Adversarial Search

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Notes

Games, man vs. algorithm

- Deep Blue
- Alpha Go
- Deep Stack
- Why Games, actually?

Games are interesting for AI *because* they are hard (to solve).

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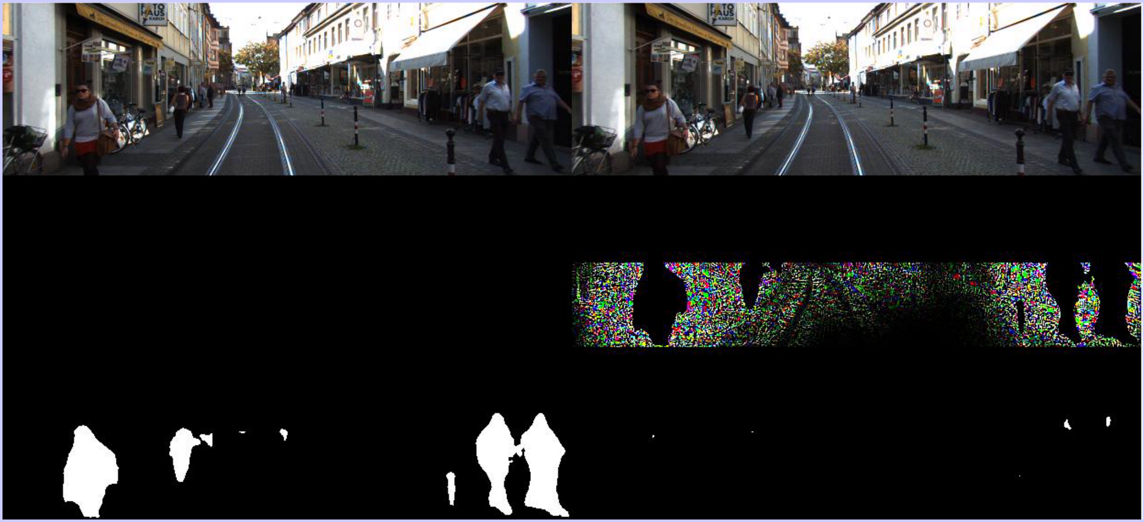
More: Adversarial Learning



Video: Adversing visual segmentation

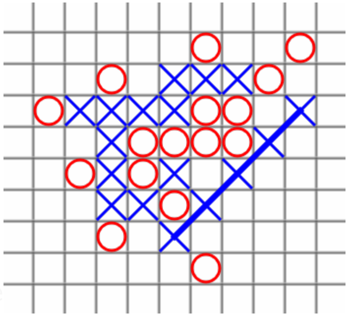
Vision for Robotics and Autonomous Systems, http://cyber.felk.cvut.cz/vras

Notes

- Fooling Tesla autopilot by adversarial attack:

Elements of the game

- s_0: The initial state
- PLAYER(s). Which player has to move in s.
- ACTIONS(s). What are the legal moves?
- RESULT(s, a). Transition, result of a move.
- TERMINAL-TEST(s). Game over?
- TERMINAL-UTILITY(s, p). What is prize? Examples for some games ...



https://commons.wikimedia.org/wiki/File:Tic-tac-toe_5.png

Notes

Defining a game as a kind of search problem:
Considering the notation, we are making slight transition from [1] to [2].

- Players: $P = \{1, 2, \ldots, N\}$ (often just $N = 2$)
- Transition functions: $S \times A \rightarrow S$.
- Terminal utilities: $S \times P \rightarrow R$. (R - as a Reward)

What are we loking for? A strategy/policy $S \rightarrow A$

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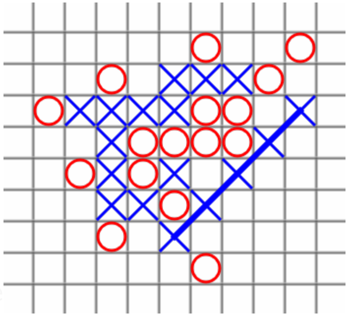

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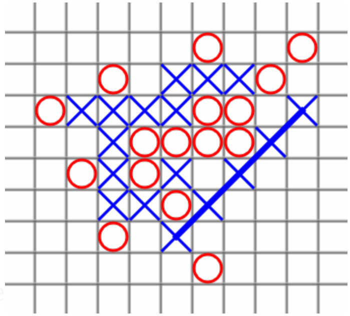

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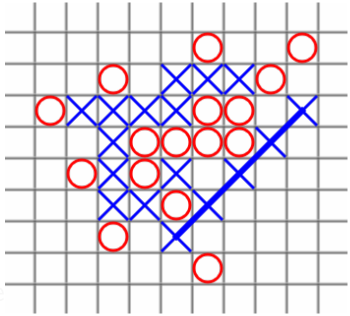

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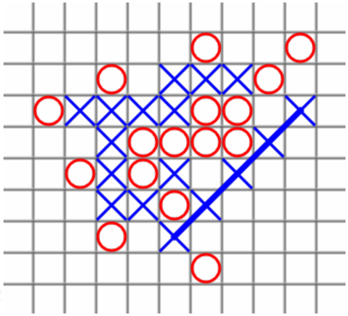

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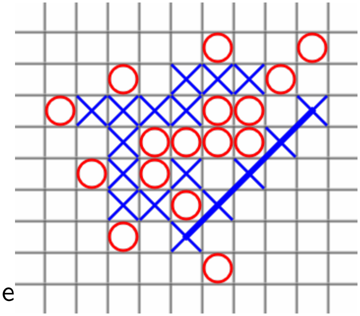

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Terminal utilitity: Zero–Sum and General games

- Zero-sum: players have opposite utilities (values)
- Zero-sum: playing against opponent
- General game: independent utilities
- General game: cooperations, competition, ...

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Most common games—such as chess—have these properties:

- two-player
- turn-taking
- deterministic with perfect information (a.k.a. deterministic, fully observable environments)

In some games, there is imperfect information (evironment is not fully observable). E.g., poker – no access to what cards opponents hold.

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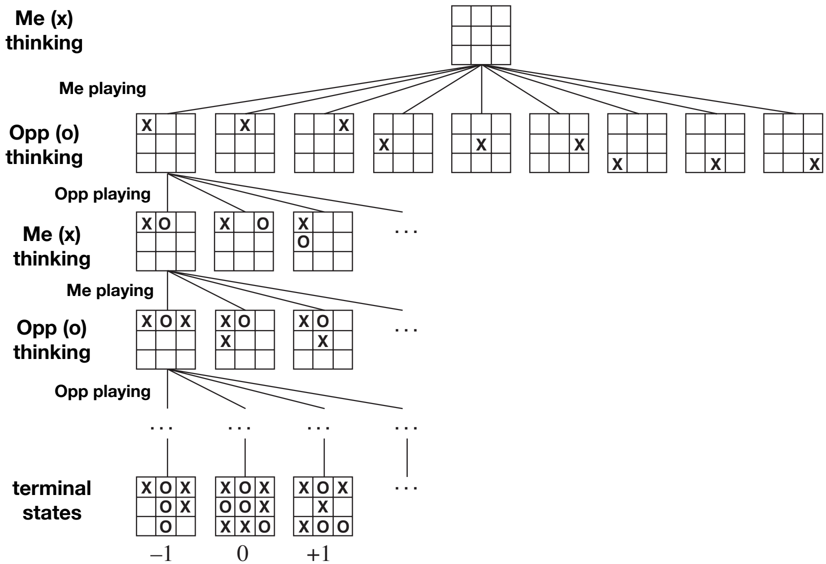
Game Tree(s)



$\textsc{Terminal-Utility}(s, \mathbf{x})$

Notes

Init state, ACTIONS function, and RESULT function defines game tree.

Note: *game tree* as opposed to *search tree*. *Game tree* are all possible evolutions of the game.
(With standard search, we similarly had *state space graph* vs. *search tree*.)

State Value $V(s)$

$V(s)$ – value V of a state s : The best utility achievable from this state.

$$V(s) = \max_{s' \in \text{children}(s)} V(s')$$

Notes

Think about the State Value. It is a theoretical construct, definition. Depending on the problem, there may be various computational algorithms.

In a game, what State Values are known? Usually, only terminal states.

Think, for a moment, you are the only player. You can control every step. How would you compute the $V(s)$ for a given state s?

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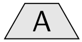
Two-ply game: **max** for me, **min** for the opponent.



$$a_1 = \underset{a \in \textsc{Actions}(A)}{\arg\max}\ \textsc{Result}(A, a)$$

Notes

One *move* consists of two *plies* (half-moves).
I'm the player that starts (state A) and want to decide what to play; actions/plies a_1, a_2, a_3 are the options. B, C, D are the possible outcomes of my moves (plies). Now the opponent is about to play. The numbers in terminal states denote *my* profit/utility.

Node evaluation: *minimax* in action.

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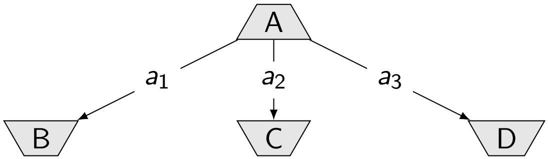

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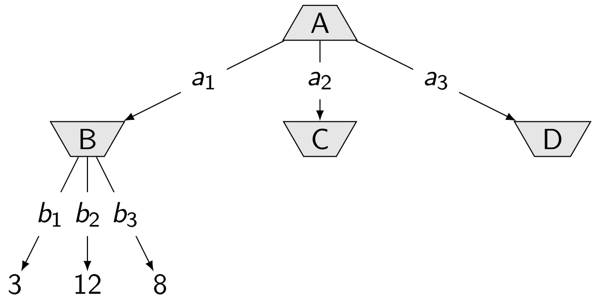

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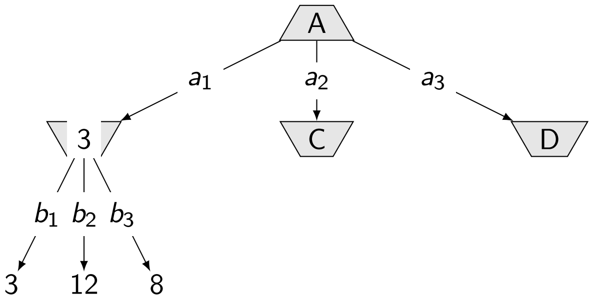

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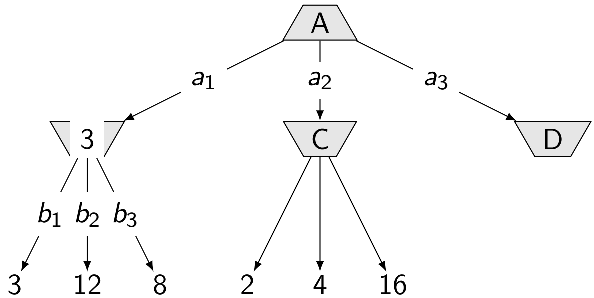

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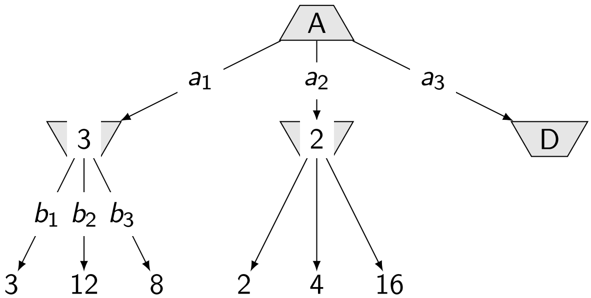

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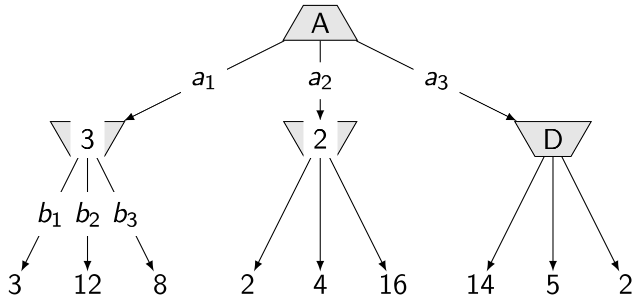

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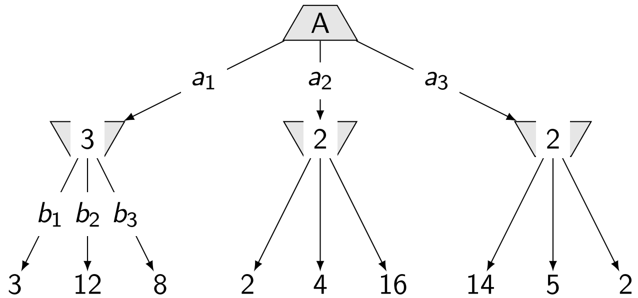

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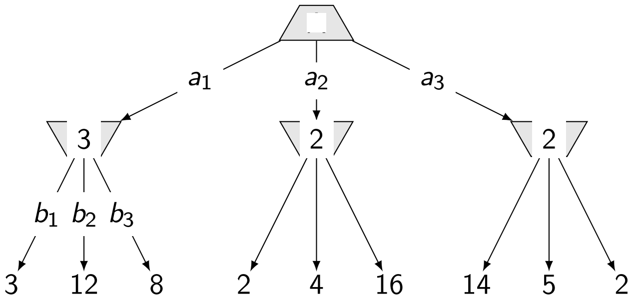

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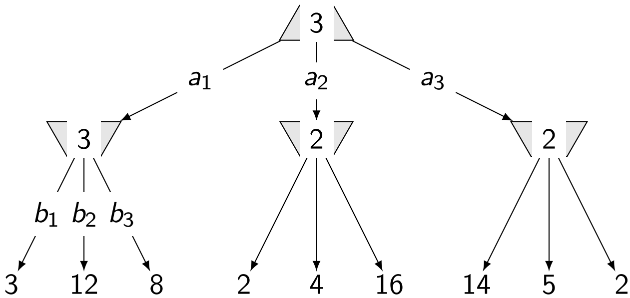

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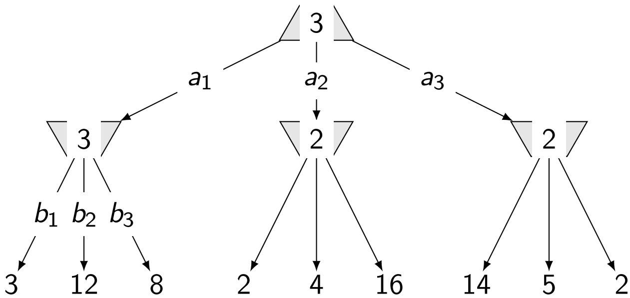

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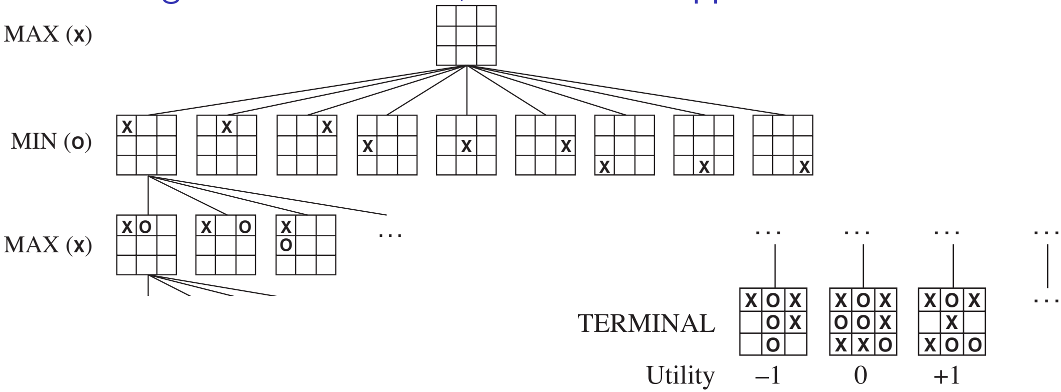

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Min step: Opponent wants to maximize his outcome which is equivalent to minimizing my outcome.

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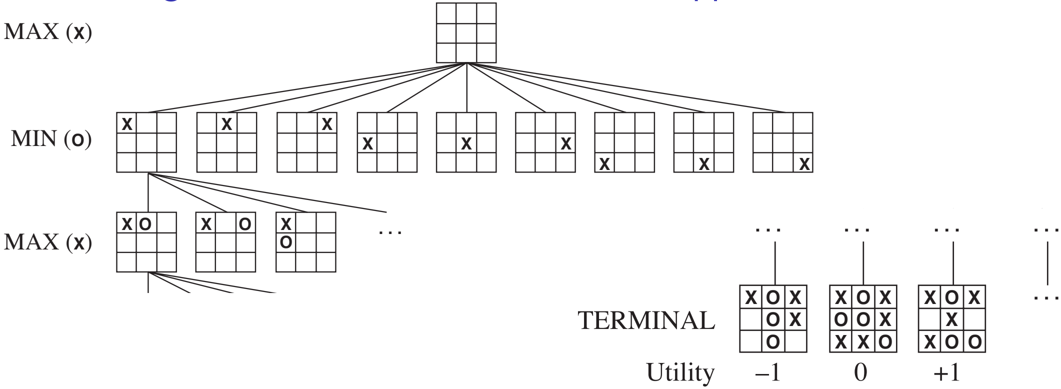

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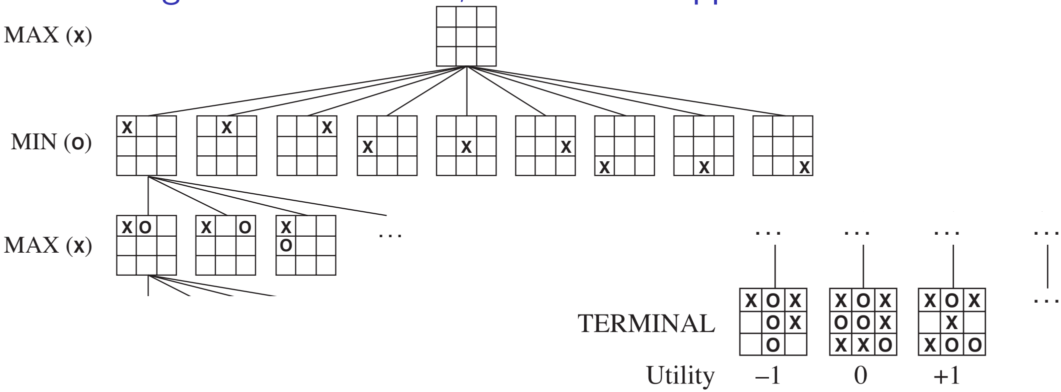

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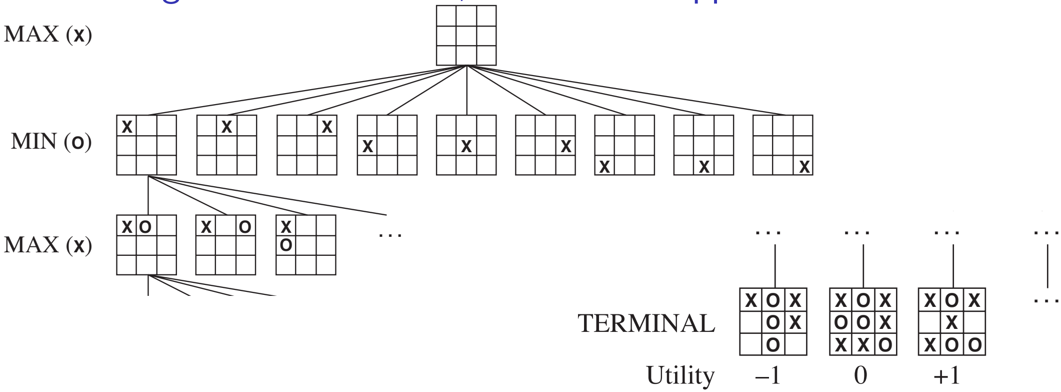

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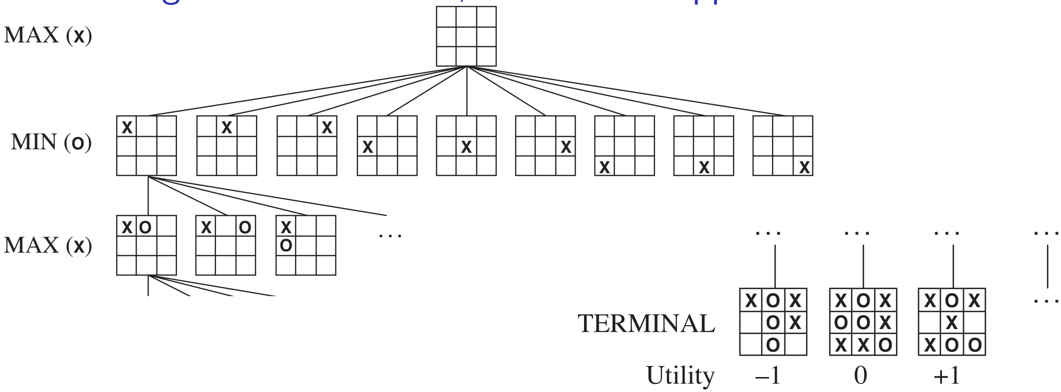

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Minimax algorithm

function MINIMAX(state) **returns** an action
  return $\underset{a \in \text{Actions}(s)}{\text{arg}\mathbf{max}}$ MIN-VALUE(RESULT(state, a))
end function

function MIN-VALUE(state) **returns** a utility value v
  if TERMINAL-TEST(state) **then return** UTILITY(state)
  end if
  $v \leftarrow \infty$
  for all ACTIONS(state) **do**
    $v \leftarrow$ **min**(v, MAX-VALUE(RESULT(state,a)))
  end for
end function

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---

function MAX-VALUE(state) **returns** a utility value v
  if TERMINAL-TEST(state) **then return** UTILITY(state)
  end if
  $v \leftarrow -\infty$
  for all ACTIONS(state) **do**
    $v \leftarrow \mathbf{max}(v,$ MIN-VALUE(RESULT(state,a)))
  end for
end function

Notes

Minimax algorithm

function MINIMAX(state) **returns** an action
  return $\underset{a \in \text{Actions}(s)}{\text{arg}\mathbf{max}}$ MIN-VALUE(RESULT(state, a))
end function

---

function MIN-VALUE(state) **returns** a utility value v
  if TERMINAL-TEST(state) **then return** UTILITY(state)
  end if
  $v \leftarrow \infty$
  for all ACTIONS(state) **do**
    $v \leftarrow$ **min**(v, MAX-VALUE(RESULT(state,a)))
  end for
end function

---

function MAX-VALUE(state) **returns** a utility value v
  if TERMINAL-TEST(state) **then return** UTILITY(state)
  end if
  $v \leftarrow -\infty$
  for all ACTIONS(state) **do**
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function MIN-VALUE(state) **returns** a utility value v
  if TERMINAL-TEST(state) **then return** UTILITY(state)
  end if
  $v \leftarrow \infty$
  for all ACTIONS(state) **do**
    $v \leftarrow$ **min**(v, MAX-VALUE(RESULT(state,a)))
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function MAX-VALUE(state) **returns** a utility value v
  if TERMINAL-TEST(state) **then return** UTILITY(state)
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  end for
end function

Notes

A two ply game, down to terminal and back again ...

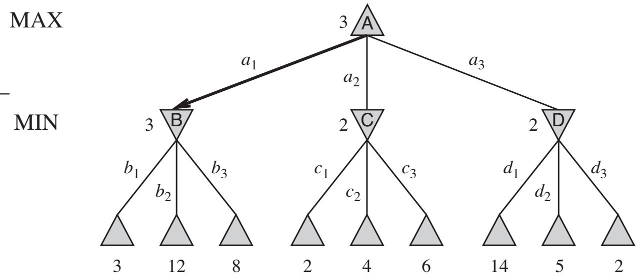
function MINIMAX(s) **returns** a
  $\underset{a \in \text{Actions}(s)}{\textbf{argmax}}$ MINVAL(RES(s, a))
end function

---

function MINVAL(s) **returns** v
  if TERMINAL(s) **then** UTIL(s)
  end if
  $v \leftarrow \infty$
  for all ACTIONS(s) **do**
    $v \leftarrow \textbf{min}(v,$ MAXVAL(RES(s, a)))
  end for
end function

---

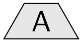
function MAXVAL(s) **returns** v
  if TERMINAL(s) **then** UTIL(s)
  end if
  $v \leftarrow -\infty$
  for all ACTIONS(s) **do**
    $v \leftarrow \textbf{max}(v,$ MINVAL(RES(s, a)))
  end for
end function



Notes

Before going to the animation on the next slide, try to follow the algorithm by a pencil and paper.

A two ply game, recursive run



Is it like DFS or BFS?

What is the complexity? How many nodes to visit?

Can we do better? How?

Notes

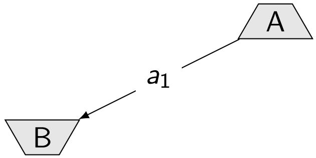
Efficiency/complexity:

- Exhaustive DFS
- Time $O(b^m)$
- Space $O(bm)$

Chess $b \approx 35, m \approx 100 \ldots$

- We cannot go(dive) to the end
- Can we save something?

A two ply game, recursive run



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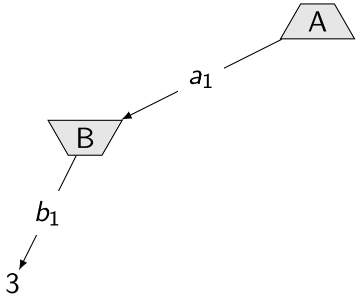

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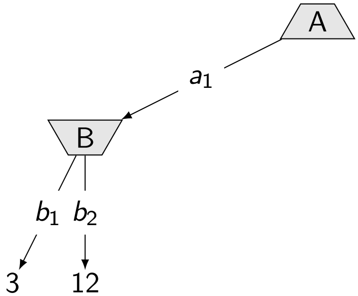

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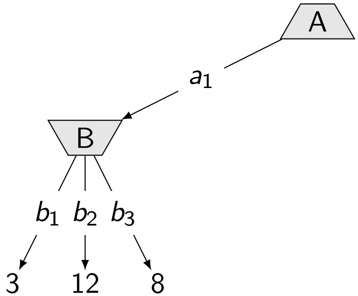

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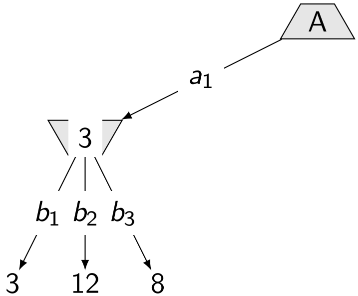

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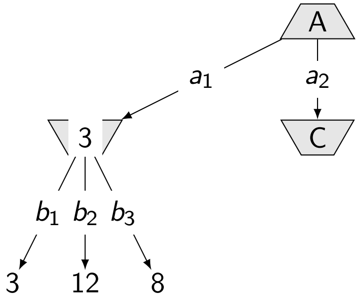

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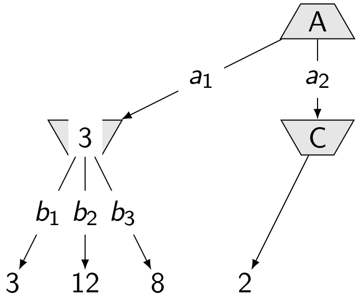

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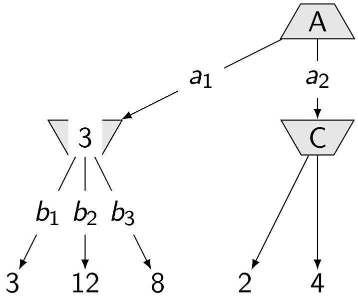

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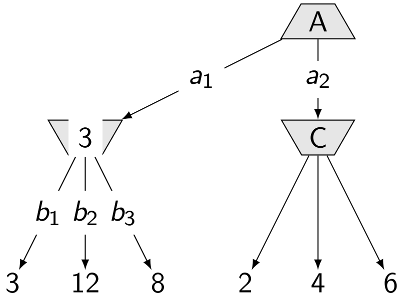

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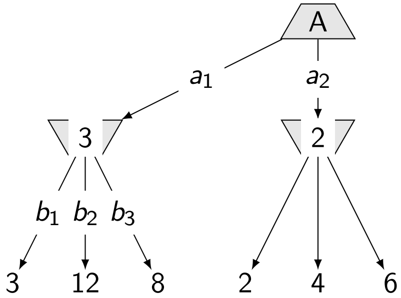

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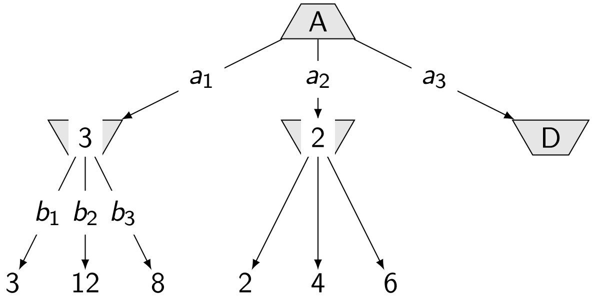

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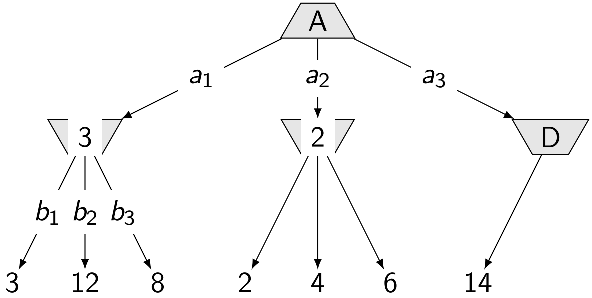

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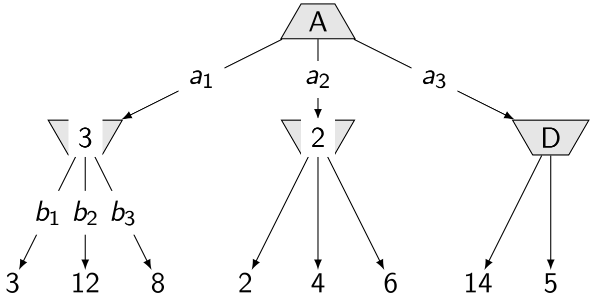

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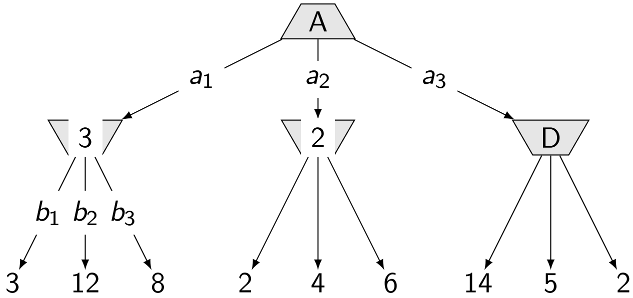

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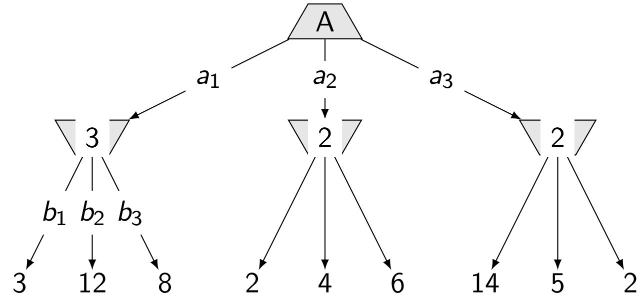

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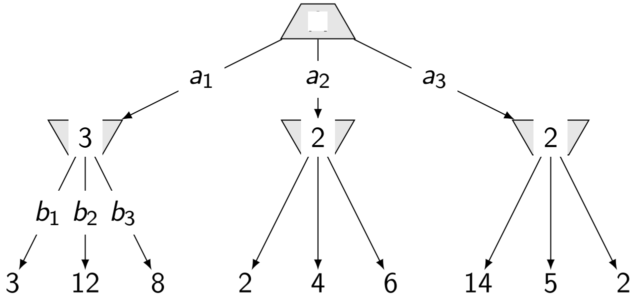

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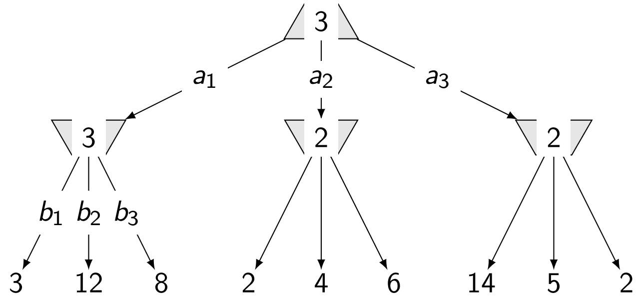

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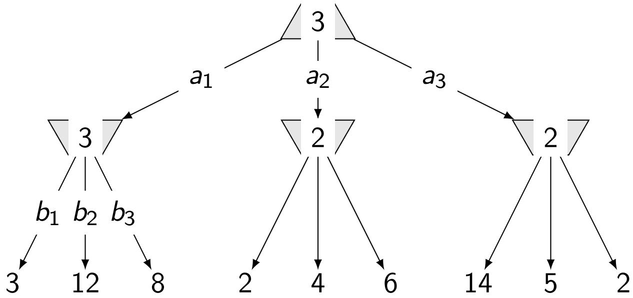

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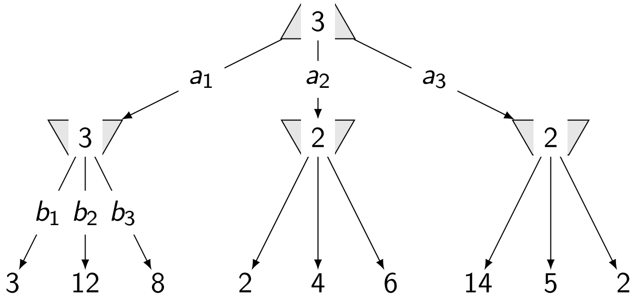

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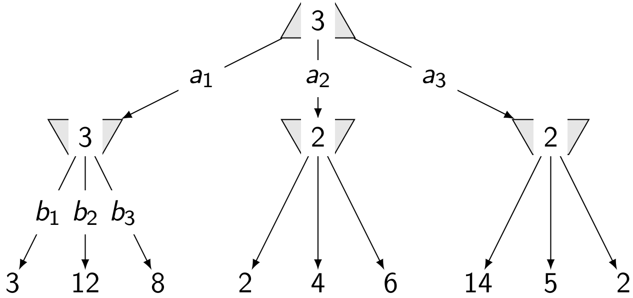

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Nodes (sub-trees) worth visiting

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Constraining the possible node values as search progresses...

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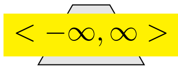

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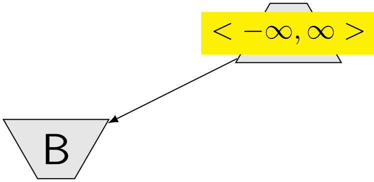

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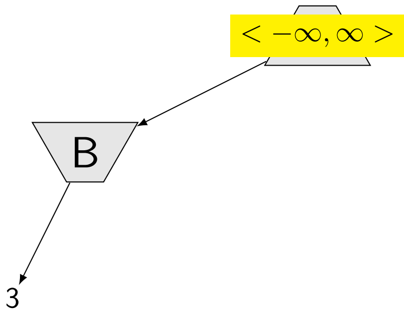

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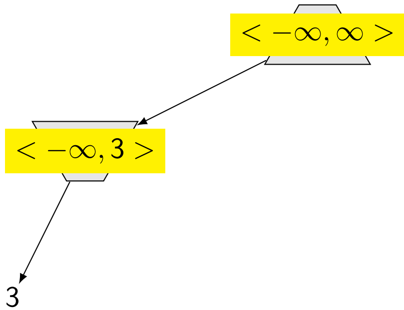

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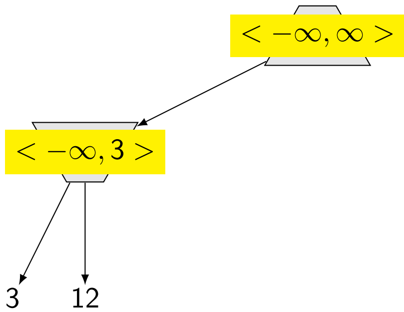

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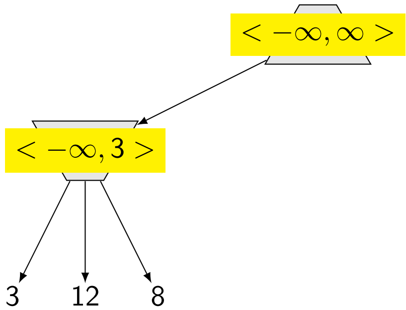

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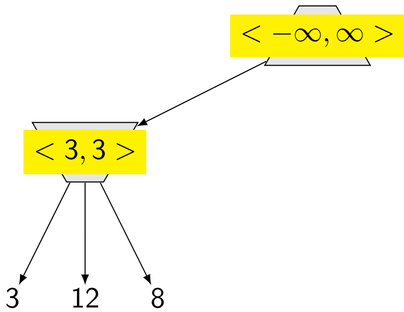

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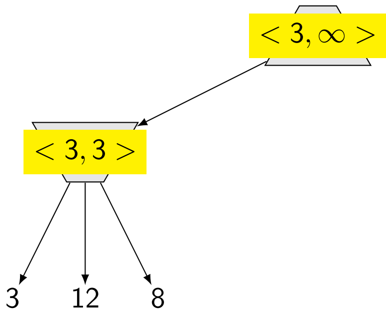

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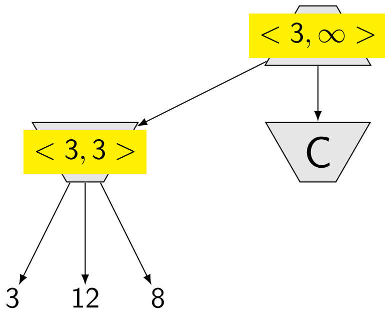

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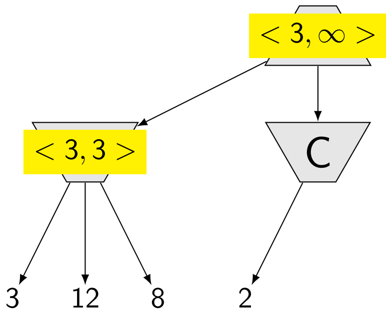

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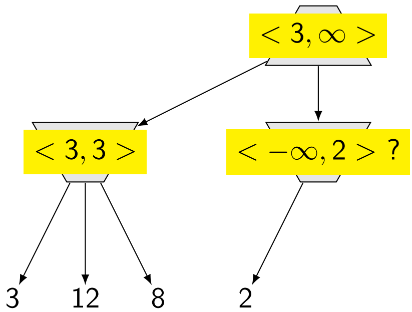

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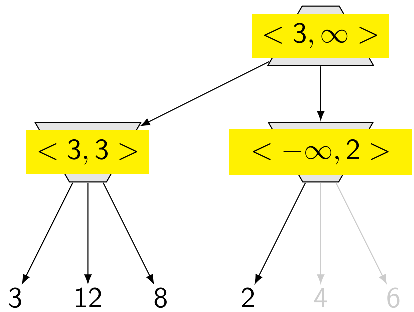

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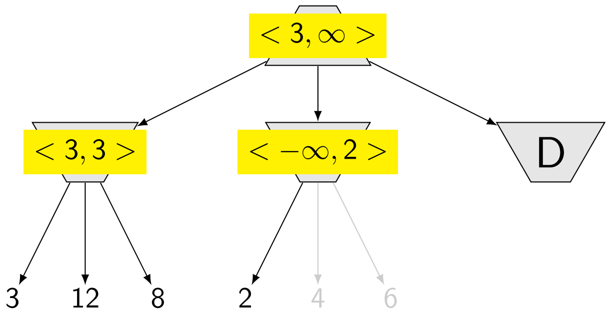

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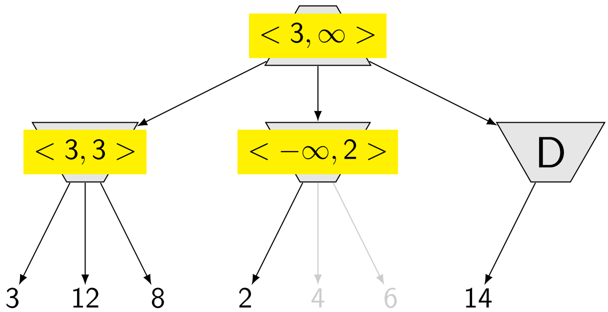

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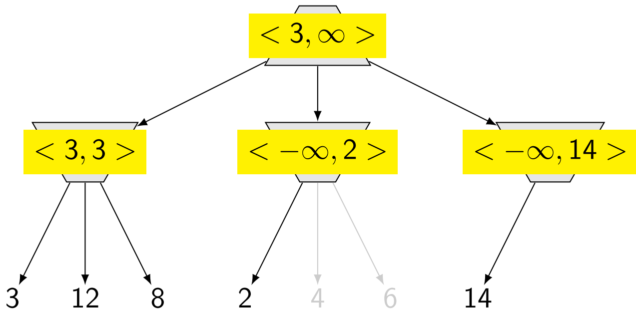

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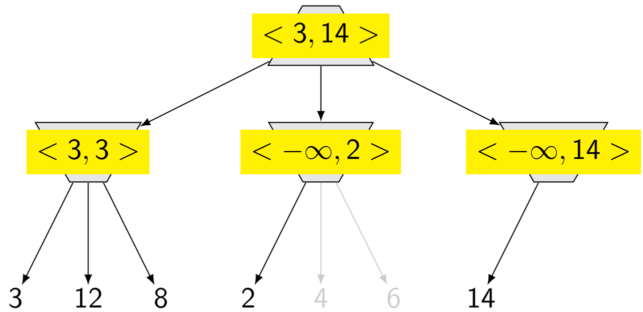

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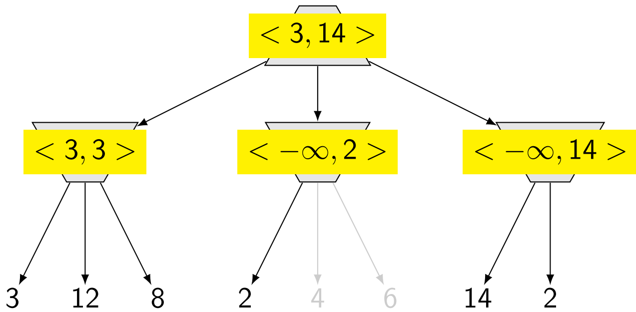

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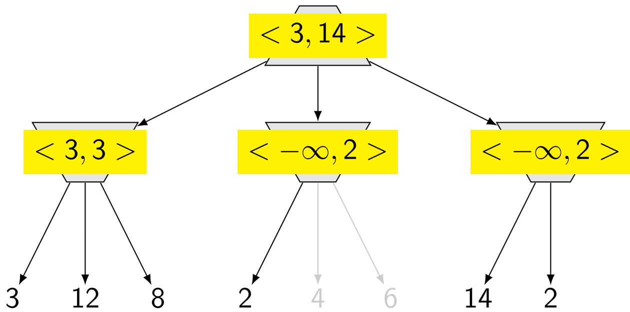

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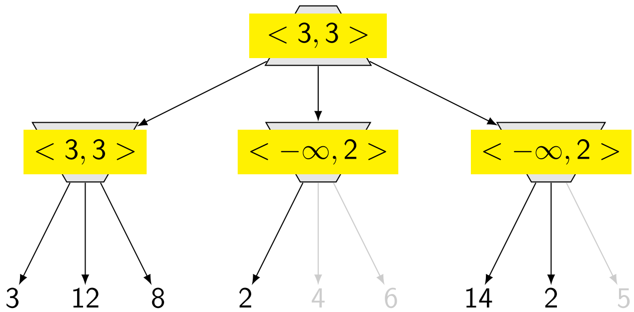

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Constraining the possible node values as search progresses...

α-β pruning

α highest (best) value choice found so far for any choice along MAX
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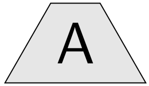

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$$\alpha = -\infty, \beta = \infty, v = ?$$

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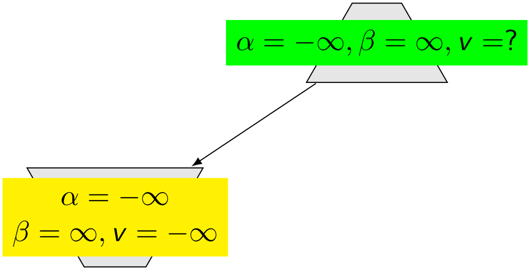

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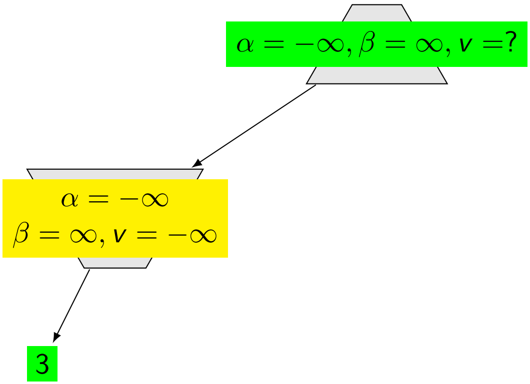

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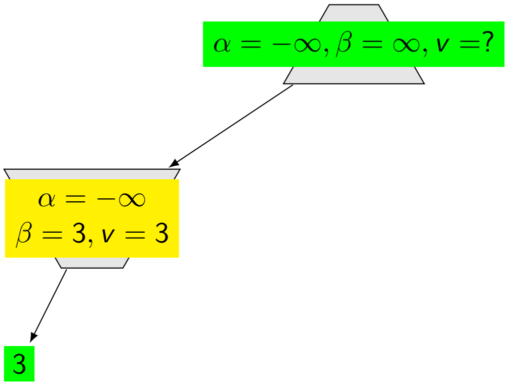

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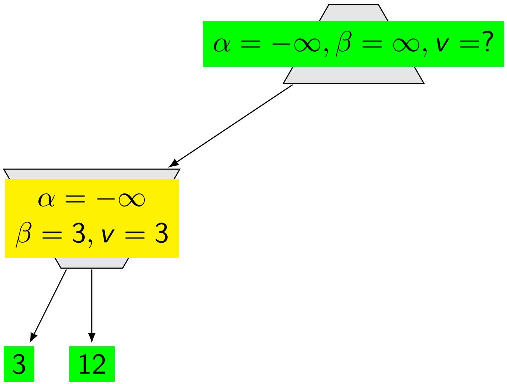

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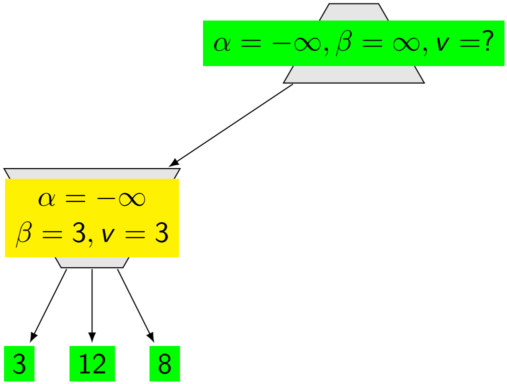

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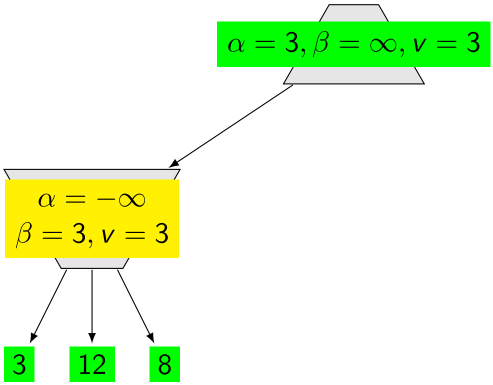

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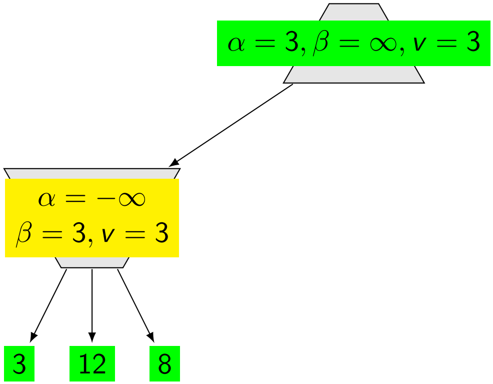

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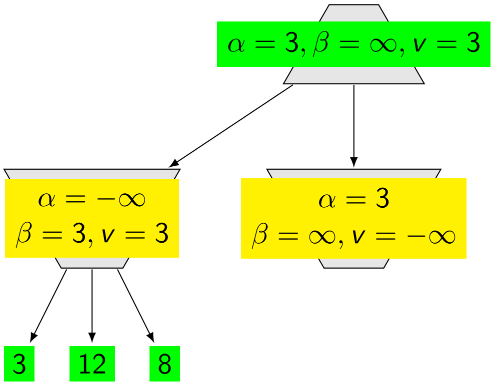

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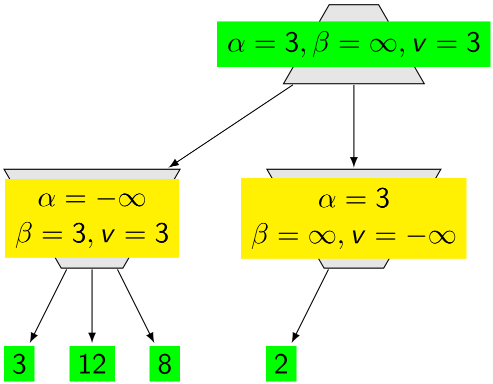

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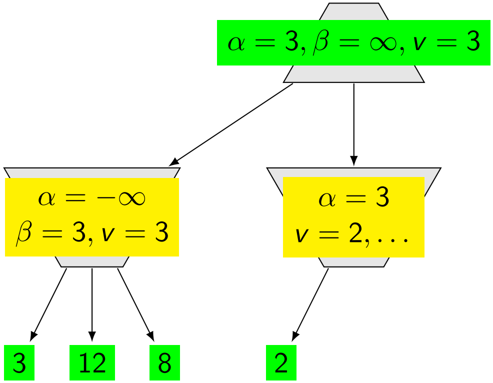

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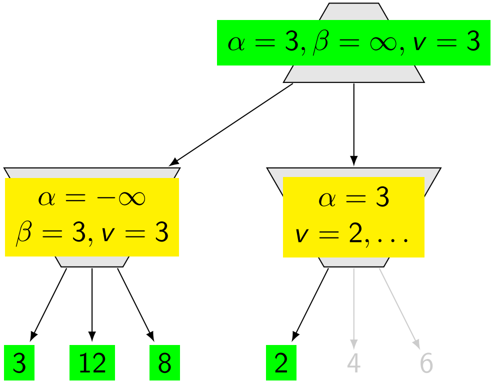

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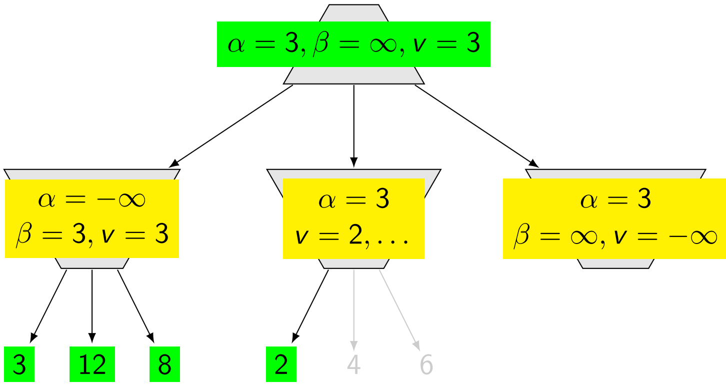

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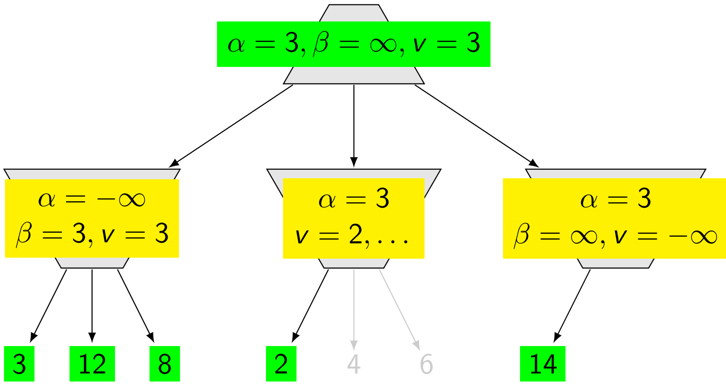

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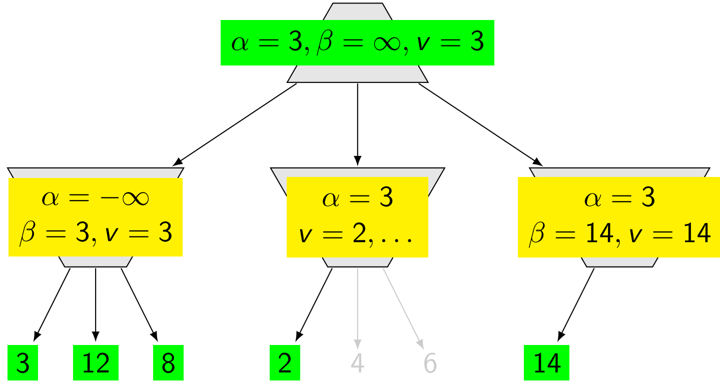

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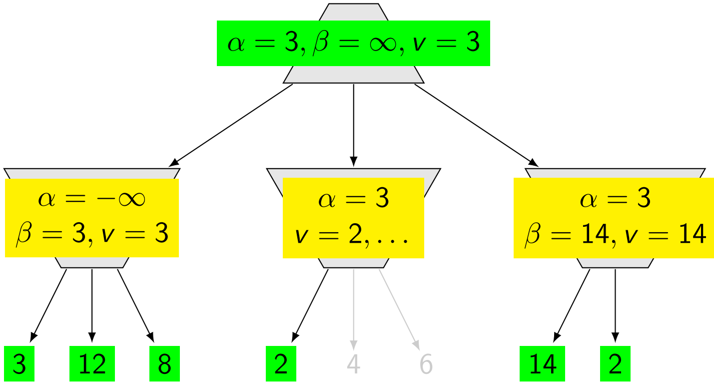

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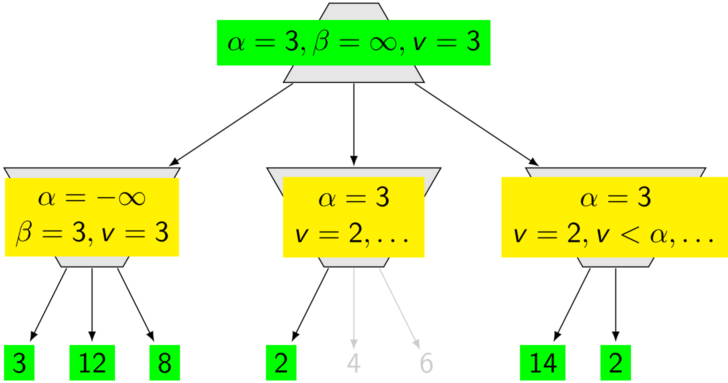

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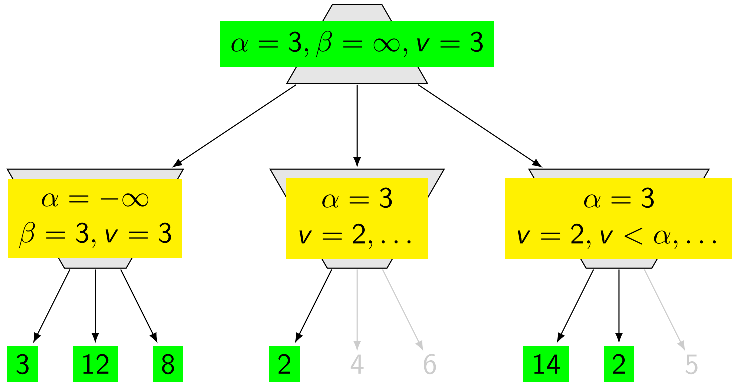

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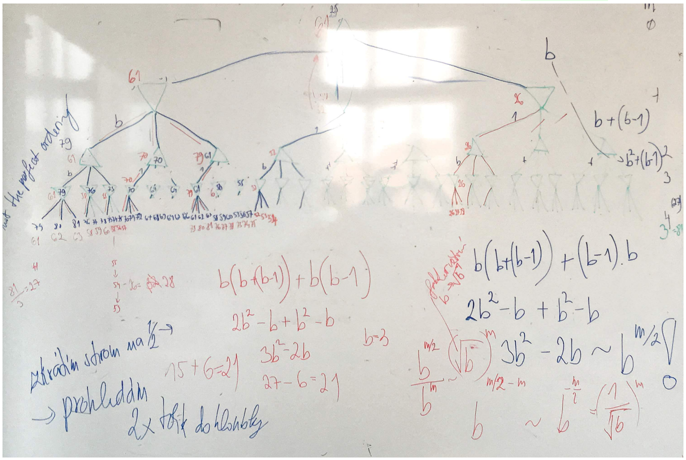
α-β prunnig – How much can we save?

original: Time: $O(b^m)$

- how to select nodes?
- perfect ordering?

Notes



It is clear that ordering of child nodes matters. Draw tree of α-β search in case of perferct ordering. Effective branching factor becomes $\sqrt{b}$ instead of b which effectively doubles the depth can be searched: Time: $O(b^{m/2})$

function ALPHA-BETA-SEARCH(state) **returns** an action
 $v \leftarrow$ MAX-VALUE(state, $\alpha = -\infty$, $\beta = \infty$)
 return action corresponding to v
end function

---

function MAX-VALUE(state, α, β) **returns** a utility value v
 if TERMINAL-TEST(state) **return** UTILITY(state)
 $v \leftarrow -\infty$
 for all ACTIONS(state) **do**
  $v \leftarrow$ **max**(v, MIN-VALUE(RESULT(state,a),α, β))
  if $v \geq \beta$ **return** v
  $\alpha \leftarrow \max(\alpha, v)$
 end for
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Notes

Take the tree from the previous slide and try to go step-by-step, watch α, β and v

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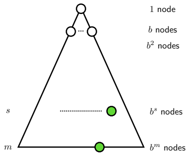
Recall: Iterative deepening DFS (ID-DFS)

- Start with `maxdepth = 1`
- Perform DFS with limited depth. Report success or failure.
- If failure, forget everything, increase `maxdepth` and repeat DFS

The "wasting" of resources is not too bad. Recall:

- Most nodes are at the deepest levels.
- Asymptotic complexity unchanged.



Bonus for α-β pruning: previous "shallower" iterations can be reused for node ordering.

Notes

α-β pruning is good. Still, in chess, for example, there is no way we can compute till the end.
Time is limited. We need to respond within a certain amount of time.
Possible solution: iterative deepening search. If I can't complete the computation for the current depth, I can use the previous shallower one that finished.

Imperfect but real-time decisions: iterative deepening

$$\textsc{h-minimax}(s,d) = \begin{cases} \textsc{eval}(s) & \text{if} \quad \textsc{cutoff-test}(s,d) \\ \max_{a \in \textsc{actions}(s)} \textsc{h-minimax}(\textsc{result}(s,a), d+1) & \text{if} \quad \textsc{player}(s) = \textsc{max} \\ \min_{a \in \textsc{actions}(s)} \textsc{h-minimax}(\textsc{result}(s,a,d+1)) & \text{if} \quad \textsc{player}(s) = \textsc{min} \end{cases}$$

Notes

Even with perfect ordering, α-β pruning does not save us.

One problem left: can't compute till then end, need to cut off, need for **Evaluation function**.

Imperfect but real-time decisions: iterative deepening

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\textsc{h-minimax}(s, d) =
$$

$$
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Cutting off search and evaluation functions

Replace
if TERMINAL-TEST(s) **then return** TERMINAL-UTILITY(s)
with:
if CUTOFF-TEST(s,d) **then return** EVAL(s)

Historical note: cutting search off earlier and use of heuristic evaluation functions proposed by Claude Shannon in *Programming a Computer for Playing Chess* (1950).

Notes

Cutting depends on d only, why we need s as the input parameter?

EVAL(s) – Evaluation functions

(estimate of) State value for non-terminal states

We need an easy-to-compute function correlated with "chance of winning". For chess:

- Material value for pieces—1 for pawn, 3 for knight/bishop, 5 for rook, 10 for queen. (minus opponent's pieces)
- Finetuning: 2 bishops are worth 6.5; knights are worth more in closed positions...
- Other features worth evaluating: controlling the center of the board, good pawn structure (no double pawns), king safety...

$$\text{EVAL}(s) = w_1 f_1(s) + w_2 f_2(s) + \cdots w_n f_n(s)$$

Notes

For many problems it is not so easy to find/construct proper function. We may try more functions and combine them conveniently.

$$f_1(s) = \text{number of white pawns} - \text{number of black pawns}$$

How to tune weights w_i?
or Deep Nets! Yeah!

How to get training data for supervised learning? More later.

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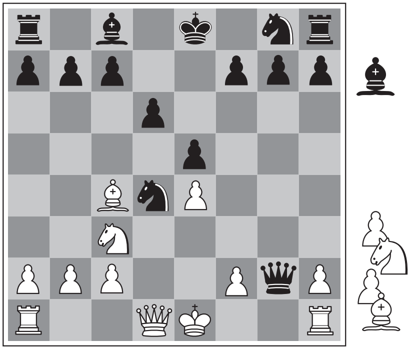
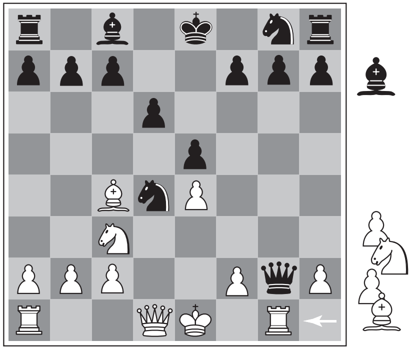
EVAL(s) – Problems

What if something important happens just beyond the current horizon of the search?



(a) White to move



(b) White to move

Additional improvements:

- "Killer moves"—capturing opponent's pieces, check etc.—should be considered first.
- *Quiescence search* – EVAL function should be applied only once things calm down. During capturing of pieces, depth should be locally increased.

Notes

Computer play vs. grandmaster play

- Computers are better since 1997 (Deep Blue defeating Garry Kasparov).
- The way they play is still very different: “dumb”, relying on “brute force”.
- Grandmasters do not excel in being able to compute very deep—many moves ahead.
  - They play based on experience: super-effective pruning and evaluation functions.
  - They consider only 2 to 3 moves in most positions (branching factor).

Notes

References

Chapter 5, "Adversarial search" in [1].

[1] Stuart Russell and Peter Norvig.
Artificial Intelligence: A Modern Approach.
Prentice Hall, 3rd edition, 2010.
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[2] Richard S. Sutton and Andrew G. Barto.
Reinforcement Learning; an Introduction.
MIT Press, 2nd edition, 2018.
http://www.incompleteideas.net/book/the-book-2nd.html.

Notes