3D Computer Vision

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Open Informatics Master's Course

Module II

Perspective Camera

covered by

[H&Z] Secs: 2.1, 2.2, 3.1, 6.1, 6.2, 8.6, 2.5, Example: 2.19

▶Basic Geometric Entities, their Representation, and Notation

- entities have names and representations
- names and their components:

entity	in 2-space	in 3-space
point	$m = (u, v)$	$X = (x, y, z)$
line	n	O
plane		$\pi,\ \varphi$

- associated vector representations

$$\mathbf{m} = \begin{bmatrix} u \\ v \end{bmatrix} = [u, v]^\top, \quad \mathbf{X} = \begin{bmatrix} x \\ y \\ z \end{bmatrix}, \quad \mathbf{n}$$

will also be written in an 'in-line' form as $\mathbf{m} = (u, v)$, $\mathbf{X} = (x, y, z)$, etc.

- vectors are always meant to be columns $\mathbf{x} \in \mathbb{R}^{n \times 1}$
- associated homogeneous representations

$$\underline{\mathbf{m}} = [m_1, m_2, m_3]^\top, \quad \underline{\mathbf{X}} = [x_1, x_2, x_3, x_4]^\top, \quad \underline{\mathbf{n}}$$

'in-line' forms: $\underline{\mathbf{m}} = (m_1, m_2, m_3)$, $\underline{\mathbf{X}} = (x_1, x_2, x_3, x_4)$, etc.

- matrices are $\mathbf{Q} \in \mathbb{R}^{m \times n}$, linear map of a $\mathbb{R}^{n \times 1}$ vector is $\mathbf{y} = \mathbf{Q}\mathbf{x}$
- j-th element of vector $\mathbf{m}_i$ is $(\mathbf{m}_i)_j$; element i, j of matrix $\mathbf{P}$ is $\mathbf{P}_{ij}$

▶Image Line (in 2D)

a finite line in the 2D (u, v) plane $\quad a\,u + b\,v + c = 0$

has a parameter (homogeneous) vector $\quad \boxed{\underline{\mathbf{n}} \simeq (a,\ b,\ c)},\quad \|\underline{\mathbf{n}}\| \neq 0$

and there is an equivalence class for $\lambda \in \mathbb{R},\ \lambda \neq 0 \quad (\lambda a,\ \lambda b,\ \lambda c) \simeq (a,\ b,\ c)$

$a \simeq b$

$\Leftrightarrow$

$a = \lambda b$

$\lambda \neq 0$

'Finite' lines

- standard representative for finite $\underline{\mathbf{n}} = (n_1, n_2, n_3)$ is $\lambda\underline{\mathbf{n}}$, where $\lambda = \frac{\mathbf{1}}{\sqrt{n_1^2 + n_2^2}}$

assuming $n_1^2 + n_2^2 \neq 0$; $\mathbf{1}$ is the unit, usually $\mathbf{1} = 1$

'Infinite' line

- we augment the set of lines for a special entity called the **line at infinity** (ideal line)

$$\underline{\mathbf{n}}_\infty \simeq (0, 0, 1) \qquad \text{(standard representative)}$$

- the set of equivalence classes of vectors in $\mathbb{R}^3 \setminus (0, 0, 0)$ forms the projective space $\mathbb{P}^2$

a set of rays →21

- line at infinity is a proper member of $\mathbb{P}^2$
- I may sometimes wrongly use $=$ instead of $\simeq$, if you are in doubt, ask me

▶Image Point

Finite point $\mathbf{m} = (u, v)$ is incident on a finite line $\underline{\mathbf{n}} = (a, b, c)$ iff   iff = works either way!

$$a\,u + b\,v + c = 0$$

can be rewritten as (with scalar product): $(u, v, \mathbf{1}) \cdot (a, b, c) = \boxed{\underline{\mathbf{m}}^\top \underline{\mathbf{n}} = 0}$

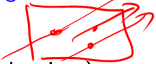
$\lambda\, \underline{\mathbf{m}}^\top \cdot \underline{\mathbf{n}} = 0$

'Finite' points

- a finite point is also represented by a homogeneous vector $\underline{\mathbf{m}} \simeq (u, v, \mathbf{1})$, $\|\underline{\mathbf{m}}\| \neq 0$
- the equivalence class for $\lambda \in \mathbb{R}, \lambda \neq 0$ is $(m_1, m_2, m_3) = \lambda\,\underline{\mathbf{m}} \simeq \underline{\mathbf{m}}$
- the standard representative for finite point $\underline{\mathbf{m}}$ is $\lambda\,\underline{\mathbf{m}}$, where $\lambda = \frac{\mathbf{1}}{m_3}$ assuming $m_3 \neq 0$
- when $\mathbf{1} = 1$ then units are pixels and $\lambda\underline{\mathbf{m}} = (u, v, 1)$ $\underline{m} = (m_1, m_2, m_3)$ | $u = \frac{m_1}{m_3}$, $v = \frac{m_2}{m_3}$, $1 = \frac{m_3}{m_3}$
- when $\mathbf{1} = f$ then all elements have a similar magnitude, $f \sim$ image diagonal

use $\mathbf{1} = 1$ unless you know what you are doing;

all entities participating in a formula must be expressed in the same units



'Infinite' points

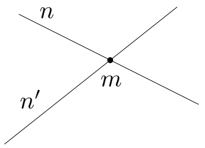
- we augment for **points at infinity** (ideal points) $\underline{\mathbf{m}}_\infty \simeq (m_1, m_2, 0)$

  proper members of $\mathbb{P}^2$
- all such points lie on the line at infinity (ideal line) $\underline{\mathbf{n}}_\infty \simeq (0, 0, 1)$, i.e. $\underline{\mathbf{m}}_\infty^\top \underline{\mathbf{n}}_\infty = 0$

▶Line Intersection and Point Join

The point of **intersection** m of image lines n and n', $n \not\simeq n'$ is

$$\boxed{\underline{\mathbf{m}} \simeq \underline{\mathbf{n}} \times \underline{\mathbf{n}}'}$$



$\underline{n}_1 = (n_{11}, n_{12}, n_{13})$

$\underline{n}_2 = (n_{21}, n_{22}, n_{23})$

$\begin{vmatrix} i & n_{11} & n_{21} \\ j & n_{12} & n_{22} \\ k & n_{13} & n_{23} \end{vmatrix}$

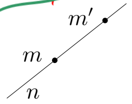
proof: If $\underline{\mathbf{m}} = \underline{\mathbf{n}} \times \underline{\mathbf{n}}'$ is the intersection point, it must be incident on both lines. Indeed, using known equivalences from vector algebra

$$\underline{\mathbf{n}}^\top \underbrace{(\underline{\mathbf{n}} \times \underline{\mathbf{n}}')}_{\underline{\mathbf{m}}} \equiv \underline{\mathbf{n}}'^\top \underbrace{(\underline{\mathbf{n}} \times \underline{\mathbf{n}}')}_{\underline{\mathbf{m}}} \equiv 0$$

$\underline{n}^T \underline{m} = \emptyset$

The **join** n of two image points m and m', $m \not\simeq m'$ is

$$\boxed{\underline{\mathbf{n}} \simeq \underline{\mathbf{m}} \times \underline{\mathbf{m}}'}$$



Paralel lines intersect (somewhere) on the line at infinity $\underline{\mathbf{n}}_\infty \simeq (0, 0, 1)$:

$$a\,u + b\,v + c = 0,$$
$$a\,u + b\,v + d = 0, \qquad d \neq c$$
$$(a, b, c) \times (a, b, d) \simeq (b, -a, 0) \quad \text{at } \infty$$

- all such intersections lie on $\underline{\mathbf{n}}_\infty$
- line at infinity therefore represents the set of (unoriented) directions in the plane
- Matlab: `m = cross(n, n_prime);`

Thank You