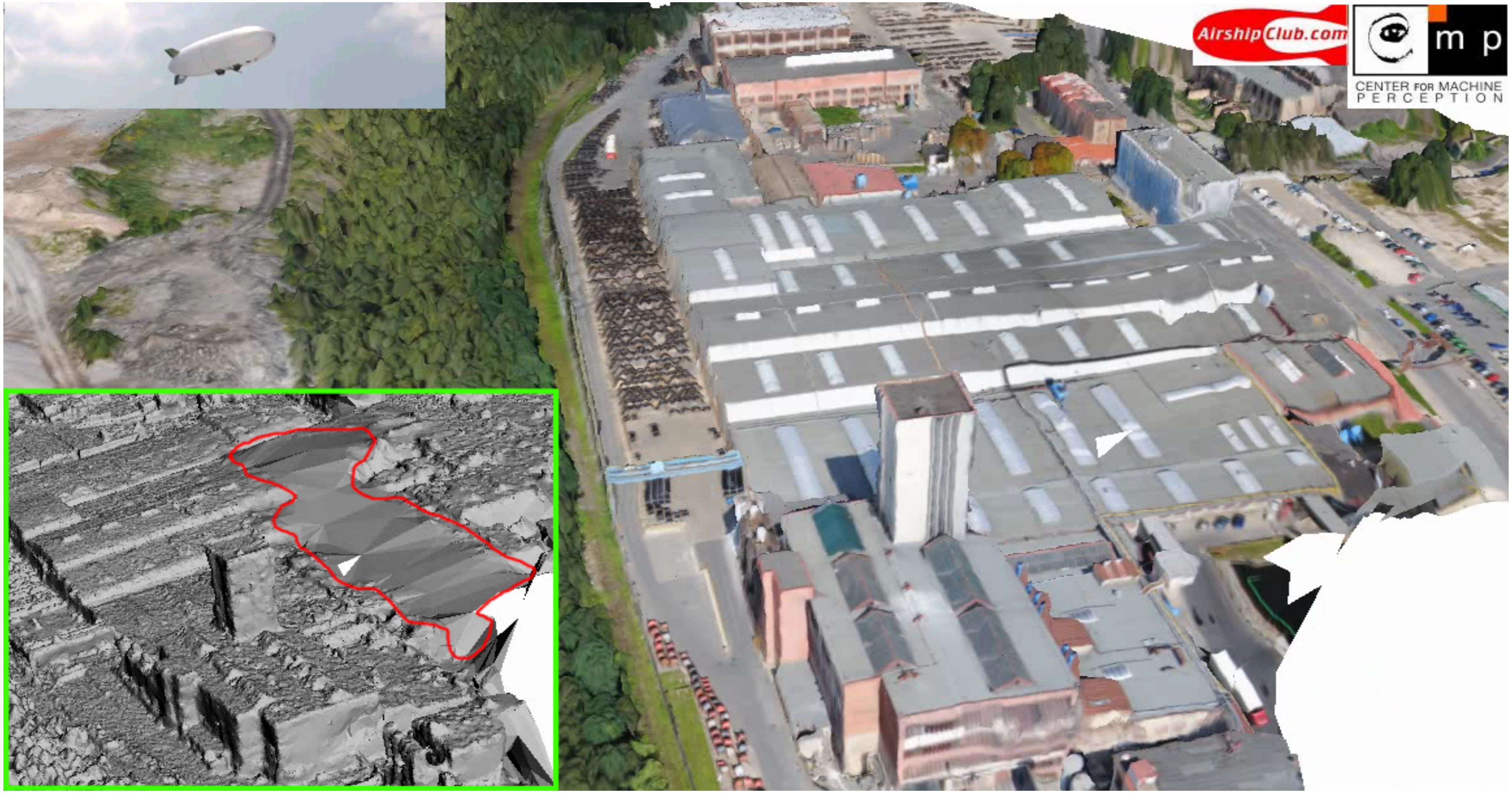
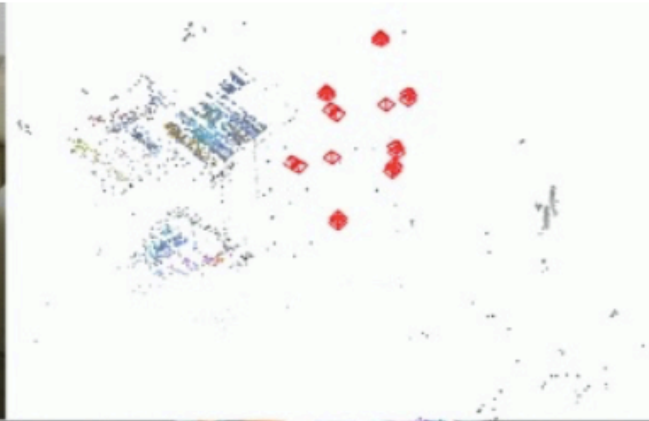
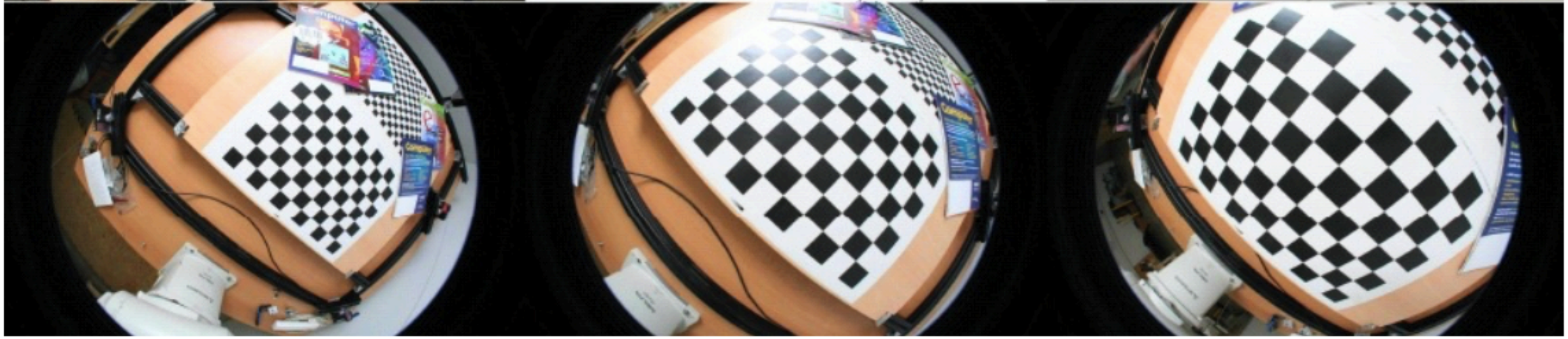
# 3D Computer Vision

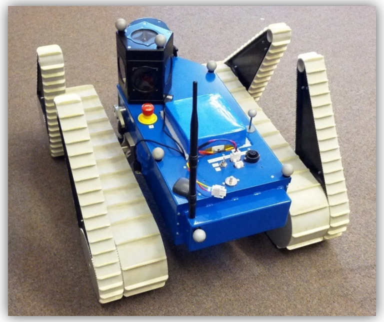


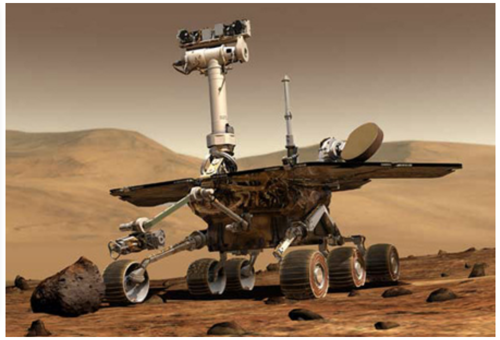
# Camera & Robot Calibration

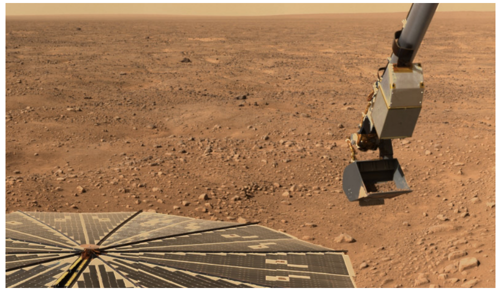


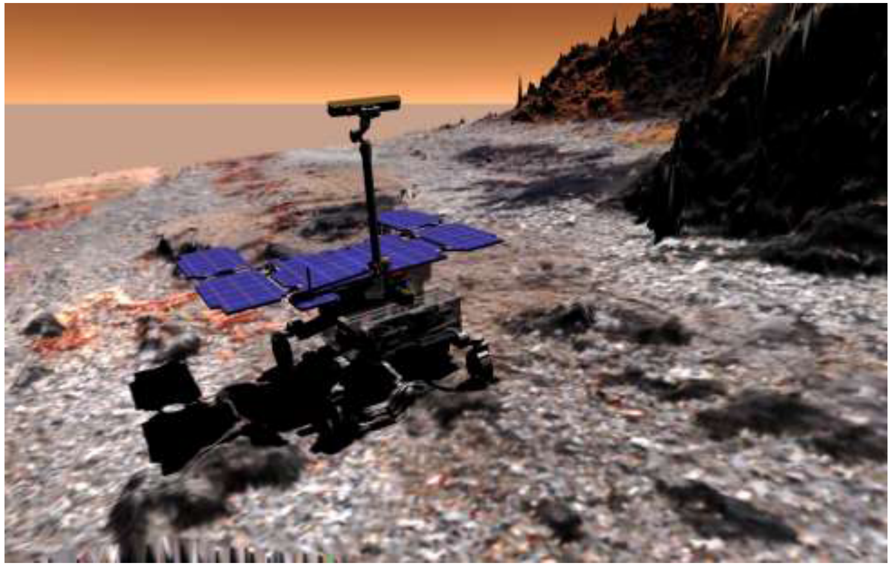


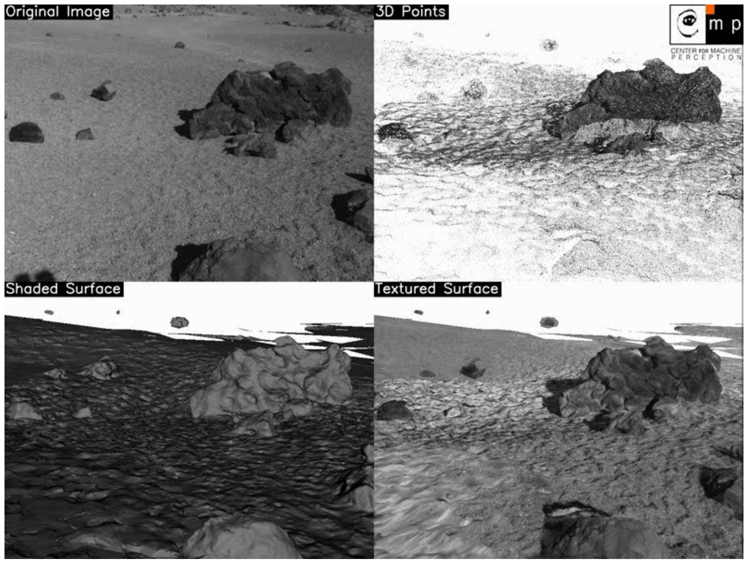
# Mobile Robotics

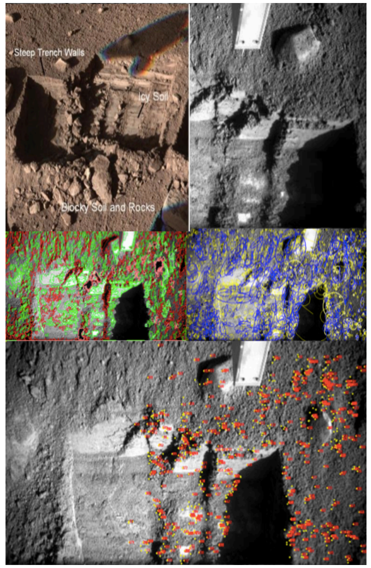







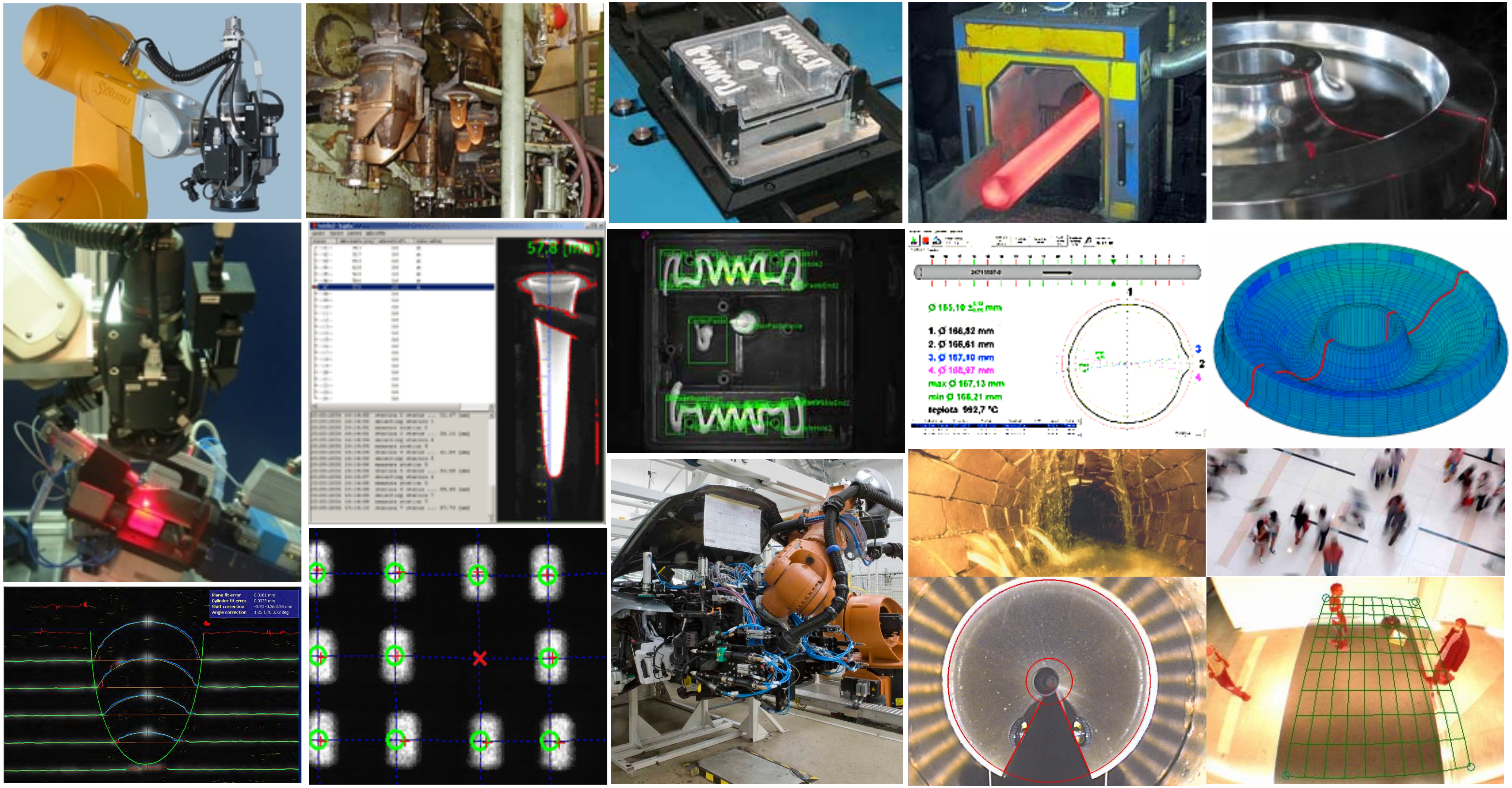




# Computer Vision for Industry

NEOVISION Industrial Vision Systems · Leica · zentity a touch of innovation · AZOS CZ s.r.o. · gisat





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# Advanced Robotics

## Lecture 1


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We will build on Robotics by V. Smutny and study more advanced robot kinematics problems, e.g.,

1. solving inverse kinematics of a general 6 DOF manipulator
2. identifying kinematic parameters of a manipultor
3. finding singular poses of a manipulator

with more advanced mathematical tools, such as

1. space rotation and motion and
2. solving algebraic equations


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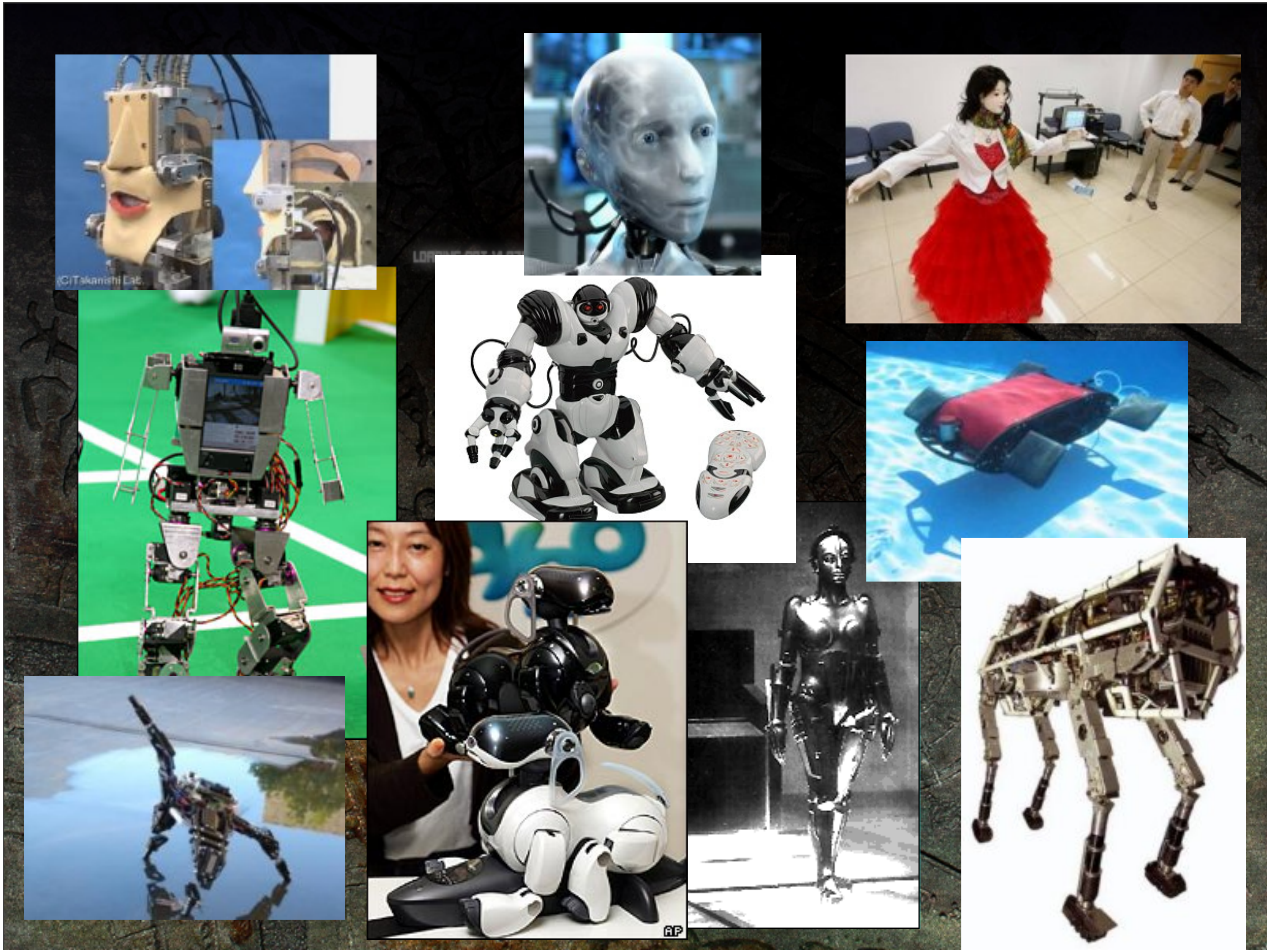
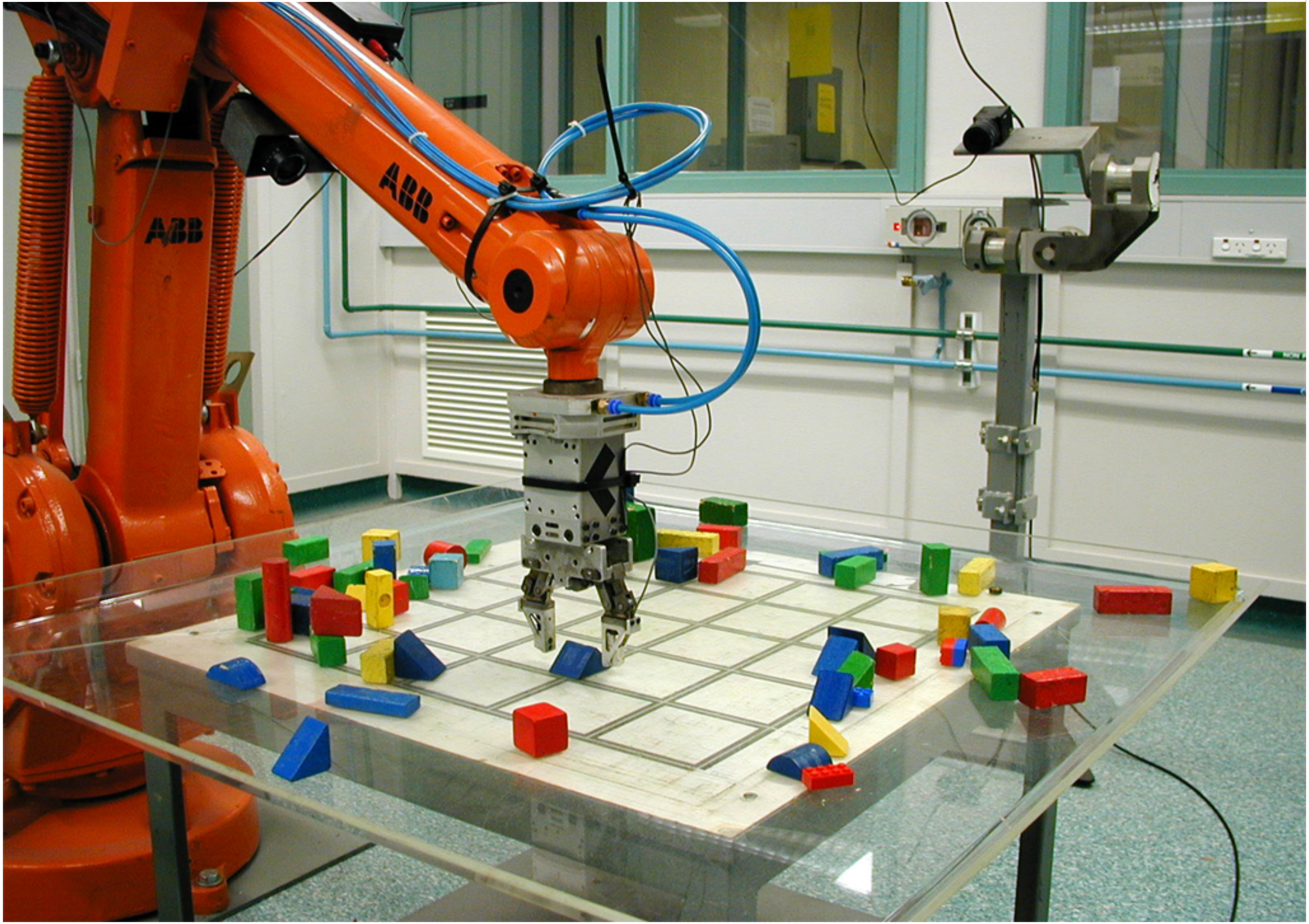

(C)Takanishi Lab.
AP

# ROBOT = A GENERAL MANIPULATOR



# Industrial Robotic Applications

ABB

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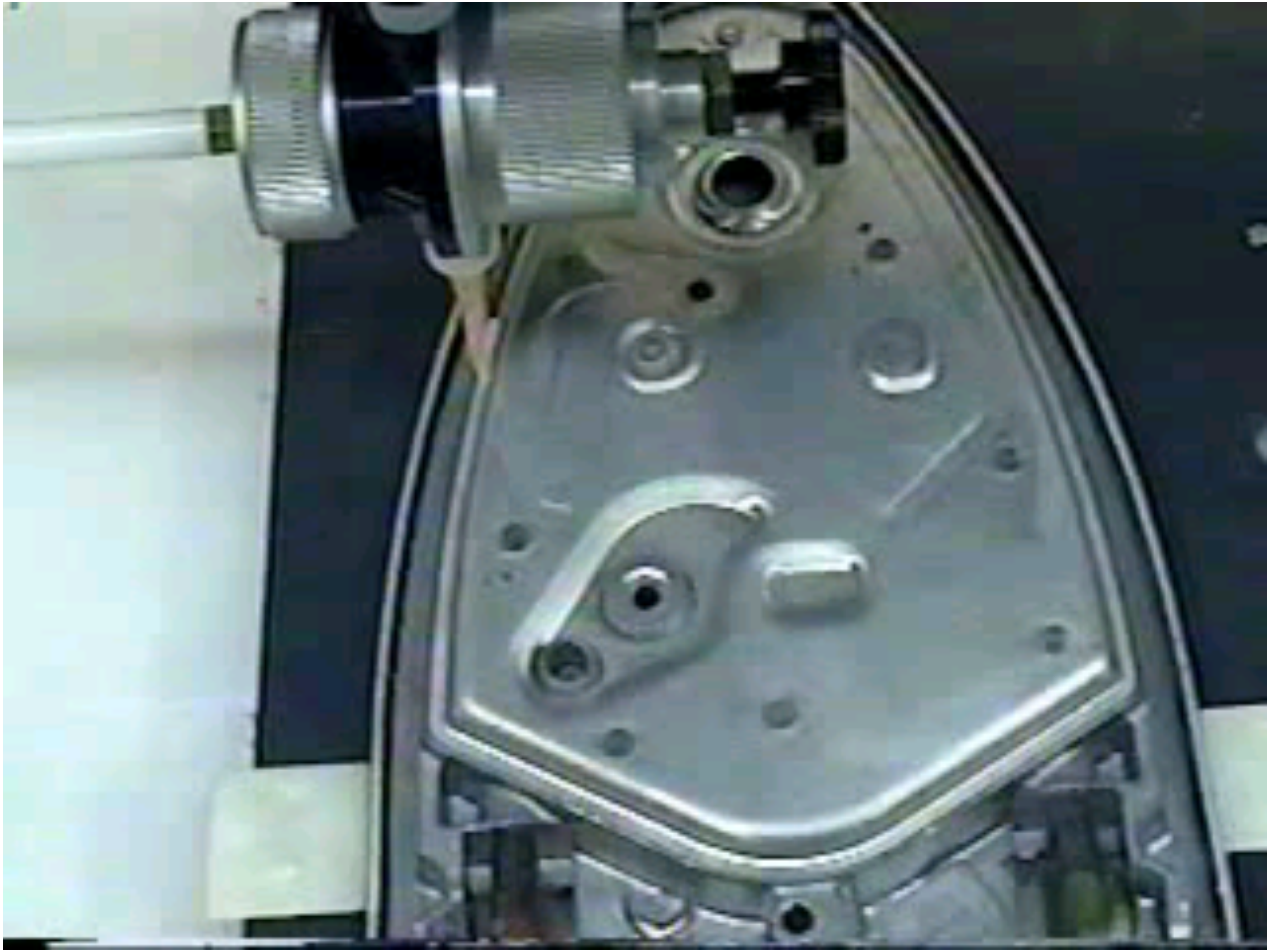
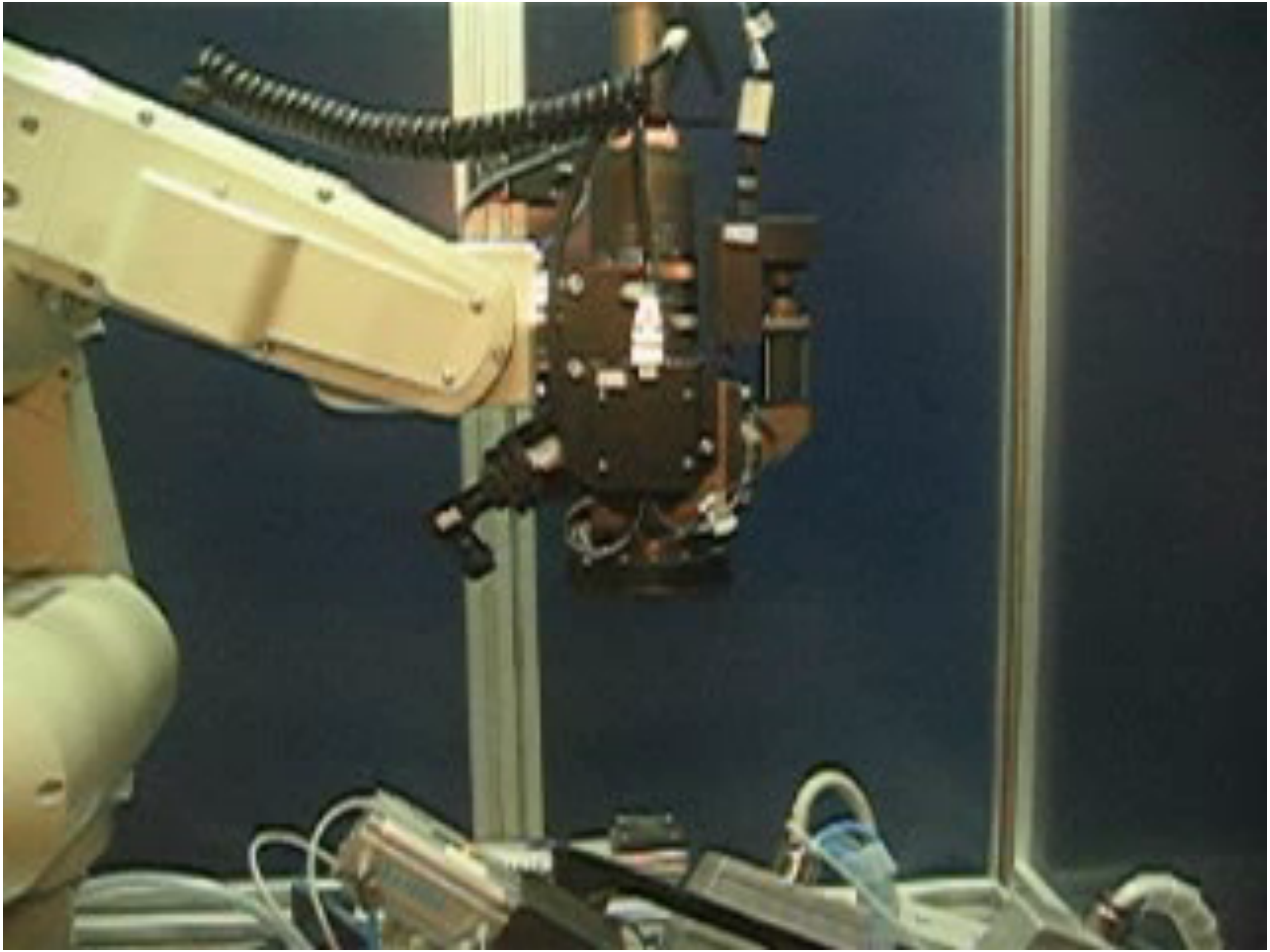
**Robotics**

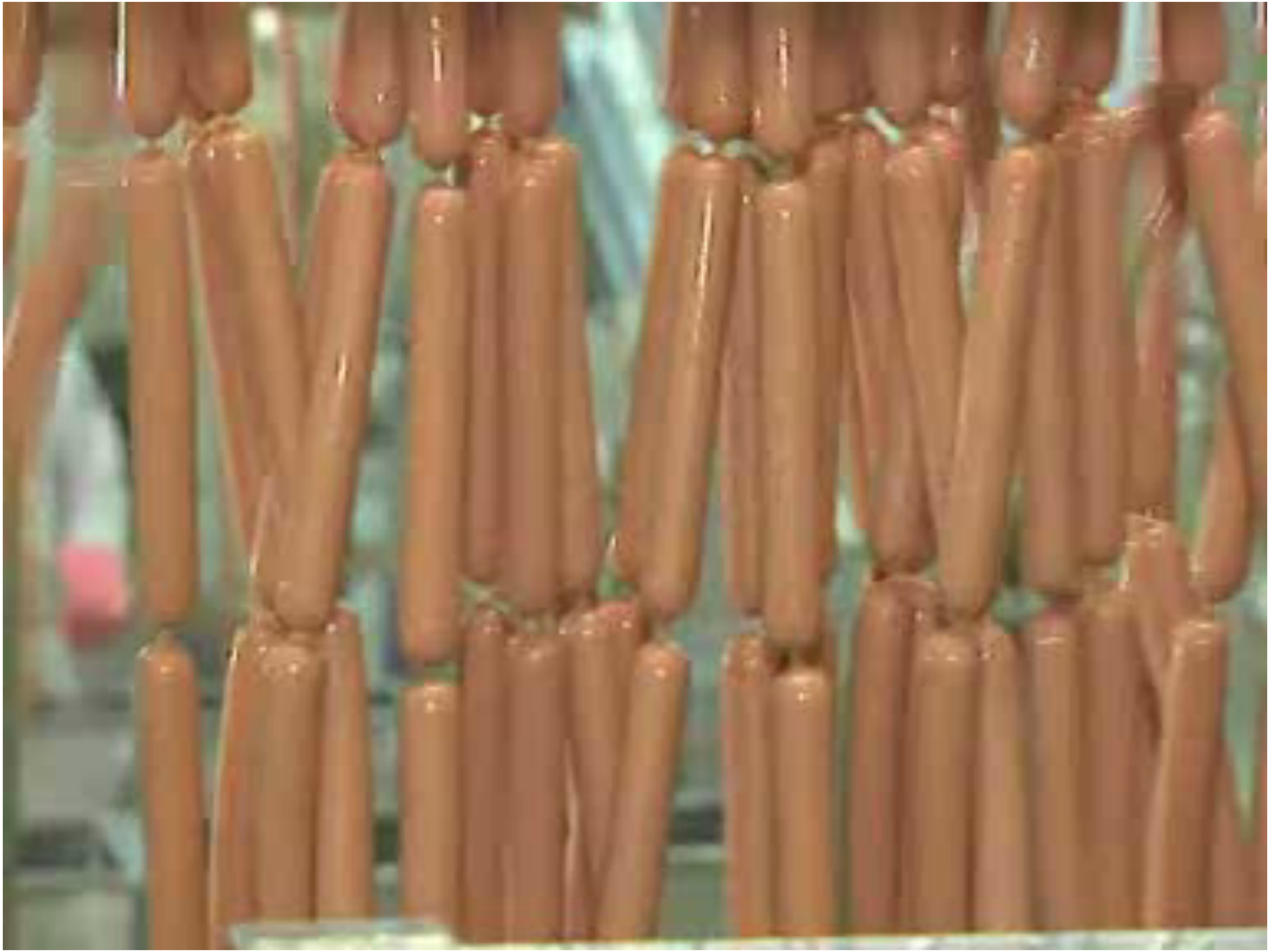
- Go to The ABB Product Guide
- Robotics startpage
- ⊞ Product range
- ⊟ Application areas
  - Arc welding
  - Assembly
  - Foundry applications
  - Gluing and Sealing
  - ⊞ Material handling and Machine Tending
  - Packing
  - Palletizing
  - Picking
  - Painting and coating
  - Spot welding
  - Waterjet cutting

## Application areas

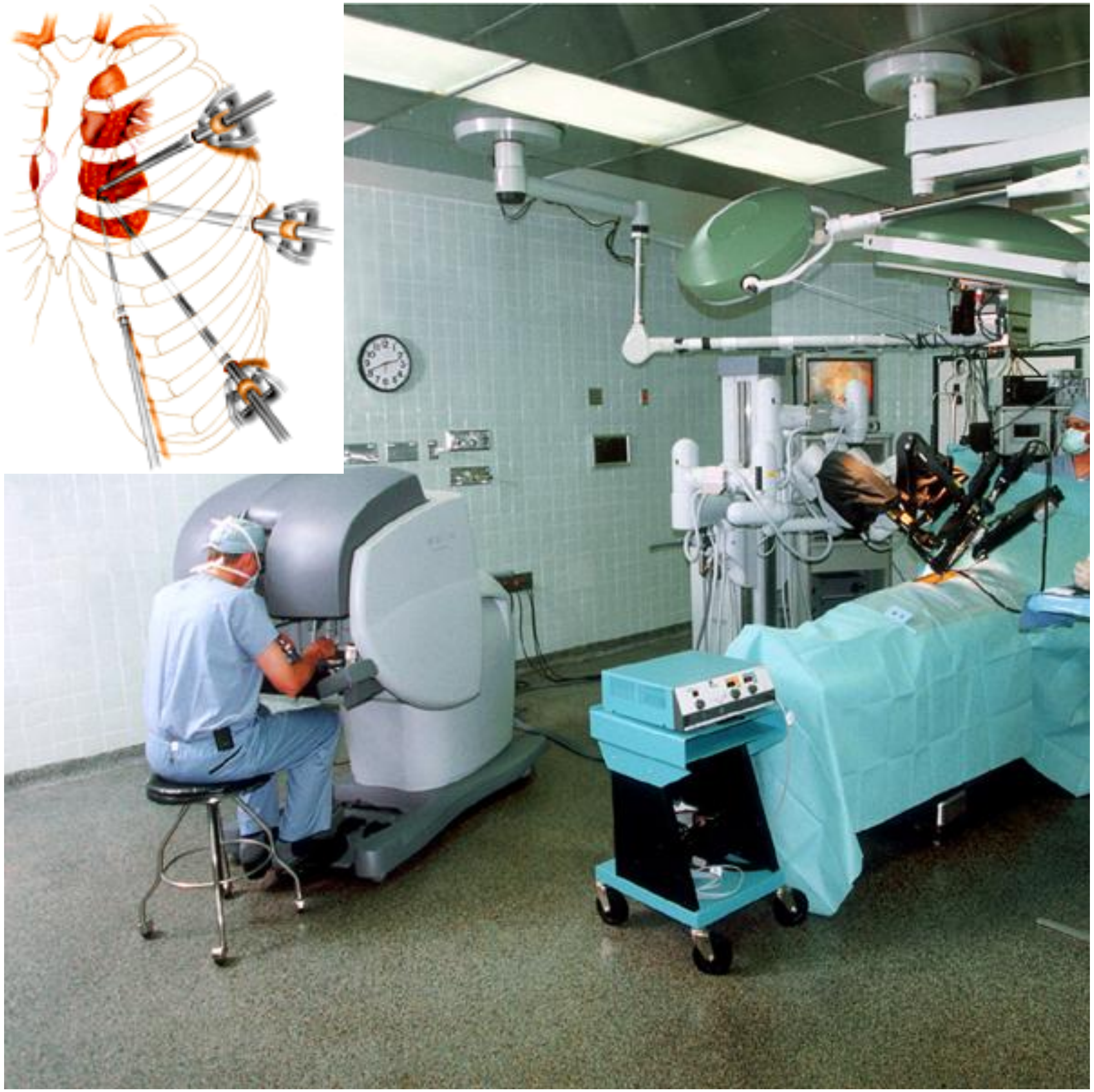


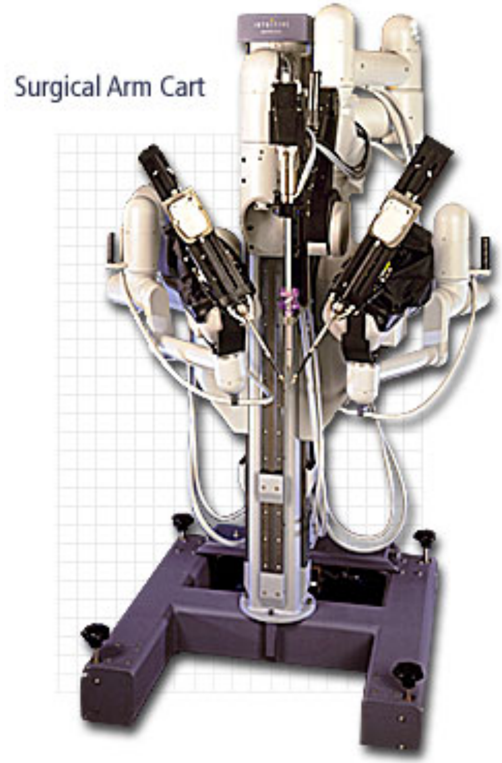


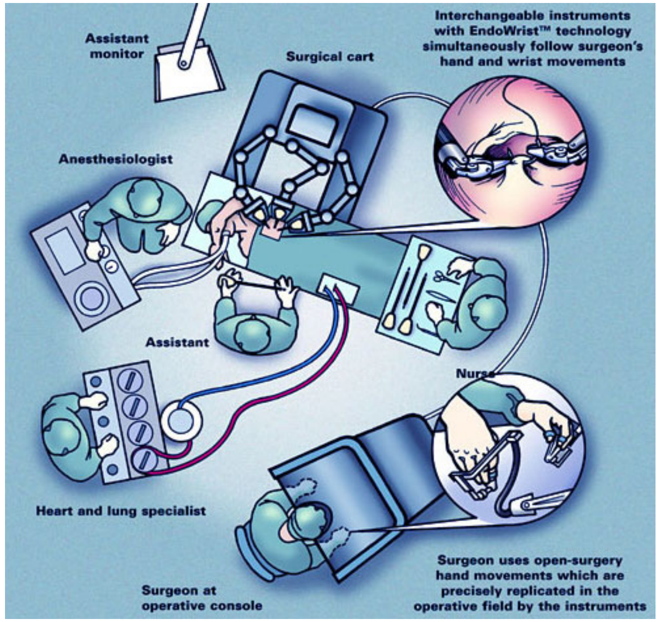

ABB



# Precision for robotic surgery

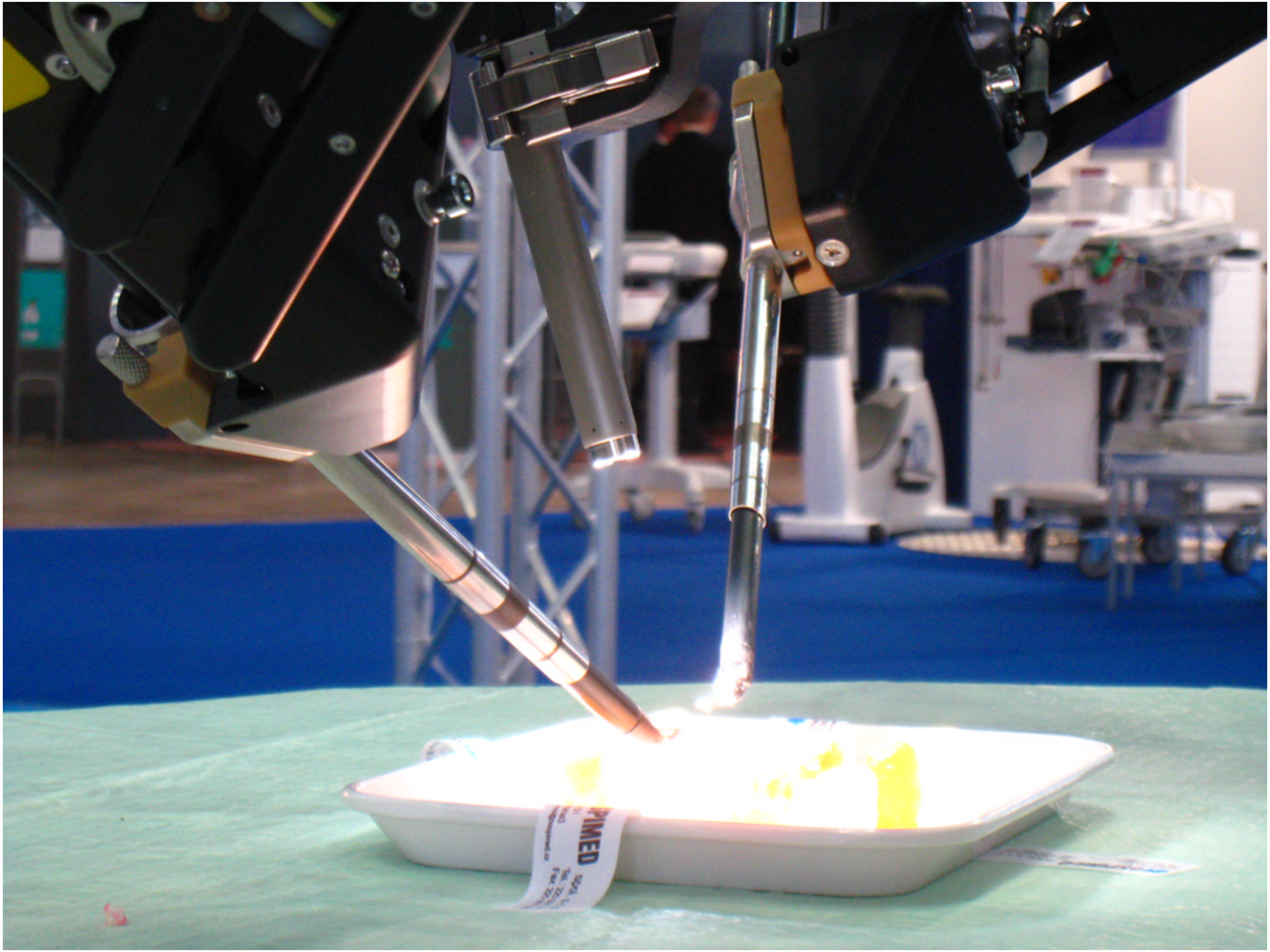


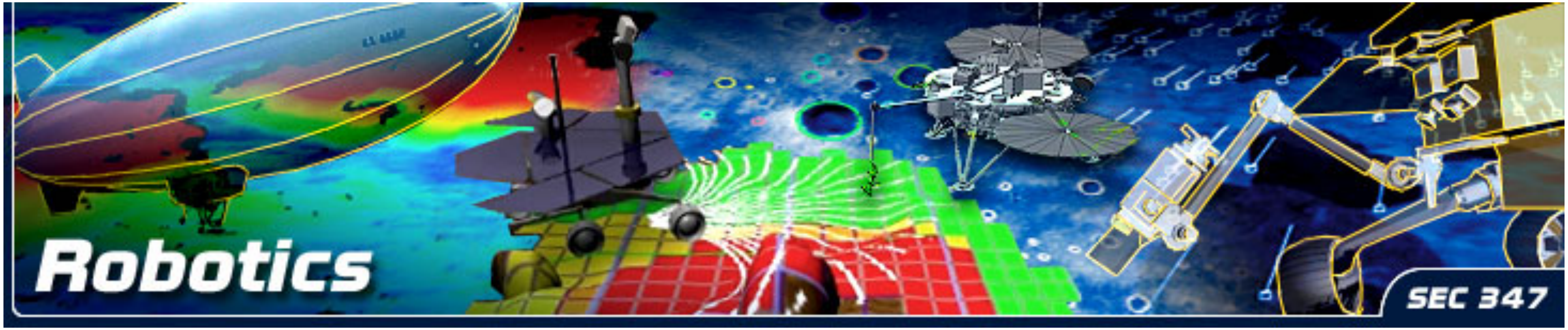



http://www.cts.usc.edu/rsi-article-robotputsuscatforefront.html

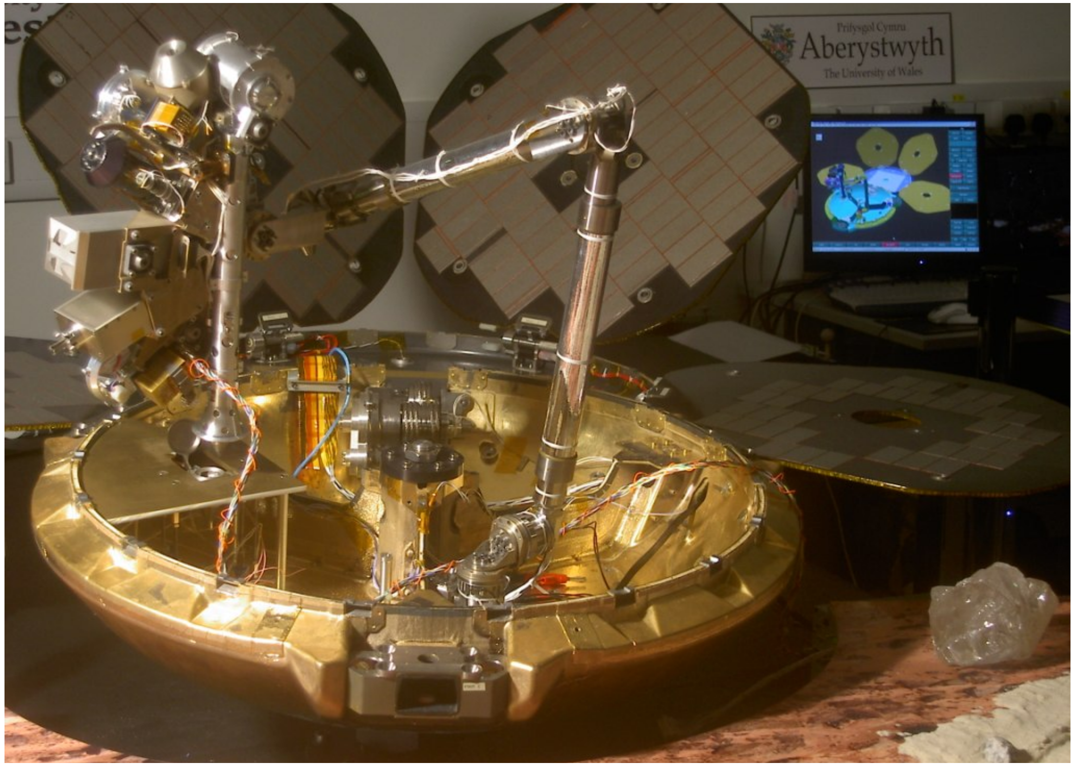

Robotický systém pro miniinvazivní chirurgii
da Vinci ®
ULTRAZVUKOVÉ SYSTÉMY
VENTILAČNÍ TECHNIKA Taema
KARDIOLOGICKÉ A MONITOROVACÍ SYSTÉMY

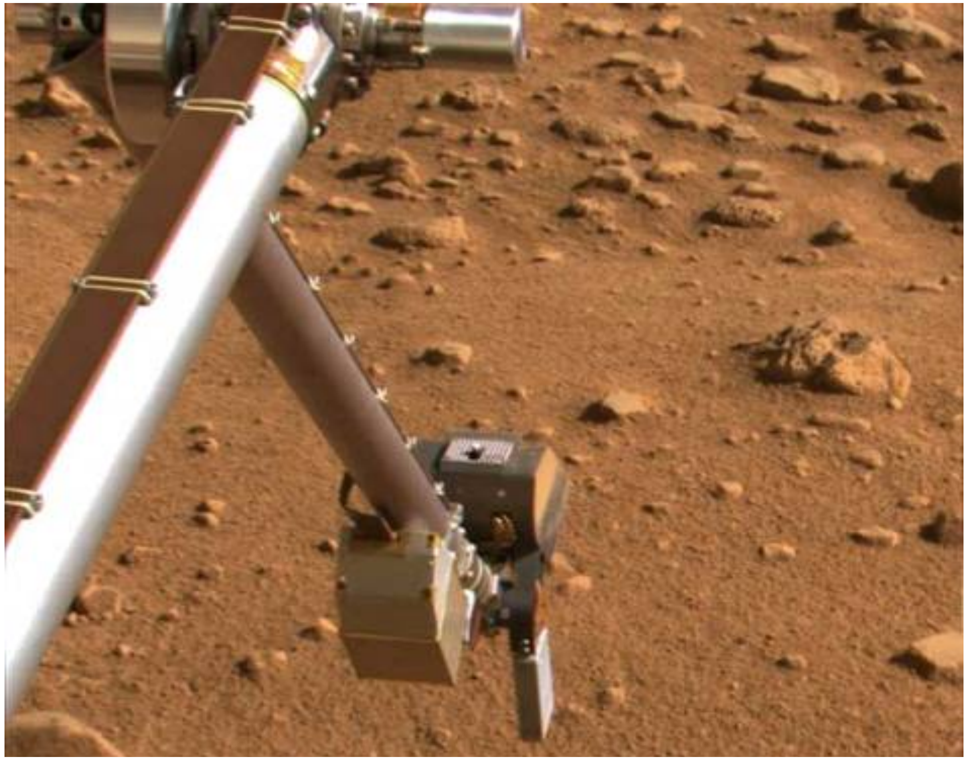



Robotics
SEC 347


BEAGLE 2
a mission to mars


PHOENIX MARS MISSION
NASA


Prifysgol Cymru
Aberystwyth
The University of Wales

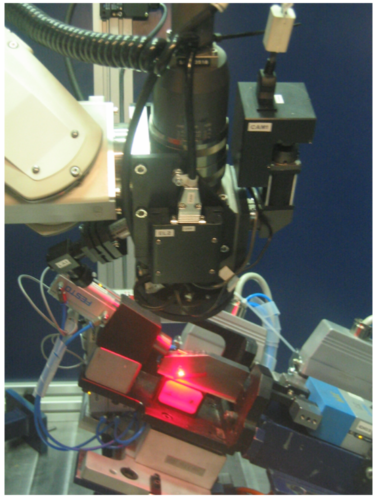


# Precision for industry

Low (e.g. manipulation)

± 5 mm in the whole working space
± 0.5 mm locally

… often available



High (e.g. laser welding)

± 0.5 mm in the whole working space
± 0.05 mm locally

… often not available



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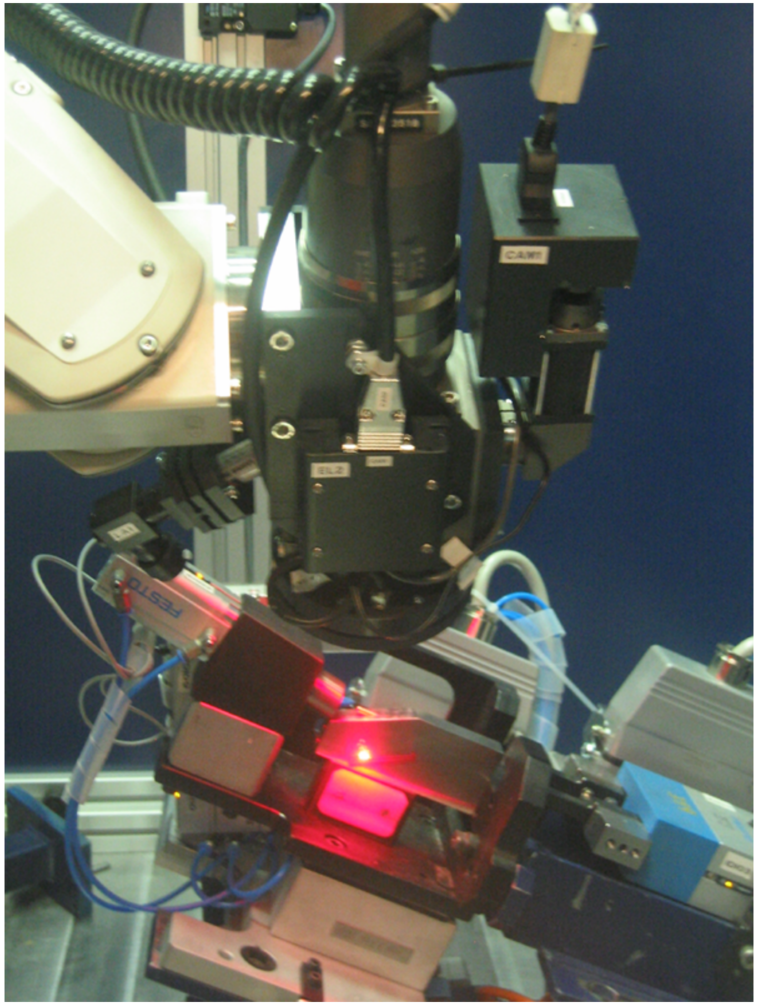
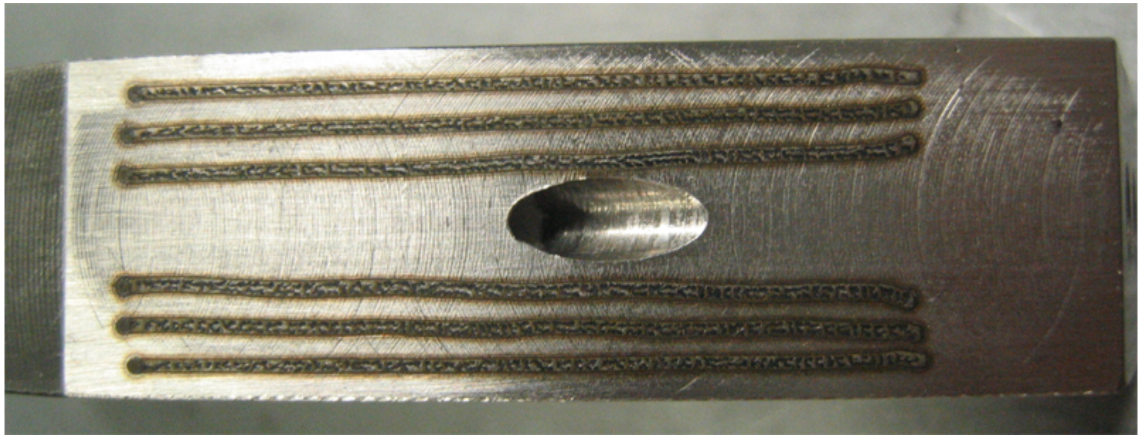
# Error ± 0.5 mm







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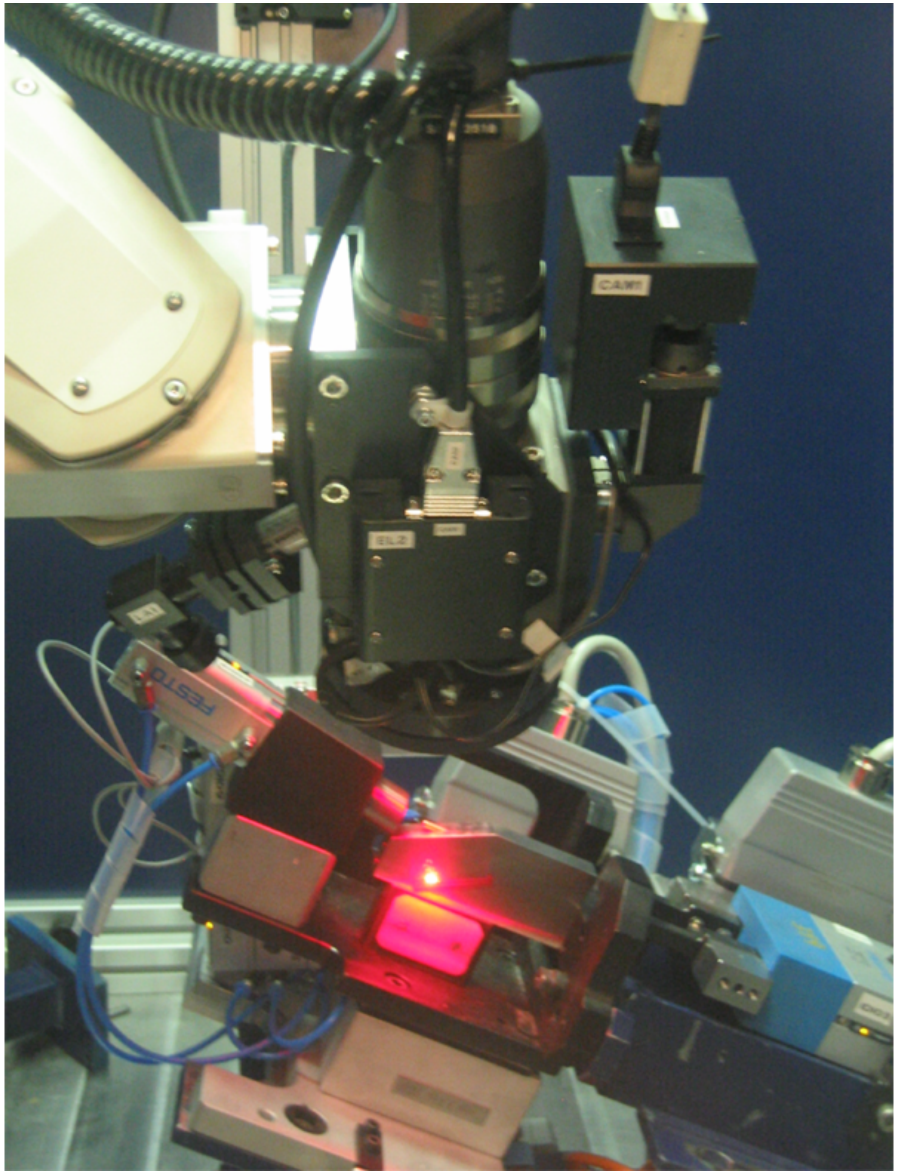
# Modeling kinematics – calibration – absolute acuracy $\pm$ 0.05 mm




Robot-Vision calibration (courtesy Neovision s.r.o.)

MITSUBISHI robot
TRUMPF welding laser
NEOVISION vision guiding



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# Two kinds of manipulators

1. Serial manipulators

2. Parallel manipulators



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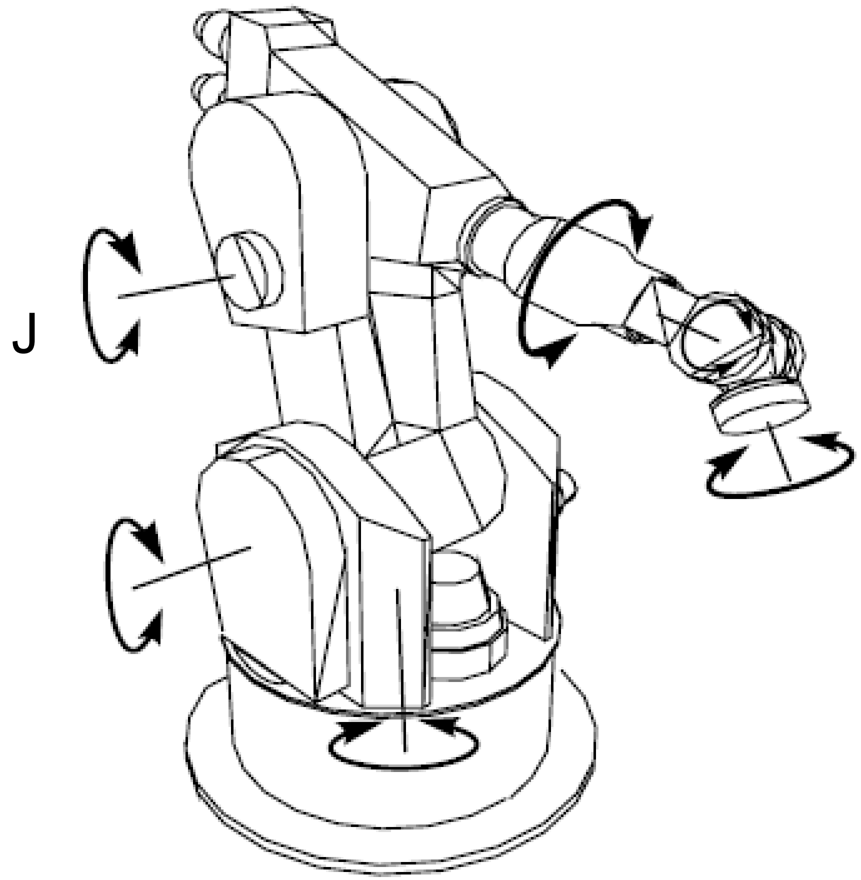
# Serial manipulators




KUKA manipulator

# Serial manipulators

Stäubli (courtesy Neovision s.r.o.)

Mitsubishi (courtesy Neovision s.r.o.)

1. Direct kinematic task – easy
2. Inverse kinematic task – difficult



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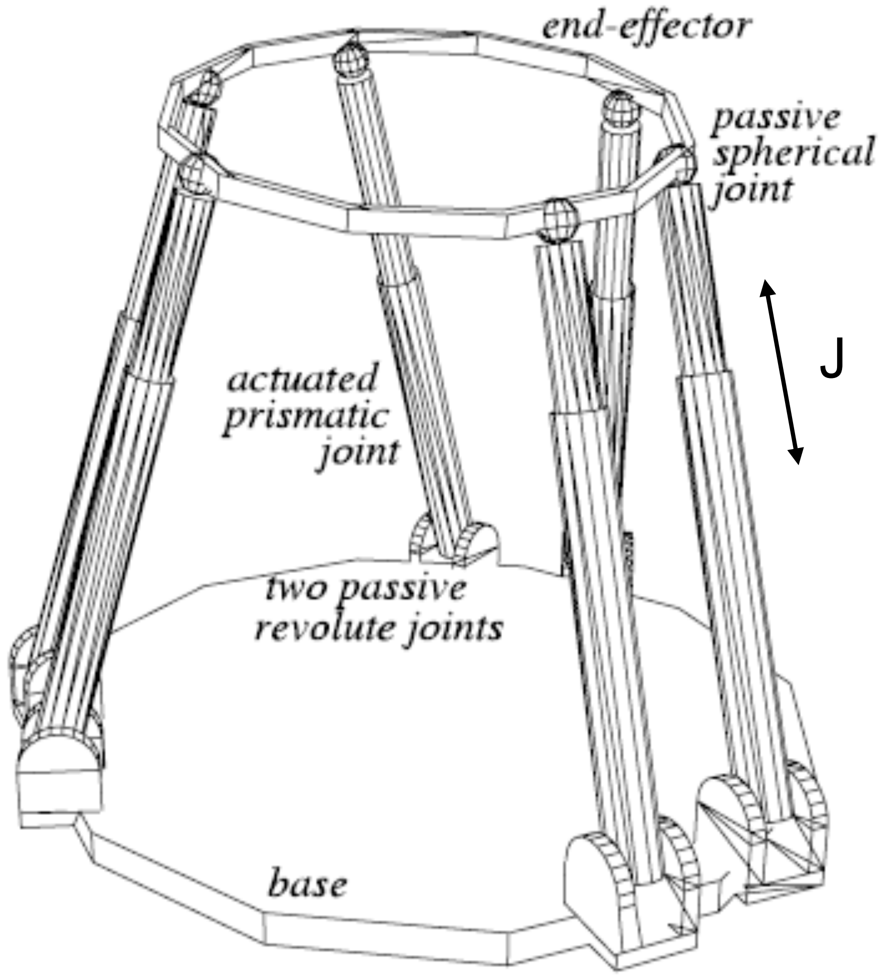
# Parallel manipulators




Stewart-Gough Platform



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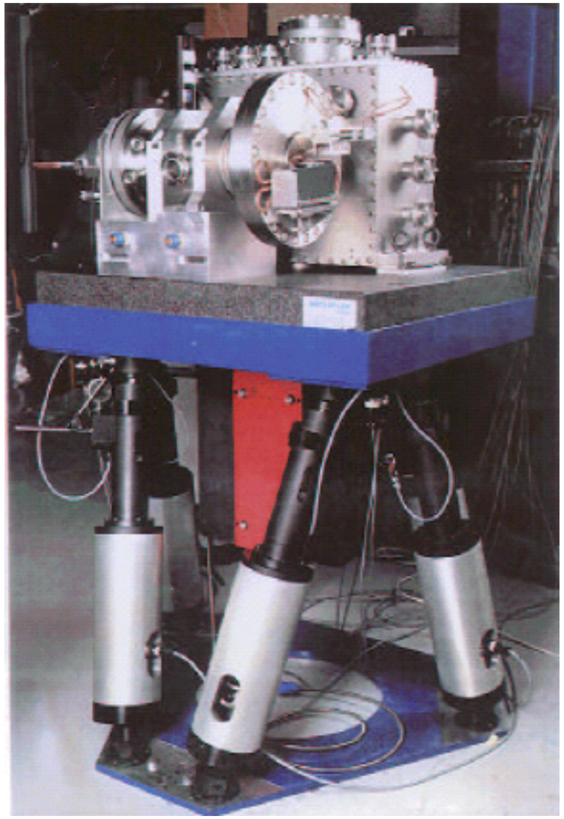
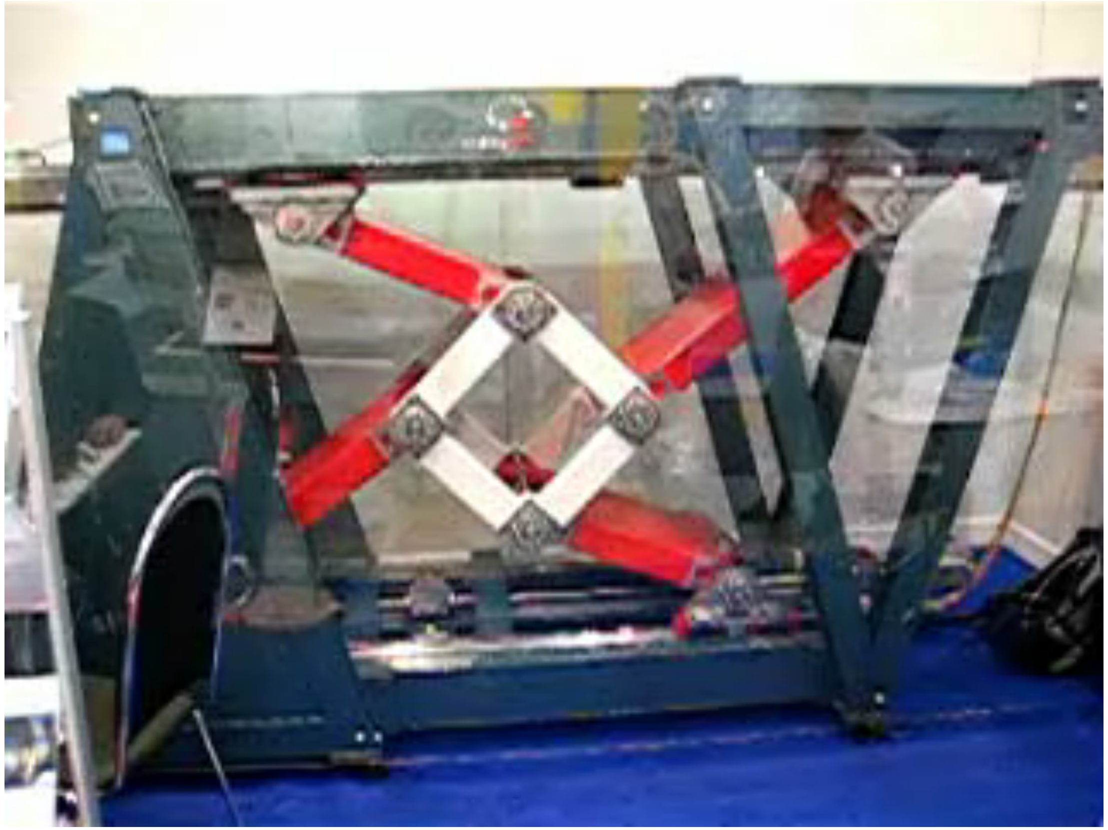
# Parallel manipulators





Sliding Star (courtesy of Prof. Valášek, CTU Prague)

1. Direct kinematic task – difficult
2. Inverse kinematic task – easy



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# Kinematics in robotics

Three main problems

1. Direct kinematic task (přímá kinematická úloha)
2. Inverse kinematic task (inverzní kinematická úloha)
3. Kinematic calibration (kalibrace kinematiky)



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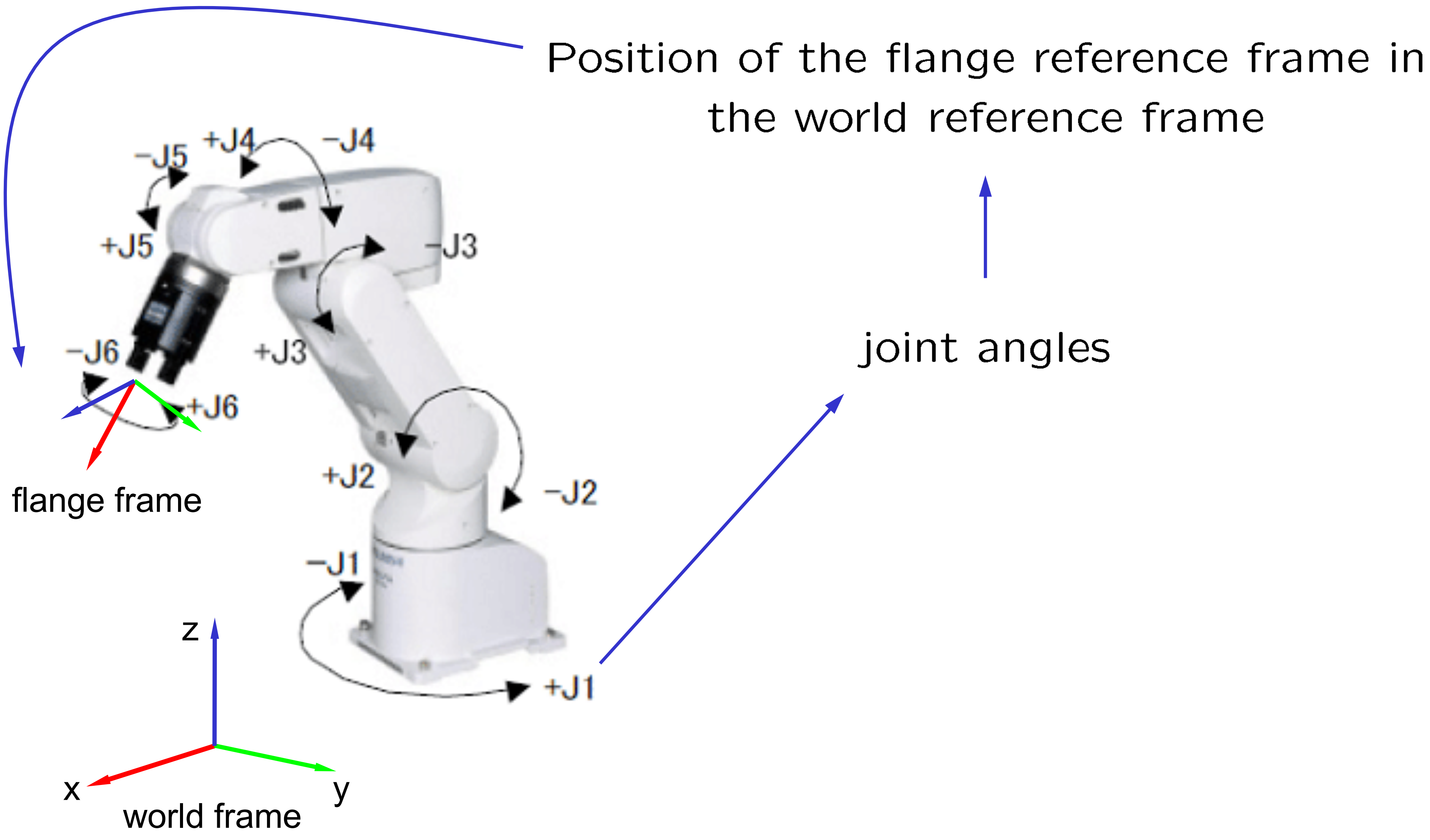
# Direct kinematic task





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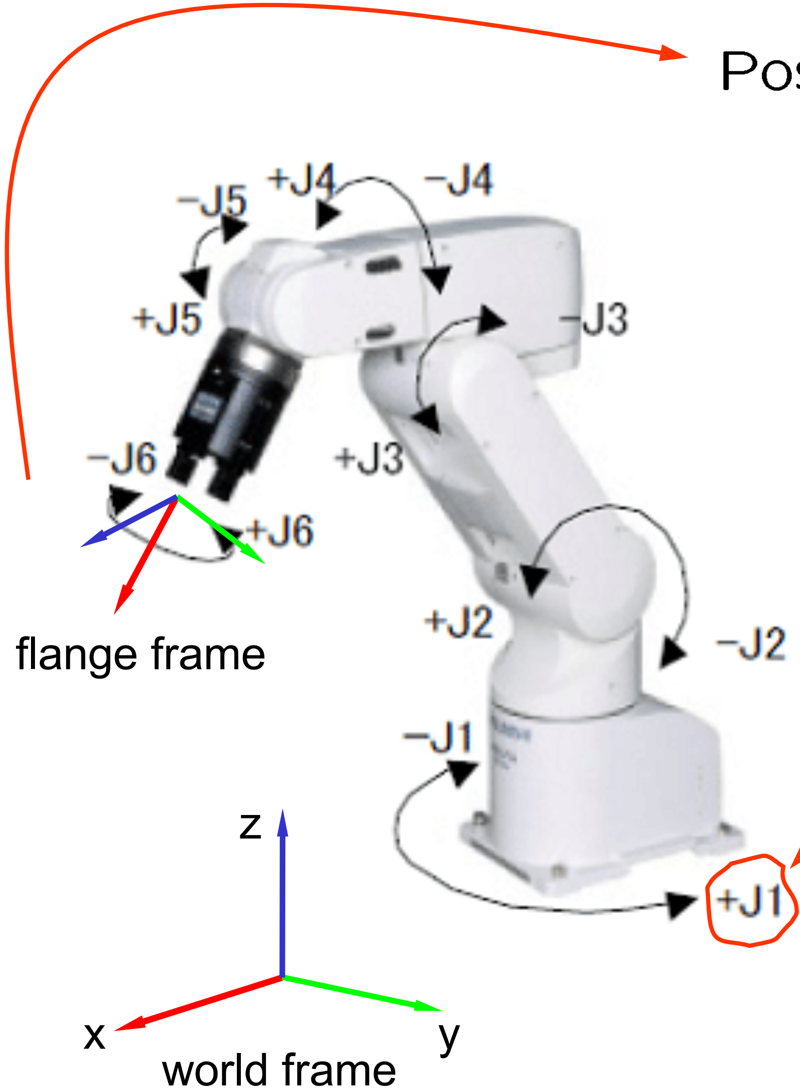
# Inverse kinematic task




Position of the flange reference frame in the world reference frame

↓

joint angles



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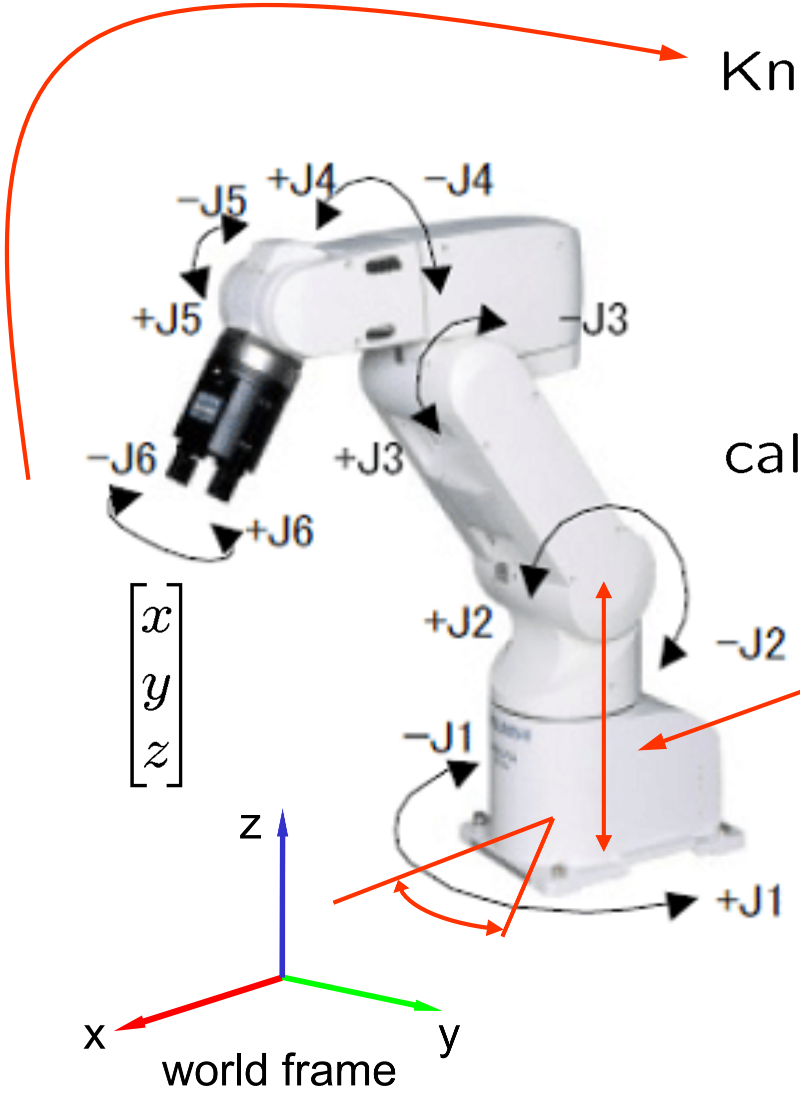
# Kinematic calibration

Known position of a point on the flange
in the world reference frame




calibration parameters (lengths, offsets)



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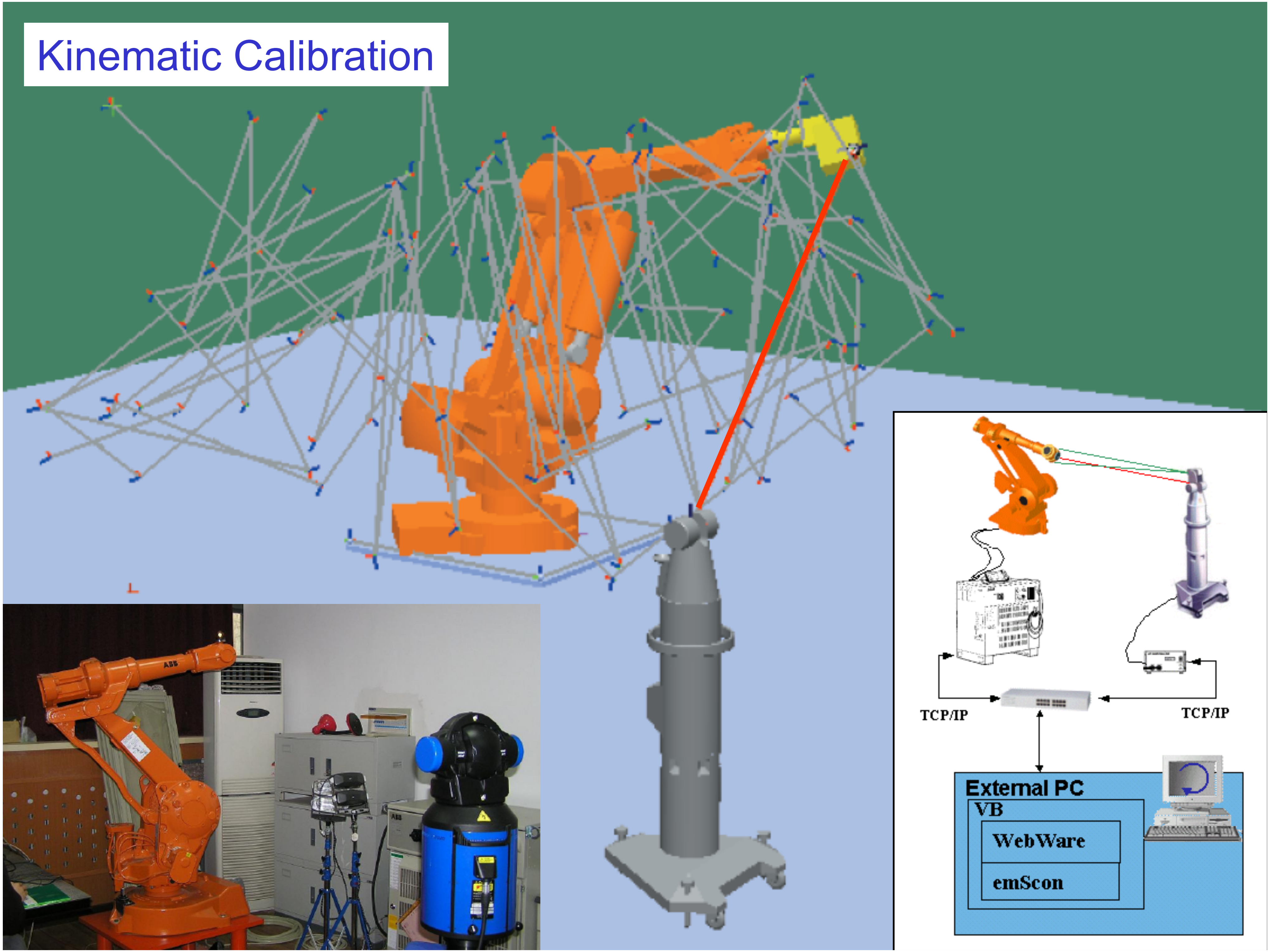
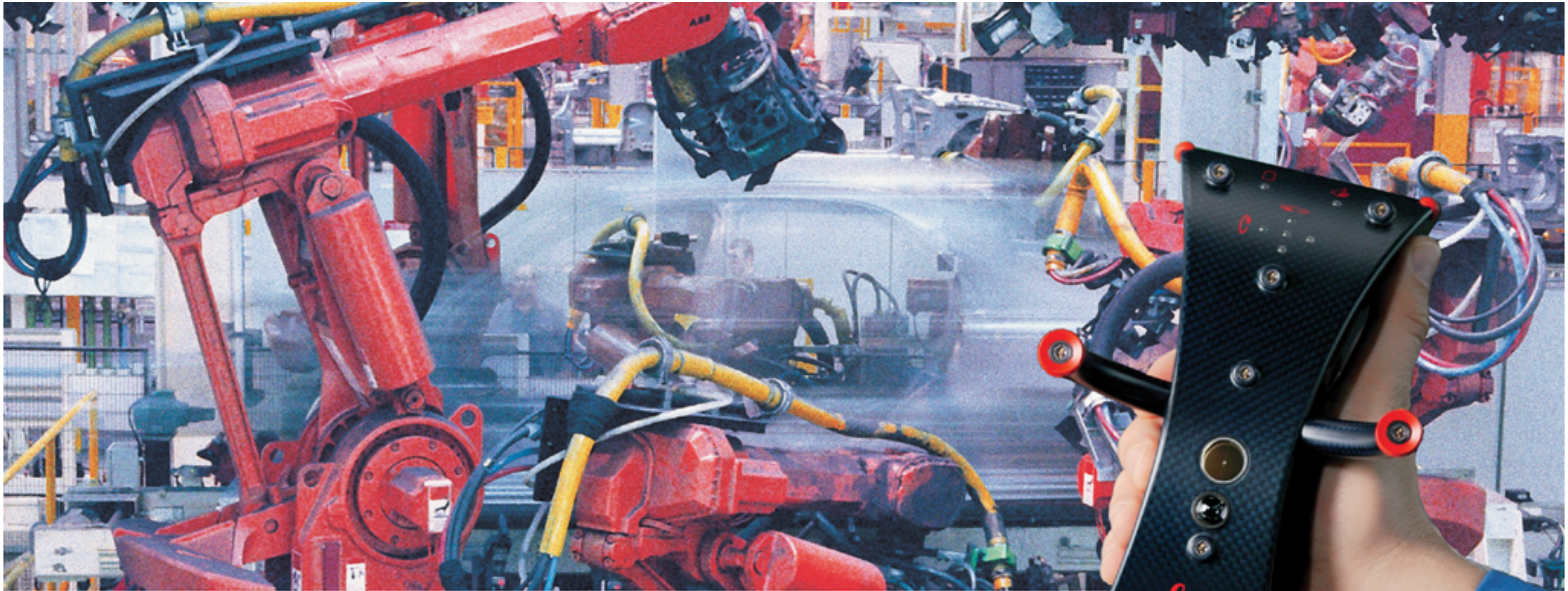
# Kinematic Calibration

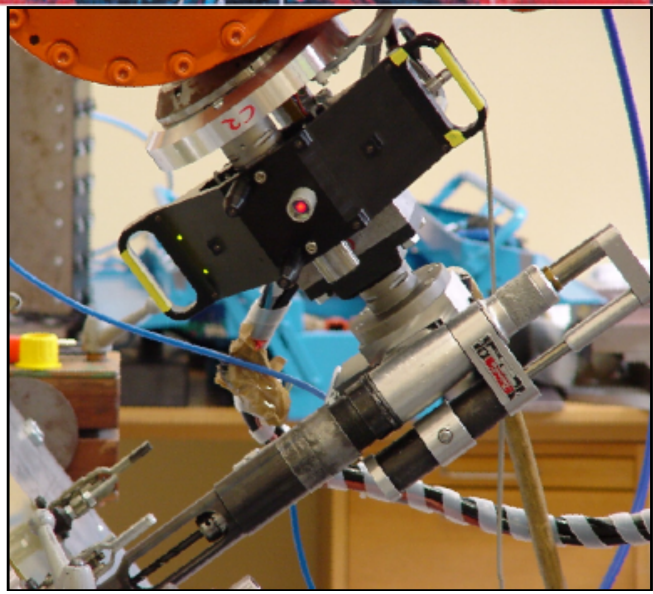


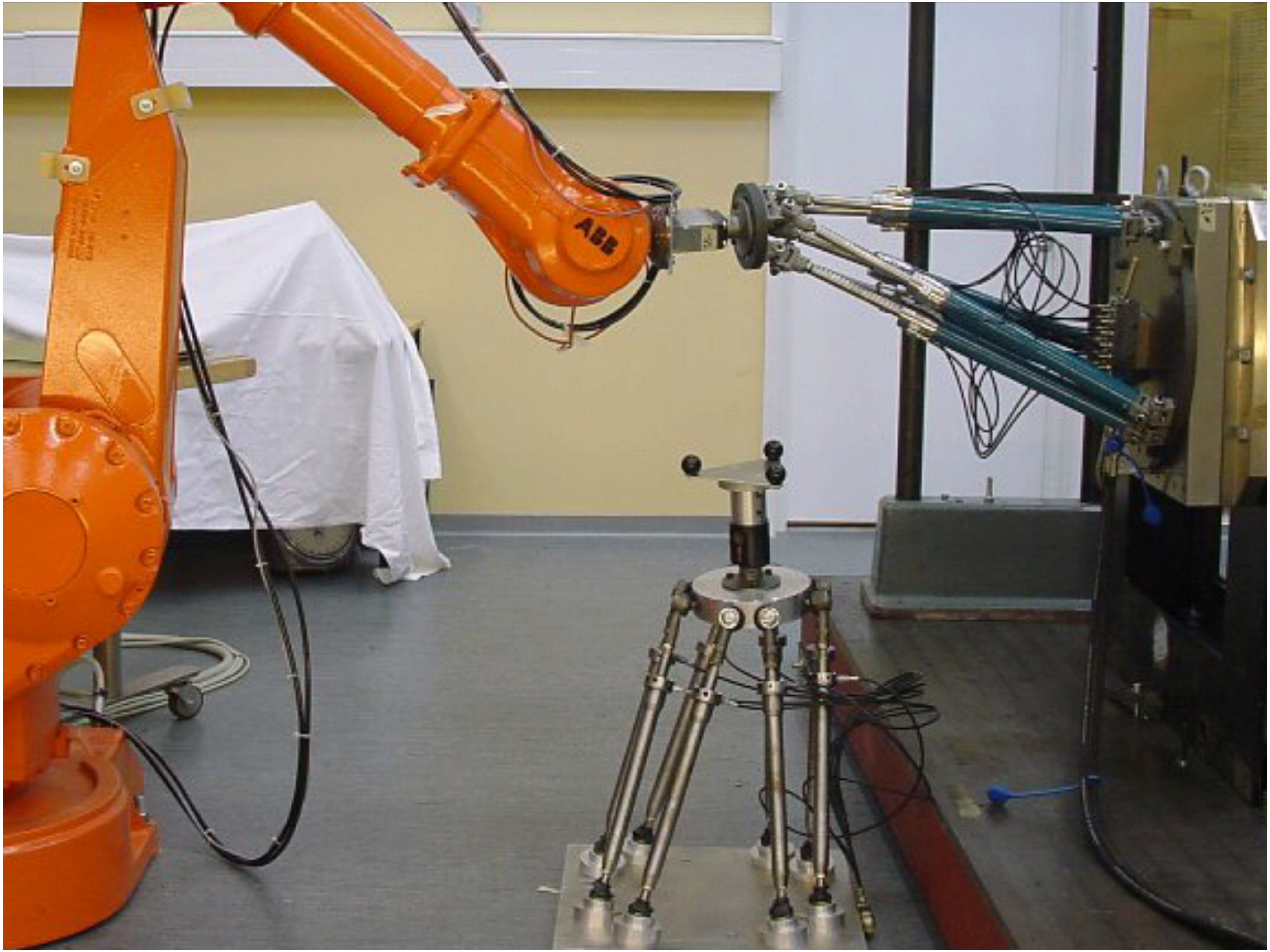
# Robot Calibration

# Kinematic Calibration

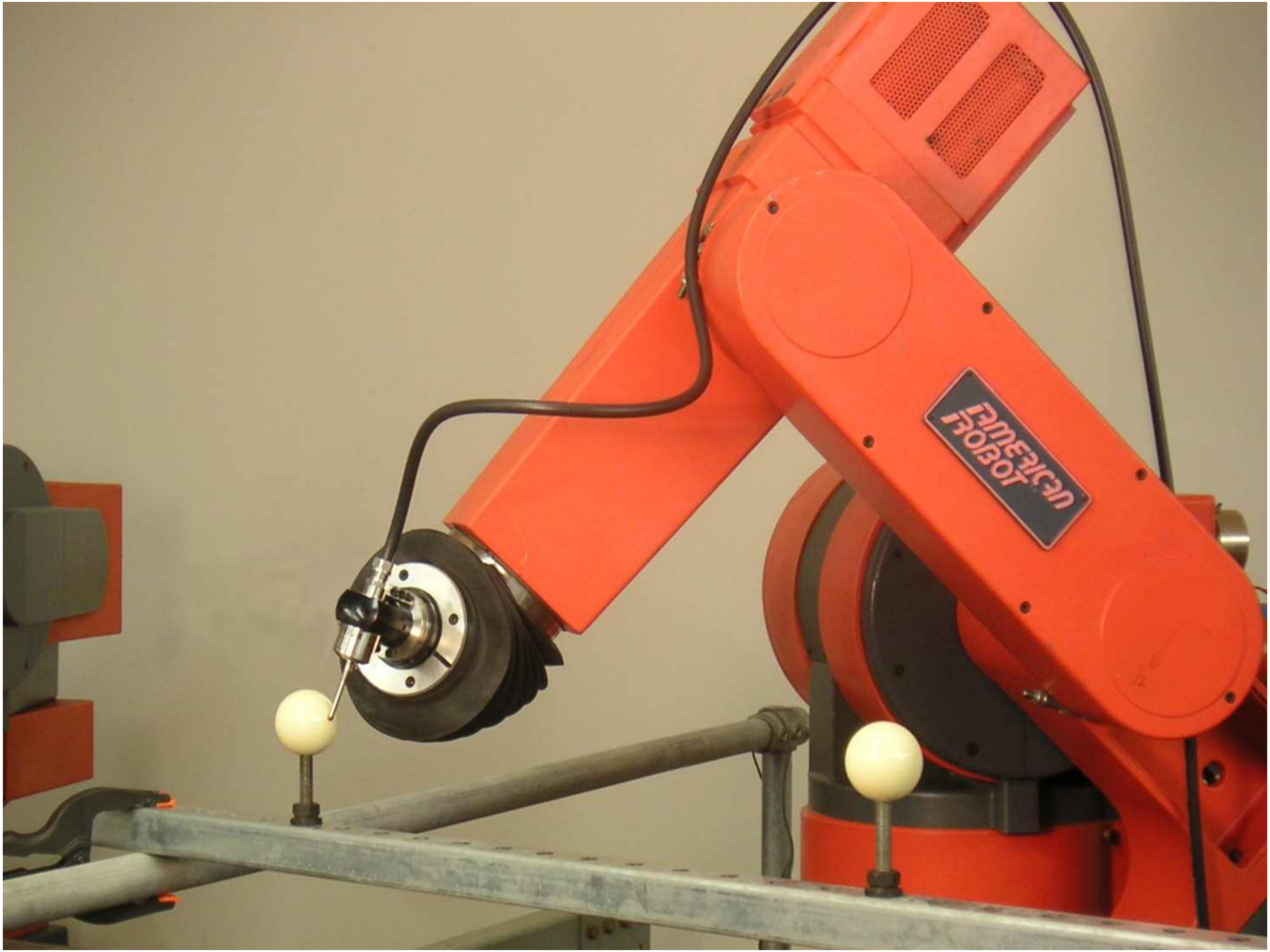
Leica Geosystems





ABB

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# Solving kinematic tasks

TASK

$\downarrow$

GEOMETRY

$\downarrow$

COORDINATE SYSTEMS

$\downarrow$

KINEMATIC MODEL

$\downarrow$

ALGEBRAIC EQUATIONS

$\downarrow$

RESULT


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# Solving kinematic tasks

1968 Donald L. Pieper (Ph.D. thesis)

The inverse kinematics of any serial manipulator with six revolute joints, and with three consecutive joints intersecting, can be solved in closed-form, i.e., analytically.

1989 M. Raghavan, B. Roth. *Kinematic Analysis of the 6R Manipulator of General Geometry.* Int. Symp. Robotics. Research. Pp. 314-320, Tokyo 1989/1990.

A general technique for computing inverse kinematics for any serial manipulator with six revolute joints.

… leads to solving an algebraic equation of degree 16.


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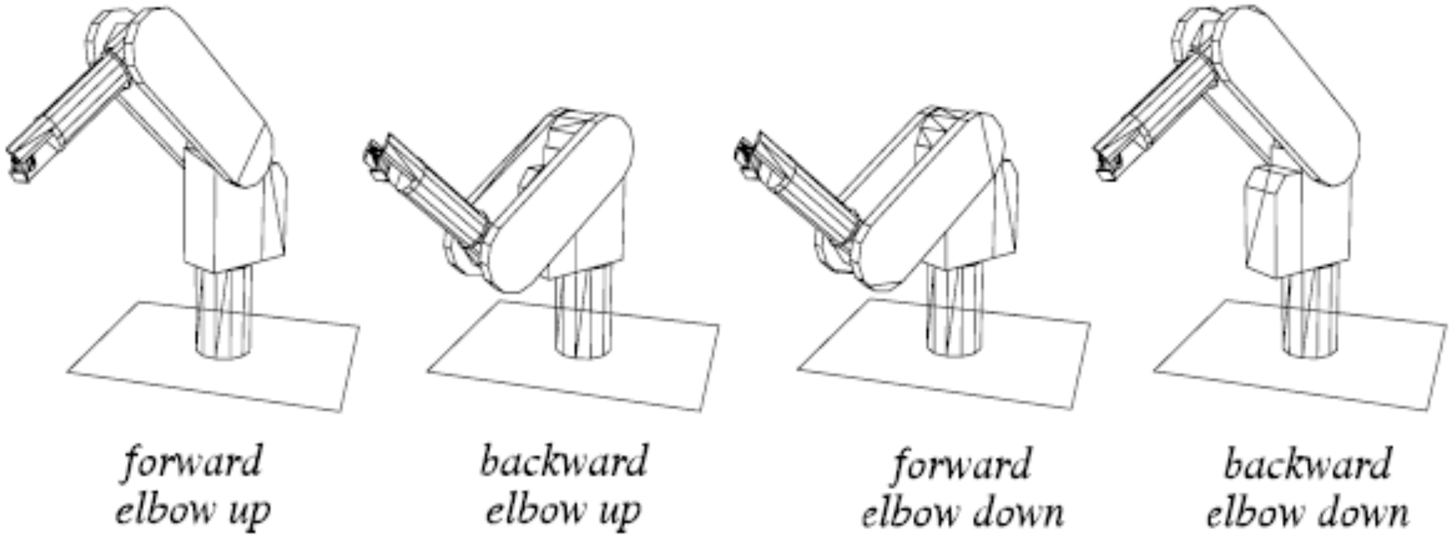
# Solving kinematic tasks

Algebraic equation of degree 16 … up to 16 solutions

4 typical solutions



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# Solving kinematic tasks

TASK

$\downarrow$

GEOMETRY

$\downarrow$

COORDINATE SYSTEMS

$\downarrow$

KINEMATIC MODEL

$\downarrow$

ALGEBRAIC EQUATIONS

$\downarrow$

RESULT



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# Stäubli TX-90 – Geometry

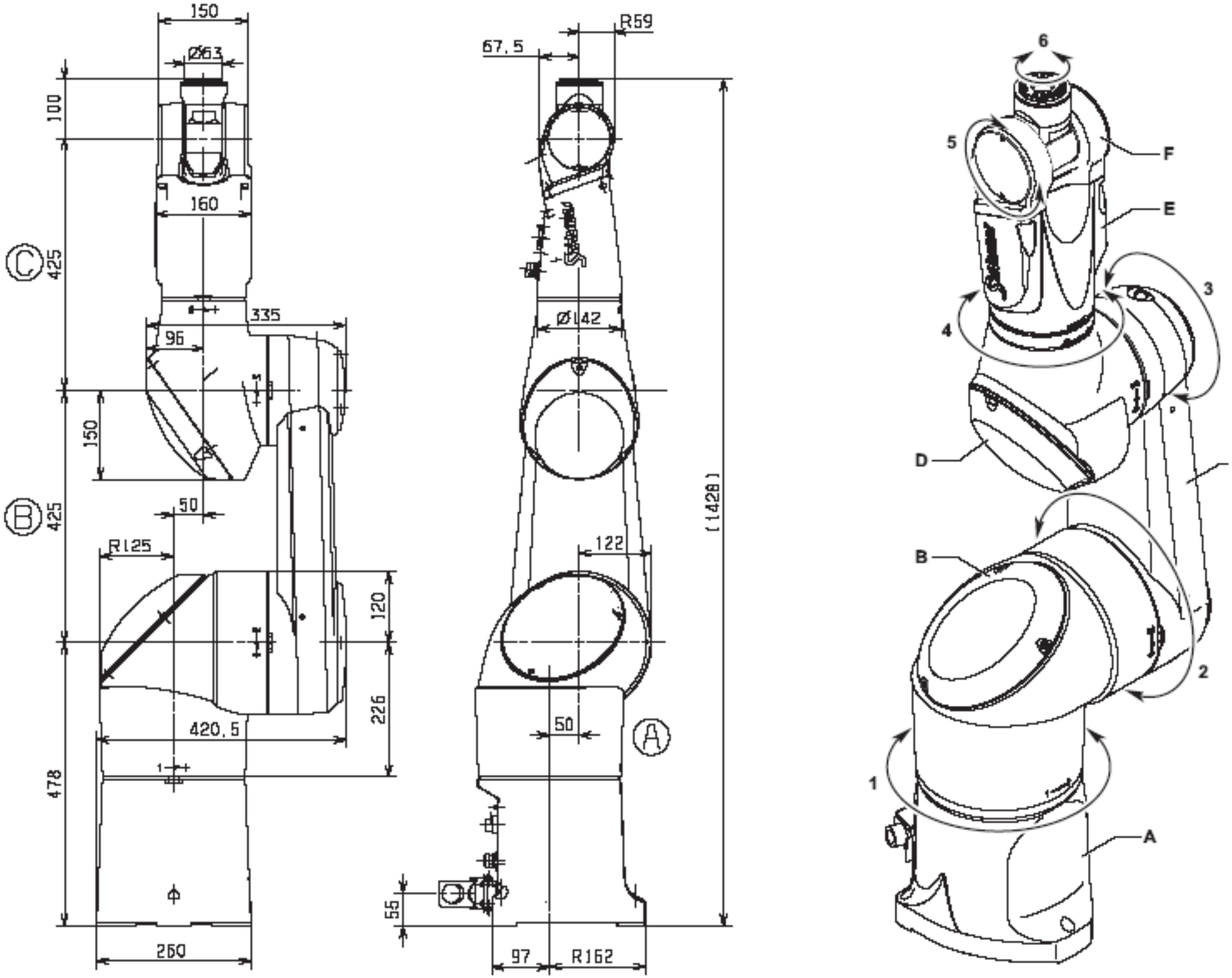


Figure 1.3 - Standard arm



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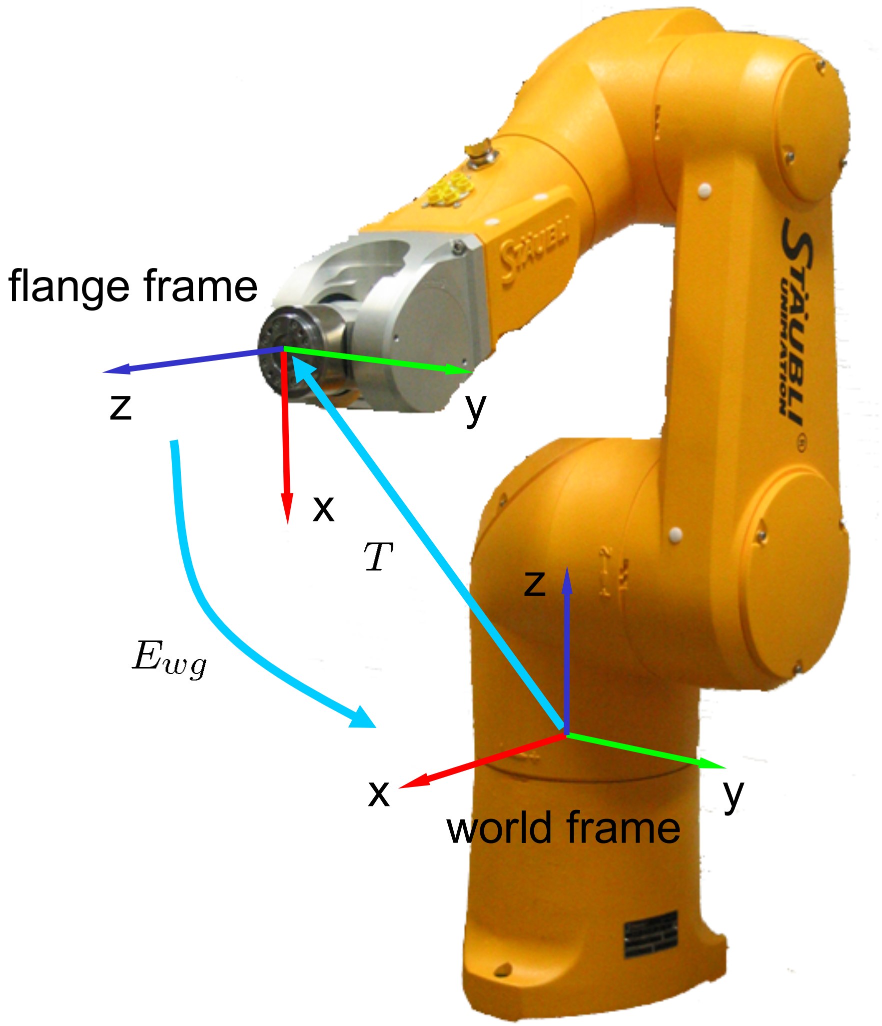
# Kinematic model



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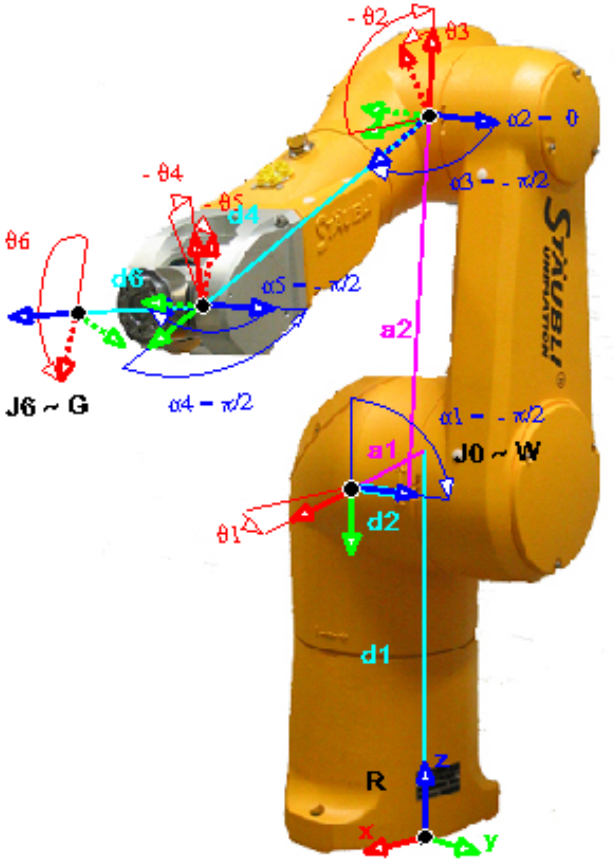
| $\alpha_i$ | $a_i$ | $\theta_i$ | $d_i$ |
|---|---|---|---|

$$A_i^{i-1} = \begin{bmatrix} \cos\theta_i & -\sin\theta_i\cos\alpha_i & \sin\theta_i\sin\alpha_i & a_i\cos\theta_i \\ \sin\theta_i & \cos\theta_i\cos\alpha_i & -\cos\theta_i\sin\alpha_i & a_i\sin\theta_i \\ 0 & \sin\alpha_i & \cos\alpha_i & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$G = A_1^0 A_2^1 A_3^2 A_4^3 A_5^4 A_6^5$$




$$\text{offset} = [0,\ -\tfrac{\pi}{2},\ -\tfrac{\pi}{2},\ 0,\ 0,\ -\pi]$$

| $\alpha_1$ | $a_1$ | $\theta_1$ | $d_1$ |
|---|---|---|---|
| $-\frac{\pi}{2}$ | $a_1$ | $\theta_1$ | 0 |

$$A_1^0 = \begin{bmatrix} \cos\theta_1 & 0 & -\sin\theta_1 & a_1 cos\theta_1 \\ \sin\theta_1 & 0 & \cos\theta_1 & a_1 sin\theta_1 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

| $\alpha_2$ | $a_2$ | $\theta_2$ | $d_2$ |
|---|---|---|---|
| 0 | $a_2$ | $\theta_2$ | $d_2$ |

$$A_2^1 = \begin{bmatrix} \cos\theta_2 & -\sin\theta_2 & 0 & a_2\cos\theta_2 \\ \sin\theta_2 & \cos\theta_2 & 0 & a_2\sin\theta_2 \\ 0 & 0 & 1 & d_2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

| $\alpha_3$ | $a_3$ | $\theta_3$ | $d_3$ |
|---|---|---|---|
| $-\frac{\pi}{2}$ | 0 | $\theta_3$ | 0 |

$$A_3^2 = \begin{bmatrix} \cos\theta_3 & 0 & -\sin\theta_3 & 0 \\ \sin\theta_3 & 0 & \cos\theta_3 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

| $\alpha_4$ | $a_4$ | $\theta_4$ | $d_4$ |
|---|---|---|---|
| $\frac{\pi}{2}$ | 0 | $\theta_4$ | $d_4$ |

$$A_4^3 = \begin{bmatrix} \cos\theta_4 & 0 & \sin\theta_4 & 0 \\ \sin\theta_4 & 0 & -\cos\theta_4 & 0 \\ 0 & 1 & 0 & d_4 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

| $\alpha_5$ | $a_5$ | $\theta_5$ | $d_5$ |
|---|---|---|---|
| $\frac{-\pi}{2}$ | 0 | $\theta_5$ | 0 |

$$A_5^4 = \begin{bmatrix} \cos\theta_5 & 0 & -\sin\theta_5 & 0 \\ \sin\theta_5 & 0 & \cos\theta_5 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

| $\alpha_6$ | $a_6$ | $\theta_6$ | $d_6$ |
|---|---|---|---|
| 0 | 0 | $\theta_6$ | $d_6$ |

$$A_6^5 = \begin{bmatrix} \cos\theta_6 & -\sin\theta_6 & 0 & 0 \\ \sin\theta_6 & \cos\theta_6 & 0 & 0 \\ 0 & 0 & 1 & d_6 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$



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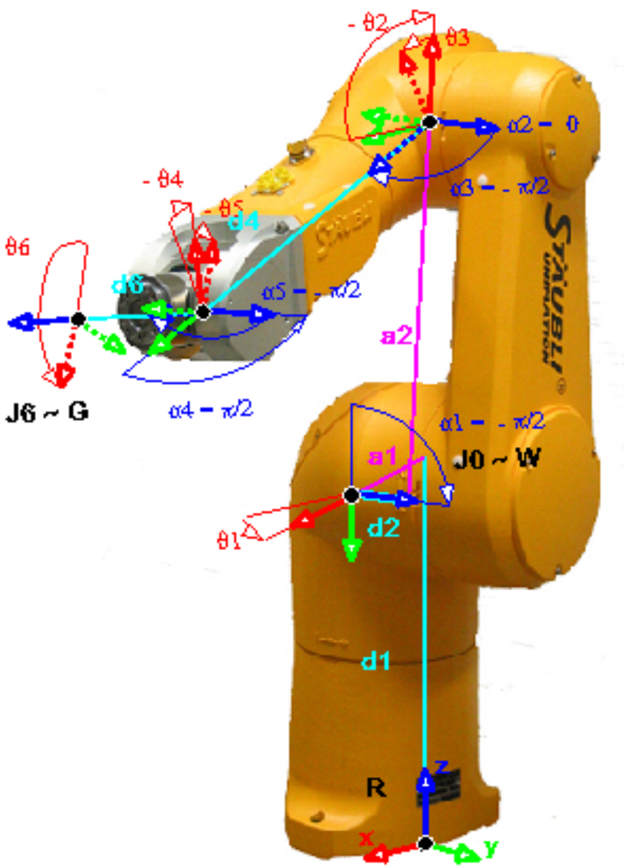
# The Standard Kinematic model in Denavit-Hartenberg Convention

## Stäubli TX 90




**TX-90 (6 axis, RRRRRR) [Staubli]**

| α | a | θ | d |
|---|---|---|---|
| -1.5708 | 50.0 | 0.0 | 350.0 |
| 0.0 | 425.0 | 0.0 | 50.0 |
| -1.5708 | 0.0 | 0.0 | 0.0 |
| 1.5708 | 0.0 | 0.0 | 425.0 |
| -1.5708 | 0.0 | 0.0 | 0.0 |
| 0.0 | 0.0 | 0.0 | 100.0 |

6 non-trivial parameteres



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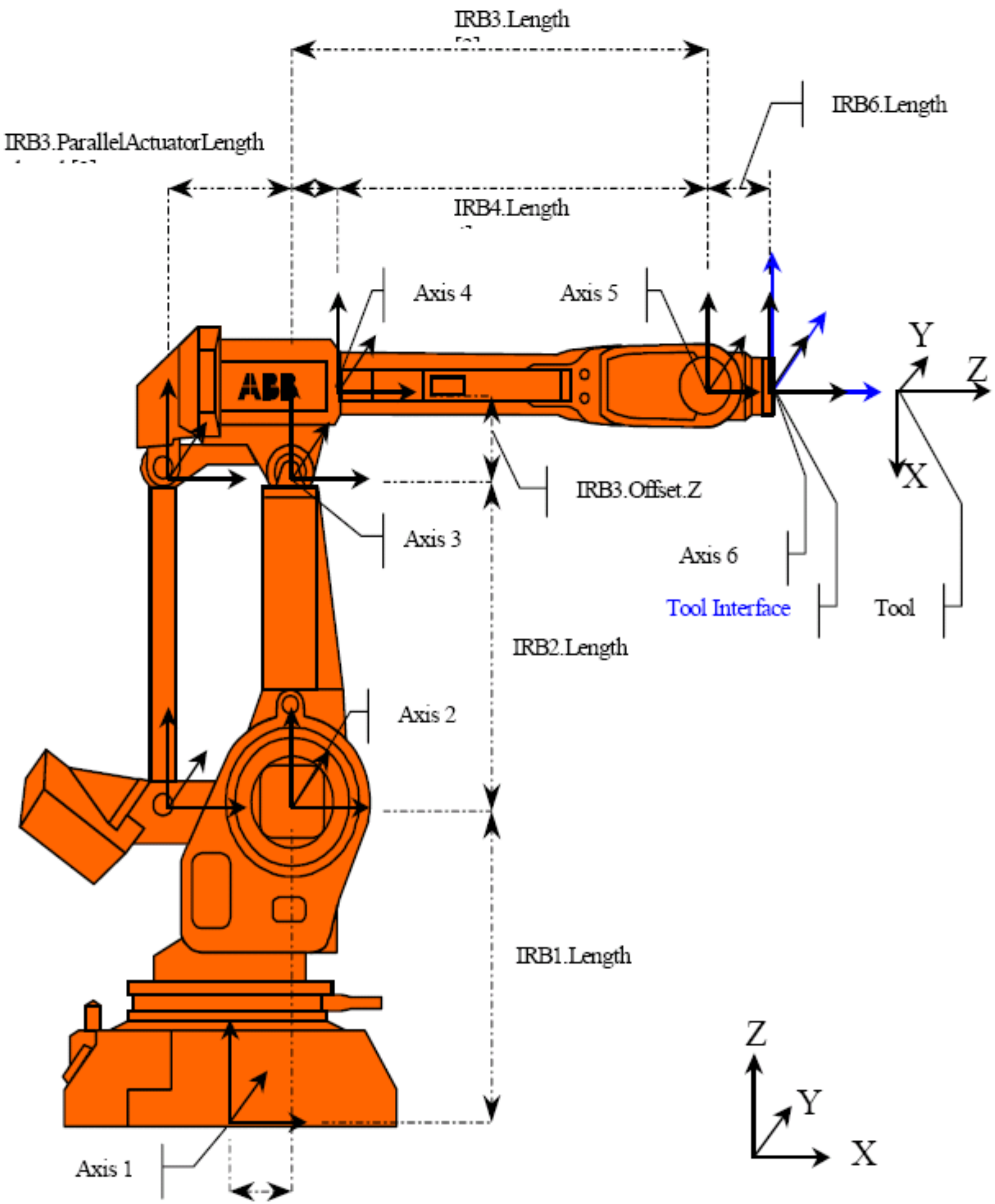
# The Standard Kinematic model in Denavit-Hartenberg Convention

ABB IBR 140



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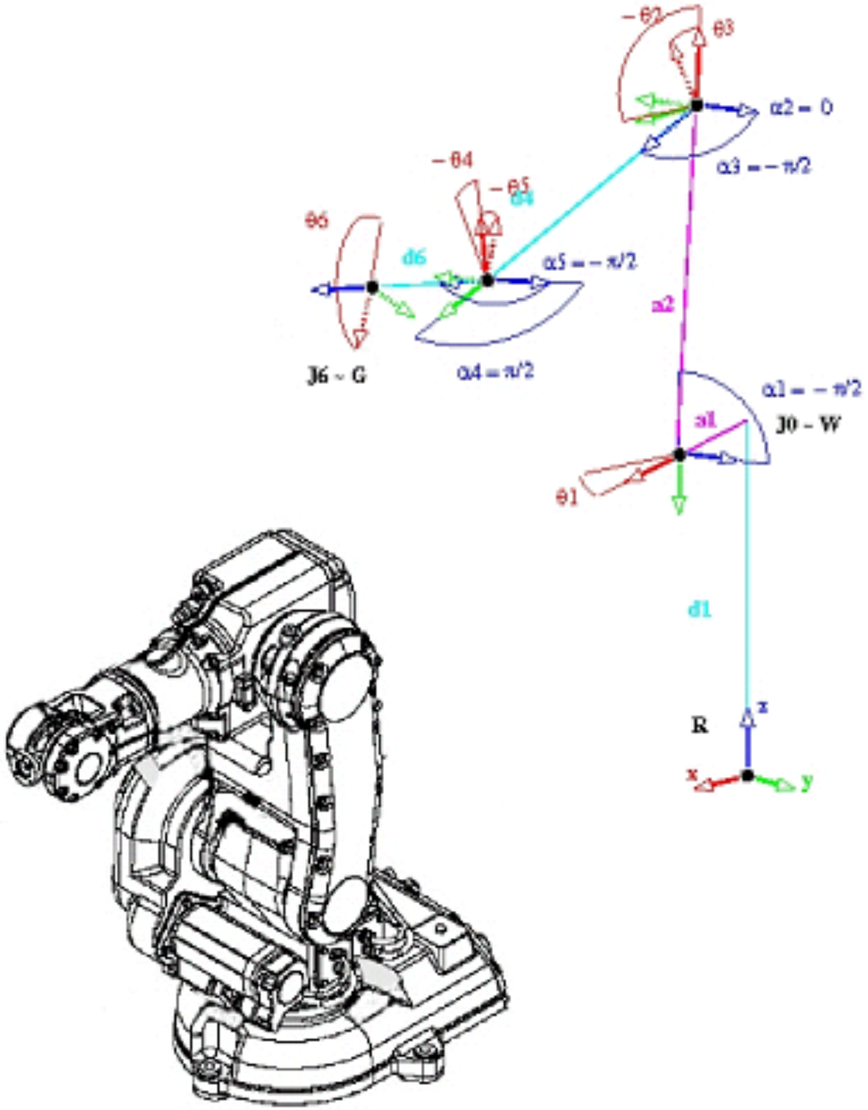
# The Standard Kinematic model in Denavit-Hartenberg Convention

## ABB IBR 140




**IBR-140 (6 axis) [ABB]**

| α | a | θ | d |
|---|---|---|---|
| -1.5708 | 70.0 | 0.0 | 352.0 |
| 0.0 | 360.0 | 0.0 | 0.0 |
| -1.5708 | 0.0 | 0.0 | 0.0 |
| 1.5708 | 0.0 | 0.0 | 380.0 |
| -1.5708 | 0.0 | 0.0 | 0.0 |
| 0.0 | 0.0 | 0.0 | 65.0 |

5 non-trivial parameteres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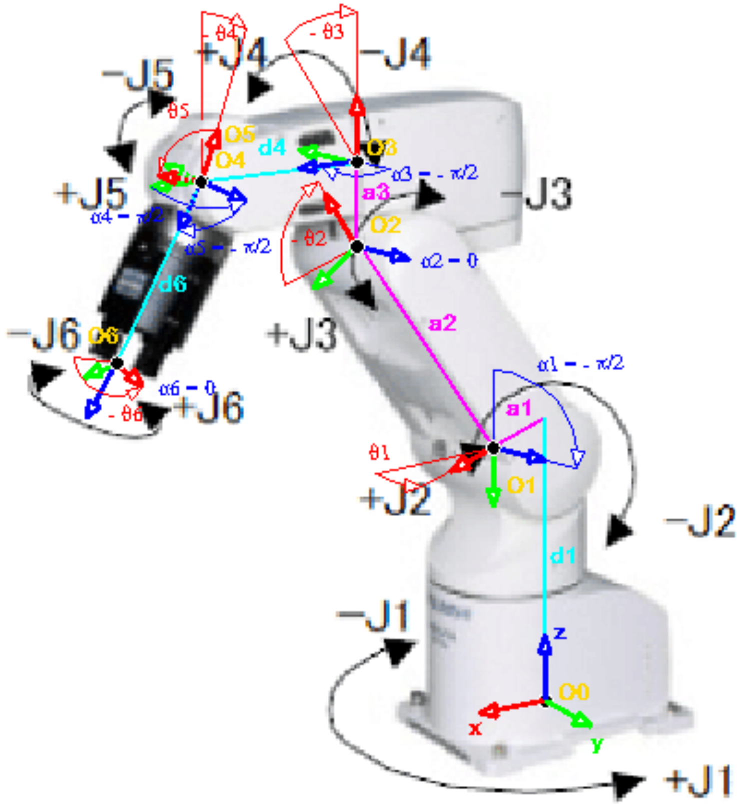
# The Standard Kinematic model in Denavit-Hartenberg Convention

Stäubli TX 90




**RV-6S (6 axis, RRRRRR) [Mitsubishi]**

| α | a | θ | d |
|---|---|---|---|
| -1.5708 | 85.0 | 0.0 | 350.0 |
| 0.0 | 280.0 | 0.0 | 0.0 |
| -1.5708 | 100.0 | 0.0 | 0.0 |
| 1.5708 | 0.0 | 0.0 | 315.0 |
| -1.5708 | 0.0 | 0.0 | 0.0 |
| 0.0 | 0.0 | 0.0 | 85.0 |

6 non-trivial parameteres



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# Special versus General Mechanisms

| Special | × | General |
|---|---|---|
| simple & tractable | | complicated & hard |

| α | a | θ | d |
|---|---|---|---|
| -1.5708 | 70.0 | - | 352.0 |
| 0.0 | 360.0 | - | 0.0 |
| -1.5708 | 0.0 | - | 0.0 |
| 1.5708 | 0.0 | - | 380.0 |
| -1.5708 | 0.0 | - | 0.0 |
| 0.0 | 0.0 | - | 65.0 |

| α | a | θ | d |
|---|---|---|---|
| -1.42 | 70.1 | - (+0.2) | 352.0 |
| 0.10 | 360.0 | - (+0.1) | 0.2 |
| -1.57 | 0.2 | - (- 0.3) | 0.3 |
| 1.58 | 0.1 | - (+0.1) | 380.2 |
| -1.59 | 0.4 | - (- 0.1) | 0.1 |
| 0.07 | 0.2 | - (- 0.2) | 65.1 |

6 non-trivial parameters × 18 (+6) non-trivial parameters

High precision → Small misalignments important → General mechanisms



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# Literature


Linear algebra

P. Pták. *Introduction to Linear Algebra*. Vydavatelství ČVUT, Praha, 2006.

Numerical linear algebra

E. Krajník. *Maticový počet*. Vydavatelství ČVUT, Praha, 2000.

The solution

M. Raghavan, B. Roth. *Kinematic Analysis of the 6R Manipulator of General Geometry.* Int. Symp. Robotics. Research. Pp. 314-320, Tokyo 1989/1990.

The numerical solution

D. Manocha, J. Canny. *Efficient Inverse Kinematics for General 6R Manipulators*. Robotics and Automation 1994.

The pedagogical solution will be developed using

D. Cox, J. Little, D. O'Shea. *Ideals, Varieties, and Algorithms*. Springer 1998.




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# Software

Matlab: www.matworks.com

Maple: www.maplesoft.com



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## One algebraic equation in one variable



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# SOLVING 1 ALGEBRAIC EQUATION

1 equation, 1 variable $\rightarrow$ companion matrix $\rightarrow$ eigenvalues

$$f(x) = x^3 + 4\,x^2 + x - 6 \;=\; -6 + 1\,x + 4\,x^2 + 1\,x^3$$

$$\mathtt{M}_x = \begin{bmatrix} 0 & 0 & 6 \\ 1 & 0 & -1 \\ 0 & 1 & -4 \end{bmatrix}$$

... a simple rule

$\mathtt{>>\ e=eig(M}_x\mathtt{)}$

$$\mathtt{e} \;=\; \begin{bmatrix} 1 \\ -2 \\ -3 \end{bmatrix} \qquad x_1 = 1,\; x_2 = -2,\; x_3 = -3$$

It works when eig works, i.e. order 100 in Matlab is often OK.



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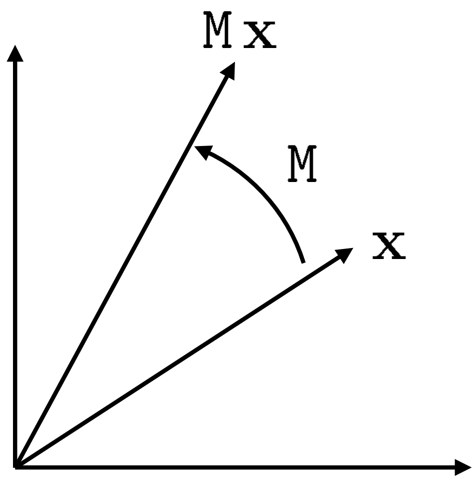
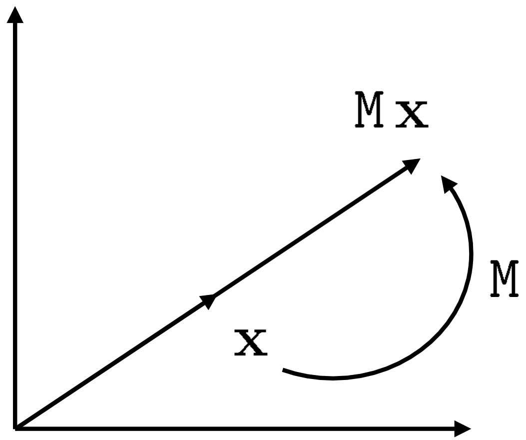
# SOLVING 1 ALGEBRAIC EQUATION

Linear maping $\mathtt{M} \in \mathbb{R}^{n \times n}$

Eigenvalues

$$\mathtt{M}\,\mathbf{x} = \lambda\,\mathbf{x}$$
$$\Updownarrow$$
$$\mathtt{M}\,\mathbf{x} - \lambda\,\mathbf{x} = 0$$
$$\Updownarrow$$
$$\mathtt{M}\,\mathbf{x} - \lambda\,\mathtt{I}\,\mathbf{x} = 0$$
$$\Updownarrow$$
$$(\mathtt{M} - \lambda\,\mathtt{I})\,\mathbf{x} = 0$$
$$\mathbf{x} \neq 0 \;\Rightarrow\; \Updownarrow$$
$$\mathrm{rank}(\mathtt{M} - \lambda\,\mathtt{I}) < n$$
$$\Updownarrow$$
$$\det(\mathtt{M} - \lambda\,\mathtt{I}) = 0$$





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# SOLVING 1 ALGEBRAIC EQUATION

algebraic equation

$$f(x) = x^4 + a_3\,x^3 + a_2\,x^2 + a_1\,x + a_0 \;= \mathrm{det}(-\mathtt{M} + x\,\mathtt{I})$$

$$-\mathtt{M} + x\,\mathtt{I} = \begin{bmatrix} & & & a_0 \\ -1 & & & a_1 \\ & -1 & & a_2 \\ & & -1 & x + a_3 \end{bmatrix}$$

$$f(x) = x^4 + a_3\,x^3 + a_2\,x^2 + a_1 \; + a_0$$

Numerical solution to f(x) is obtained by

```
>> x = eig(M);
```



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