▶Camera Orientation from Two Finite Vanishing Points

Problem: Given $\mathbf{K}$ and two vanishing points corresponding to two known orthogonal directions $\mathbf{d}_1$, $\mathbf{d}_2$, compute camera orientation $\mathbf{R}$ with respect to the plane.

- 3D coordinate system choice, e.g.:

$$\mathbf{d}_1 = (1, 0, 0), \quad \mathbf{d}_2 = (0, 1, 0)$$

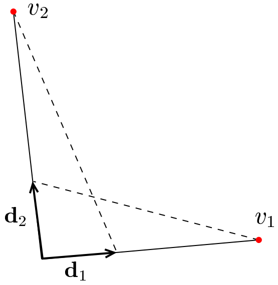
- we know that

$$\mathbf{d}_i \simeq \mathbf{Q}^{-1}\underline{\mathbf{v}}_i = (\mathbf{K}\mathbf{R})^{-1}\underline{\mathbf{v}}_i = \mathbf{R}^{-1}\underbrace{\mathbf{K}^{-1}\underline{\mathbf{v}}_i}_{\underline{\mathbf{w}}_i}$$

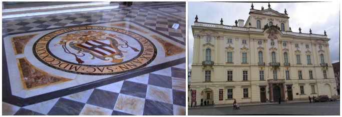
$$\mathbf{R}\mathbf{d}_i \simeq \underline{\mathbf{w}}_i$$

- knowing $\mathbf{d}_{1,2}$ we conclude that $\underline{\mathbf{w}}_i/\|\underline{\mathbf{w}}_i\|$ is the i-th column $\mathbf{r}_i$ of $\mathbf{R}$
- the third column is orthogonal: $\mathbf{r}_3 \simeq \mathbf{r}_1 \times \mathbf{r}_2$

$$\mathbf{R} = \begin{bmatrix} \frac{\underline{\mathbf{w}}_1}{\|\underline{\mathbf{w}}_1\|} & \frac{\underline{\mathbf{w}}_2}{\|\underline{\mathbf{w}}_2\|} & \frac{\underline{\mathbf{w}}_1 \times \underline{\mathbf{w}}_2}{\|\underline{\mathbf{w}}_1 \times \underline{\mathbf{w}}_2\|} \end{bmatrix}$$



some suitable scenes



Application: Planar Rectification

Principle: Rotate camera parallel to the plane of interest.

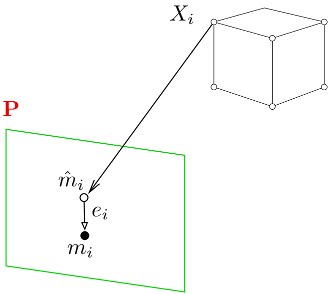
$$\underline{\mathbf{m}} \simeq \mathbf{K}\mathbf{R}\begin{bmatrix}\mathbf{I} & -\mathbf{C}\end{bmatrix}\underline{\mathbf{X}} \qquad\qquad \underline{\mathbf{m}}' \simeq \mathbf{K}\begin{bmatrix}\mathbf{I} & -\mathbf{C}\end{bmatrix}\underline{\mathbf{X}}$$

$$\underline{\mathbf{m}}' \simeq \mathbf{K}(\mathbf{K}\mathbf{R})^{-1}\,\underline{\mathbf{m}} = \mathbf{K}\mathbf{R}^{\top}\mathbf{K}^{-1}\,\underline{\mathbf{m}} = \mathbf{H}\,\underline{\mathbf{m}}$$

- $\mathbf{H}$ is the rectifying homography
- both $\mathbf{K}$ and $\mathbf{R}$ can be calibrated from two finite vanishing points assuming ORUA →56
- not possible when one (or both) of them are infinite
- without ORUA we would need 4 additional views to calibrate $\mathbf{K}$ as on →53

▶Camera Resection

Camera calibration and orientation from a known set of $k \geq 6$ reference points and their images $\{(X_i, m_i)\}_{i=1}^{6}$.

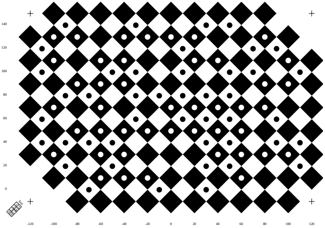


- X_i are considered exact
- m_i is a measurement subject to detection error

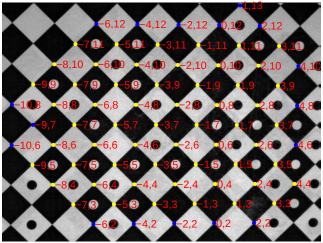
$$\mathbf{m}_i = \hat{\mathbf{m}}_i + \mathbf{e}_i \qquad \text{Cartesian}$$

- where $\underline{\hat{\mathbf{m}}}_i \simeq \mathbf{P}\underline{\mathbf{X}}_i$

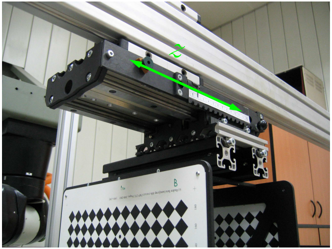
Resection Targets



calibration chart



automatic calibration point detection



resection target with translation stage

- target translated at least once
- by a calibrated (known) translation
- X_i point locations looked up in a table based on their code

▶The Minimal Problem for Camera Resection

Problem: Given $k = 6$ corresponding pairs $\{(X_i,\, m_i)\}_{i=1}^{k}$, find $\mathbf{P}$

$$\lambda_i \underline{\mathbf{m}}_i = \mathbf{P}\underline{\mathbf{X}}_i, \qquad \mathbf{P} = \begin{bmatrix} \mathbf{q}_1^\top & q_{14} \\ \mathbf{q}_2^\top & q_{24} \\ \mathbf{q}_3^\top & q_{34} \end{bmatrix} \qquad \begin{aligned} \underline{\mathbf{X}}_i &= (x_i, y_i, z_i, 1), \quad i = 1, 2, \ldots, k,\ k = 6 \\ \underline{\mathbf{m}}_i &= (u_i, v_i, 1), \quad \lambda_i \in \mathbb{R},\ \lambda_i \neq 0 \end{aligned}$$

easy to modify for infinite points X_i but be aware of →64

expanded: $\lambda_i u_i = \mathbf{q}_1^\top \mathbf{X}_i + q_{14}, \quad \lambda_i v_i = \mathbf{q}_2^\top \mathbf{X}_i + q_{24}, \quad \lambda_i = \mathbf{q}_3^\top \mathbf{X}_i + q_{34}$

after elimination of λ_i: $(\mathbf{q}_3^\top \mathbf{X}_i + q_{34})u_i = \mathbf{q}_1^\top \mathbf{X}_i + q_{14}, \quad (\mathbf{q}_3^\top \mathbf{X}_i + q_{34})v_i = \mathbf{q}_2^\top \mathbf{X}_i + q_{24}$

Then

$$\mathbf{A}\,\mathbf{q} = \begin{bmatrix} \mathbf{X}_1^\top & 1 & \mathbf{0}^\top & 0 & -u_1\mathbf{X}_1^\top & -u_1 \\ \mathbf{0}^\top & 0 & \mathbf{X}_1^\top & 1 & -v_1\mathbf{X}_1^\top & -v_1 \\ \vdots & & & & & \vdots \\ \mathbf{X}_k^\top & 1 & \mathbf{0}^\top & 0 & -u_k\mathbf{X}_k^\top & -u_k \\ \mathbf{0}^\top & 0 & \mathbf{X}_k^\top & 1 & -v_k\mathbf{X}_k^\top & -v_k \end{bmatrix} \cdot \begin{bmatrix} \mathbf{q}_1 \\ q_{14} \\ \mathbf{q}_2 \\ q_{24} \\ \mathbf{q}_3 \\ q_{34} \end{bmatrix} = \mathbf{0} \tag{9}$$

- we need 11 indepedent parameters for $\mathbf{P}$
- $\mathbf{A} \in \mathbb{R}^{2k,12}, \ \mathbf{q} \in \mathbb{R}^{12}$
- 6 points in a general position give $\operatorname{rank} \mathbf{A} = 12$ and there is no non-trivial null space
- drop one row to get rank 11 matrix, then the basis vector of the null space of $\mathbf{A}$ gives $\mathbf{q}$

▶The Jack-Knife Solution for $k = 6$

- given the 6 correspondences, we have 12 equations for the 11 parameters
- can we use all the information present in the 6 points?

Jack-knife estimation

1. $n := 0$
2. for $i = 1, 2, \ldots, 2k$ do
   a) delete i-th row from $\mathbf{A}$, this gives $\mathbf{A}_i$
   b) if $\dim \operatorname{null} \mathbf{A}_i > 1$ continue with the next i
   c) $n := n + 1$
   d) compute the right null-space $\mathbf{q}_i$ of $\mathbf{A}_i$ — e.g. by 'economy-size' SVD
   e) $\hat{\mathbf{q}}_i := \mathbf{q}_i$ normalized to $q_{34} = 1$ and dimension-reduced — assuming finite cam. with $P_{3,4} = 1$
3. from all n vectors $\hat{\mathbf{q}}_i$ collected in Step 1d compute

$$\mathbf{q} = \frac{1}{n} \sum_{i=1}^{n} \hat{\mathbf{q}}_i, \qquad \operatorname{var}[\mathbf{q}] = \frac{n-1}{n} \operatorname{diag} \sum_{i-1}^{n} (\hat{\mathbf{q}}_i - \mathbf{q})(\hat{\mathbf{q}}_i - \mathbf{q})^\top$$

regular for $n \geq 11$

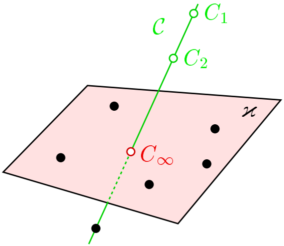
- have a solution + an error estimate, per individual elements of $\mathbf{P}$ (except P_{34})
- at least 5 points must be in a general position (→64)
- large error indicates near degeneracy
- computation not efficient with $k > 6$ points, needs $\binom{2k}{11}$ draws, e.g. $k = 7 \Rightarrow 364$ draws
- better error estimation method: decompose $\mathbf{P}_i$ to $\mathbf{K}_i$, $\mathbf{R}_i$, $\mathbf{t}_i$ (→32), represent $\mathbf{R}_i$ with 3 parameters (e.g. Euler angles, or in Cayley representation →139) and compute the errors for the parameters

▶Degenerate (Critical) Configurations for Camera Resection

Let $\mathcal{X} = \{X_i;\, i = 1, \ldots\}$ be a set of points and $\mathbf{P}_1 \not\simeq \mathbf{P}_j$ be two regular (rank-3) cameras. Then two configurations $(\mathbf{P}_1, \mathcal{X})$ and $(\mathbf{P}_j, \mathcal{X})$ are image-equivalent if

$$\mathbf{P}_1 \underline{\mathbf{X}}_i \simeq \mathbf{P}_j \underline{\mathbf{X}}_i \quad \text{for all} \quad X_i \in \mathcal{X}$$

there is a non-trivial set of other cameras that see the same image



Case 4

- **importantly**: If all calibration points $X_i \in \mathcal{X}$ lie on a plane $\varkappa$ then camera resection is non-unique and all image-equivalent camera centers lie on a spatial line $\mathcal{C}$ with the $C_\infty = \varkappa \cap \mathcal{C}$ excluded

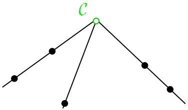
  this also means we cannot resect if all X_i are infinite
- by adding points $X_i \in \mathcal{X}$ to $\mathcal{C}$ we gain nothing
- there are additional image-equivalent configurations, see next

proof sketch in [H&Z, Sec. 22.1.2]

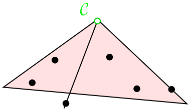
Note that if $\mathbf{Q}$, $\mathbf{T}$ are suitable homographies then $\mathbf{P}_1 \simeq \mathbf{Q}\mathbf{P}_0\mathbf{T}$, where $\mathbf{P}_0$ is canonical and the analysis can be made with $\hat{\mathbf{P}}_j \simeq \mathbf{Q}^{-1}\mathbf{P}_j$

$$\mathbf{P}_0 \underbrace{\mathbf{T}\underline{\mathbf{X}}_i}_{\underline{\mathbf{Y}}_i} \simeq \hat{\mathbf{P}}_j \underbrace{\mathbf{T}\underline{\mathbf{X}}_i}_{\underline{\mathbf{Y}}_i} \quad \text{for all} \quad Y_i \in \mathcal{Y}$$

cont'd (all cases)

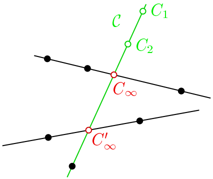


Case 5

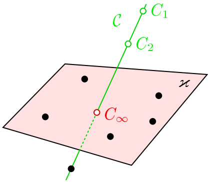


Case 6

- cameras C_1, C_2 co-located at point $\mathcal{C}$
- points on three optical rays or one optical ray and one optical plane
- Case 5: camera sees 3 isolated point images
- Case 6: cam. sees a line of points and an isolated point



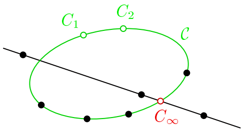
Case 3



Case 4

- cameras lie on a line $\mathcal{C} \setminus \{C_\infty, C'_\infty\}$
- points lie on $\mathcal{C}$ and
  1. on two lines meeting $\mathcal{C}$ at C_∞, C'_∞
  2. or on a plane meeting $\mathcal{C}$ at C_∞
- Case 3: camera sees 2 lines of points

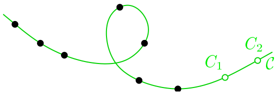
Case 2



- cameras lie on a planar conic $\mathcal{C} \setminus \{C_\infty\}$

  not necessarily an ellipse
- points lie on $\mathcal{C}$ and an additional line meeting the conic at C_∞
- Case 2: camera sees 2 lines of points

Case 1



- cameras and points all lie on a twisted cubic $\mathcal{C}$
- Case 1: camera sees points on a conic

►Three-Point Exterior Orientation Problem (P3P)

Calibrated camera rotation and translation from Perspective images of 3 reference Points.

Problem: Given $\mathbf{K}$ and three corresponding pairs $\{(m_i, X_i)\}_{i=1}^3$, find $\mathbf{R}$, $\mathbf{C}$ by solving

$$\lambda_i \underline{\mathbf{m}}_i = \mathbf{K}\mathbf{R}\,(\mathbf{X}_i - \mathbf{C}), \qquad i = 1, 2, 3$$

1. Transform $\underline{\mathbf{v}}_i \overset{\text{def}}{=} \mathbf{K}^{-1}\underline{\mathbf{m}}_i$. Then

$$\lambda_i \underline{\mathbf{v}}_i = \mathbf{R}\,(\mathbf{X}_i - \mathbf{C}). \tag{10}$$

2. Eliminate $\mathbf{R}$ by taking rotation preserves length: $\|\mathbf{R}\mathbf{x}\| = \|\mathbf{x}\|$

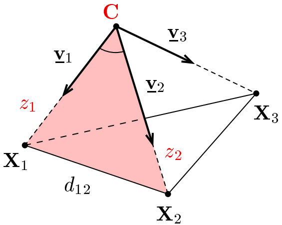
$$|\lambda_i| \cdot \|\underline{\mathbf{v}}_i\| = \|\mathbf{X}_i - \mathbf{C}\| \overset{\text{def}}{=} z_i \tag{11}$$

3. Consider only angles among $\underline{\mathbf{v}}_i$ and apply Cosine Law per triangle $(\mathbf{C}, \mathbf{X}_i, \mathbf{X}_j)$ $i, j = 1, 2, 3,\ i \neq j$

$$d_{ij}^2 = z_i^2 + z_j^2 - 2\, z_i\, z_j\, c_{ij},$$

$$z_i = \|\mathbf{X}_i - \mathbf{C}\|,\ \ d_{ij} = \|\mathbf{X}_j - \mathbf{X}_i\|,\ \ c_{ij} = \cos(\angle \underline{\mathbf{v}}_i\, \underline{\mathbf{v}}_j)$$

configuration w/o rotation in (11)

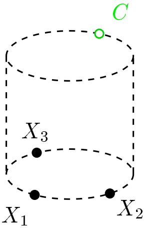


4. Solve system of 3 quadratic eqs in 3 unknowns z_i [Fischler & Bolles, 1981]
   there may be no real root; there are up to 4 solutions that cannot be ignored (verify on additional points)

5. Compute $\mathbf{C}$ by trilateration (3-sphere intersection) from $\mathbf{X}_i$ and z_i; then λ_i from (11) and $\mathbf{R}$ from (10)

Similar problems (P4P with unknown f) at http://cmp.felk.cvut.cz/minimal/ (with code)

Degenerate (Critical) Configurations for Exterior Orientation



unstable solution

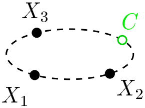
- center of projection C located on the orthogonal circular cylinder with base circumscribing the three points X_i

unstable: a small change of X_i results in a large change of C
can be detected by error propagation

degenerate

- camera C is coplanar with points (X_1, X_2, X_3) but is not on the circumscribed circle of (X_1, X_2, X_3)

camera sees point on a line



no solution

1. C cocyclic with (X_1, X_2, X_3) camera sees points on a line

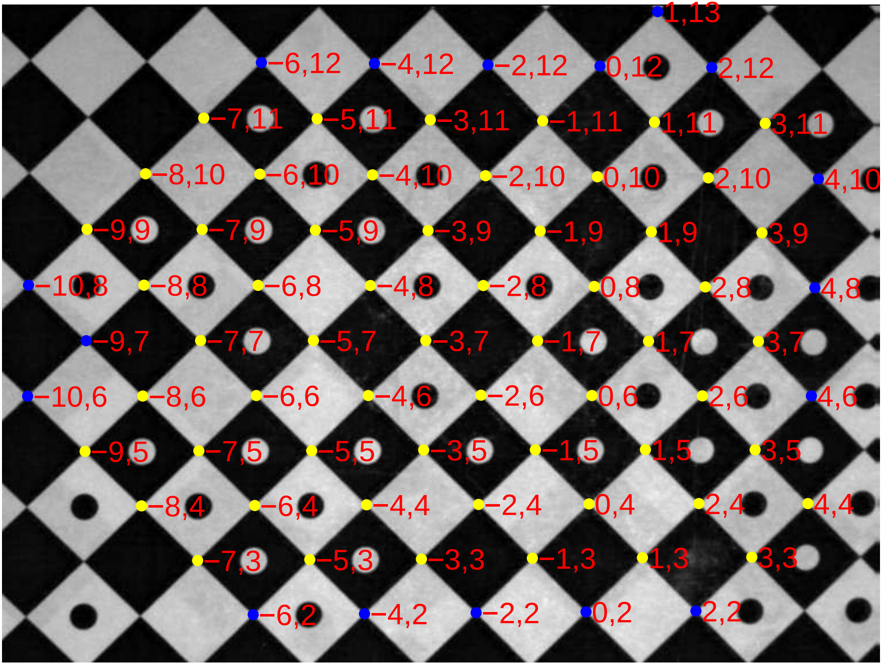
---

- additional critical configurations depend on the method to solve the quadratic equations

[Haralick et al. IJCV 1994]

Thank You


BOOKS
FILMS
TICKETS


1,13
−6,12
−4,12
−2,12
0,12
2,12
−7,11
−5,11
−3,11
−1,11
1,11
3,11
−8,10
−6,10
−4,10
−2,10
0,10
2,10
4,10
−9,9
−7,9
−5,9
−3,9
−1,9
1,9
3,9
−10,8
−8,8
−6,8
−4,8
−2,8
0,8
2,8
4,8
−9,7
−7,7
−5,7
−3,7
−1,7
1,7
3,7
−10,6
−8,6
−6,6
−4,6
−2,6
0,6
2,6
4,6
−9,5
−7,5
−5,5
−3,5
−1,5
1,5
3,5
−8,4
−6,4
−4,4
−2,4
0,4
2,4
4,4
−7,3
−5,3
−3,3
−1,3
1,3
3,3
−6,2
−4,2
−2,2
0,2
2,2