What Can We Do with An 'Uncalibrated' Perspective Camera?

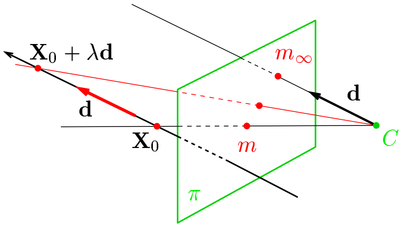
How far is the engine?

distance between sleepers (ties) 0.806m but we cannot count them, image resolution is too low

We will review some life-saving theory. . .
. . . and build a bit of geometric intuition. . .

▶Vanishing Point

Vanishing point: the limit of the projection of a point that moves along a space line infinitely in one direction. the image of the point at infinity on the line



$$\underline{\mathbf{m}}_\infty \simeq \lim_{\lambda \to \pm\infty} \mathbf{P} \begin{bmatrix} \mathbf{X}_0 + \lambda\mathbf{d} \\ 1 \end{bmatrix} = \cdots \simeq \mathbf{Q}\,\mathbf{d}$$

⊛ P1; 1pt: Prove (use Cartesian coordinates and L'Hôpital's rule)

- the V.P. of a spatial line with directional vector $\mathbf{d}$ is $\underline{\mathbf{m}}_\infty \simeq \mathbf{Q}\,\mathbf{d}$
- V.P. is independent on line position $\mathbf{X}_0$, it depends on its directional vector only
- all parallel lines share the same V.P., including the optical ray defined by m_∞

Some Vanishing Point "Applications"

where is the sun?

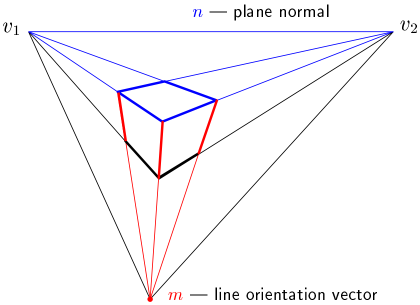
what is the wind direction?
(must have video)

fly above the lane,
at constant altitude!

▶Vanishing Line

Vanishing line: The set of vanishing points of all lines in a plane

the image of the line at infinity in the plane
and in all parallel planes



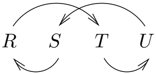
- V.L. n corresponds to spatial plane of normal vector $\mathbf{p} = \mathbf{Q}^\top \underline{\mathbf{n}}$

  because this is the normal vector of a parallel optical plane (!) →38
- a spatial plane of normal vector $\mathbf{p}$ has a V.L. represented by $\underline{\mathbf{n}} = \mathbf{Q}^{-\top}\mathbf{p}$.

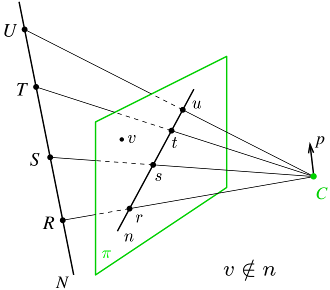
▸Cross Ratio

Four distinct collinear spatial points R, S, T, U define cross-ratio

$$[RSTU] = \frac{|\overrightarrow{RT}|}{|\overrightarrow{SR}|} \frac{|\overrightarrow{US}|}{|\overrightarrow{TU}|}$$



a mnemonic (∞)



$|\overrightarrow{RT}|$ – distance from R to T in the arrow direction

6 cross-ratios from four points:

$$[SRUT] = [RSTU], \ [RSUT] = \frac{1}{[RSTU]}, \ [RTSU] = 1 - [RSTU], \ \cdots$$

Obs: $$[RSTU] = \frac{|\underline{\mathbf{r}} \ \underline{\mathbf{t}} \ \underline{\mathbf{v}}|}{|\underline{\mathbf{s}} \ \underline{\mathbf{r}} \ \underline{\mathbf{v}}|} \cdot \frac{|\underline{\mathbf{u}} \ \underline{\mathbf{s}} \ \underline{\mathbf{v}}|}{|\underline{\mathbf{t}} \ \underline{\mathbf{u}} \ \underline{\mathbf{v}}|}, \quad |\underline{\mathbf{r}} \ \underline{\mathbf{t}} \ \underline{\mathbf{v}}| = \det [\underline{\mathbf{r}} \ \underline{\mathbf{t}} \ \underline{\mathbf{v}}] = (\underline{\mathbf{r}} \times \underline{\mathbf{t}})^\top \underline{\mathbf{v}} \qquad (1)$$

Corollaries:

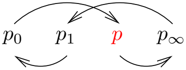
- cross ratio is invariant under homographies $\underline{\mathbf{x}}' \simeq \mathbf{H}\underline{\mathbf{x}}$ plug $\mathbf{H}\underline{\mathbf{x}}$ in (1): $(\mathbf{H}^{-\top}(\underline{\mathbf{r}} \times \underline{\mathbf{t}}))^\top \mathbf{H}\underline{\mathbf{v}}$
- cross ratio is invariant under perspective projection: $[RSTU] = [r \, s \, t \, u]$

- 4 collinear points: any perspective camera will “see” the same cross-ratio of their images
- we measure the same cross-ratio in image as on the world line
- one of the points R, S, T, U may be at infinity (we take the limit, in effect $\frac{\infty}{\infty} = 1$)

▶1D Projective Coordinates

The 1-D projective coordinate of a point P is defined by the following cross-ratio:

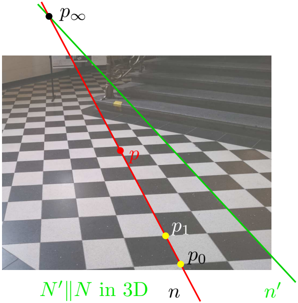
$$[P] = [P_0\,P_1\,P\,P_\infty] = [p_0\,p_1\,p\,p_\infty] = \frac{|\overrightarrow{p_0\,p}|}{|\overrightarrow{p_1\,p_0}|}\,\frac{|\overrightarrow{p_\infty\,p_1}|}{|\overrightarrow{p\,p_\infty}|} = [p]$$



naming convention:

P_0 – the origin	$[P_0] = 0$
P_1 – the unit point	$[P_1] = 1$
P_∞ – the supporting point	$[P_\infty] = \pm\infty$

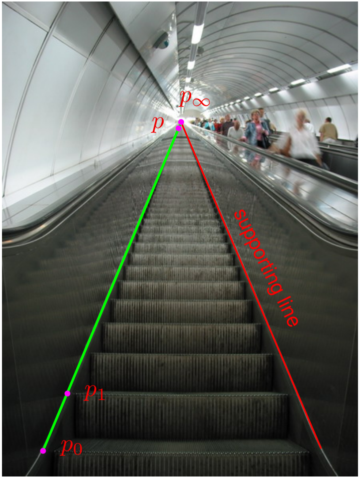
$[P]$ is equal to Euclidean coordinate along N
$[p]$ is its measurement in the image plane



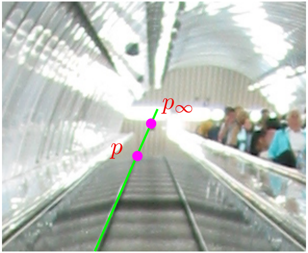
Applications

- Given the image of a 3D line N, the origin, the unit point, and the vanishing point, then the Euclidean coordinate of any point $P \in N$ can be determined →47
- Finding v.p. of a line through a regular object →48

Application: Counting Steps



- Namesti Miru underground station in Prague

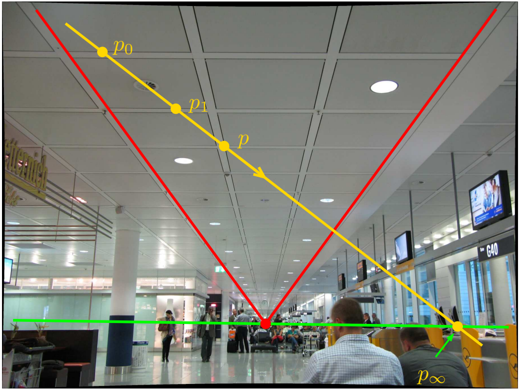


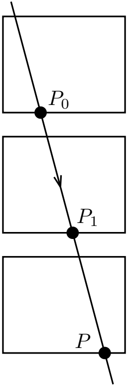
detail around the vanishing point

Result: $[P] = 214$ steps (correct answer is 216 steps)

4Mpx camera

Application: Finding the Horizon from Repetitions





in 3D: $|P_0 P| = 2|P_0 P_1|$ then [H&Z, p. 218]

$$[P_0 P_1 P P_\infty] = \frac{|P_0 P|}{|P_1 P_0|} = 2 \quad \Rightarrow \quad x_\infty = \frac{x_0\,(2\,x - x_1) - x\,x_1}{x + x_0 - 2\,x_1}$$

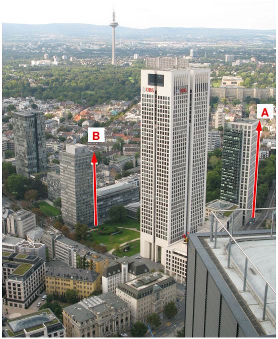
- x – 1D coordinate along the yellow line, positive in the arrow direction
- could be applied to counting steps (→47) if there was no supporting line

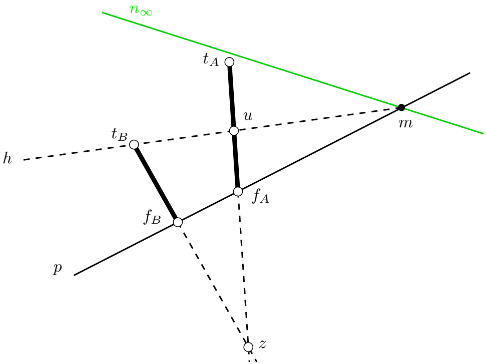
⊛ P1; 1pt: How high is the camera above the floor?

Homework Problem

⊛ H2; 3pt: What is the ratio of heights of Building A to Building B?

- expected: conceptual solution; use notation from this figure
- deadline: LD+2 weeks

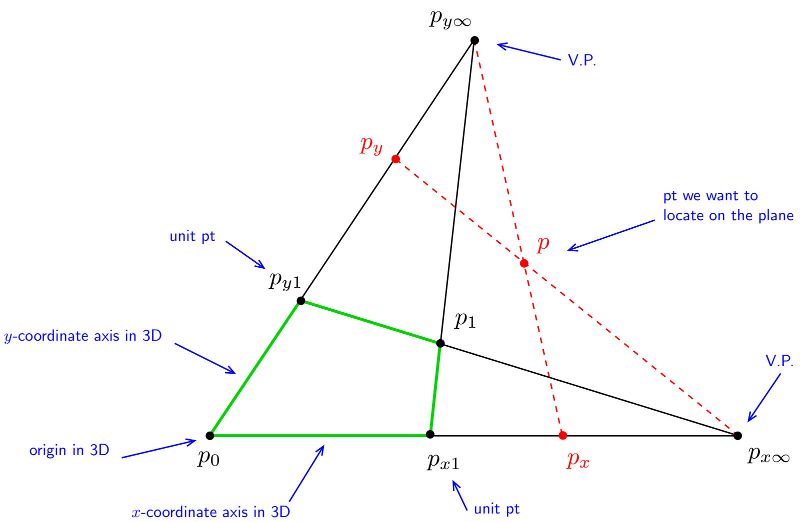




Hints

1. What are the interesting properties of line h connecting the top t_B of Buiding B with the point m at which the horizon intersects the line p joining the foots f_A, f_B of both buildings? [1 point]
2. How do we actually get the horizon n_∞? (we do not see it directly, there are some hills there...) [1 point]
3. Give the formula for measuring the length ratio. [formula = 1 point]

2D Projective Coordinates



$$[P_x] = [P_0 \; P_{x1} \; P_x \; P_{x\infty}] \qquad [P_y] = [P_0 \; P_{y1} \; P_y \; P_{y\infty}]$$

Application: Measuring on the Floor (Wall, etc)

San Giovanni in Laterano, Rome

- measuring distances on the floor in terms of tile units
- what are the dimensions of the seal? Is it circular (assuming square tiles)?
- needs no explicit camera calibration

because we can see the calibrating object (vanishing points)

Module III

Computing with a Single Camera

3.1 Calibration: Internal Camera Parameters from Vanishing Points and Lines

3.2 Camera Resection: Projection Matrix from 6 Known Points

3.3 Exterior Orientation: Camera Rotation and Translation from 3 Known Points

covered by

[1] [H&Z] Secs: 8.6, 7.1, 22.1

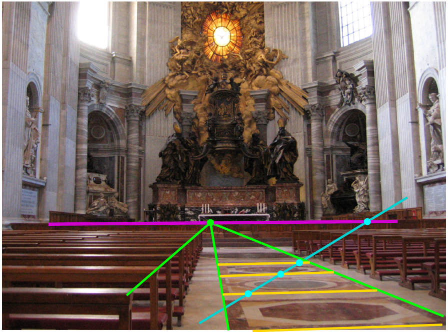
[2] Fischler, M.A. and Bolles, R.C . Random Sample Consensus: A Paradigm for Model Fitting with Applications to Image Analysis and Automated Cartography. *Communications of the ACM* 24(6):381–395, 1981

[3] [Golub & van Loan 2013, Sec. 2.5]

Obtaining Vanishing Points and Lines

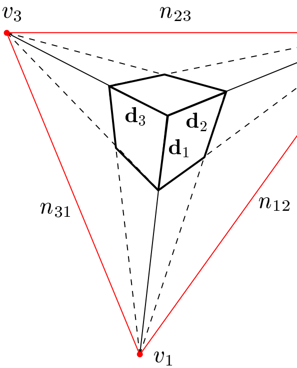
- orthogonal direction pairs can be collected from more images by camera rotation

- vanishing line can be obtained without vanishing points (→48)



▸Camera Calibration from Vanishing Points and Lines

Problem: Given finite vanishing points and/or vanishing lines, compute $\mathbf{K}$



$$\mathbf{d}_i \simeq \mathbf{Q}^{-1}\underline{\mathbf{v}}_i, \qquad i = 1,2,3 \quad \to 42$$

$$\mathbf{p}_{ij} \simeq \mathbf{Q}^{\top}\underline{\mathbf{n}}_{ij}, \quad i,j = 1,2,3,\ i \neq j \quad \to 38 \qquad (2)$$

- simple method: solve (2) after eliminating nuisance pars.

Special Configurations

1. orthogonal rays $\mathbf{d}_1 \perp \mathbf{d}_2$ in space then

$$0 = \mathbf{d}_1^{\top}\mathbf{d}_2 = \underline{\mathbf{v}}_1^{\top}\mathbf{Q}^{-\top}\mathbf{Q}^{-1}\underline{\mathbf{v}}_2 = \underline{\mathbf{v}}_1^{\top}\underbrace{(\mathbf{K}\mathbf{K}^{\top})^{-1}}_{\boldsymbol{\omega}\ \text{(IAC)}}\underline{\mathbf{v}}_2$$

2. orthogonal planes $\mathbf{p}_{ij} \perp \mathbf{p}_{ik}$ in space

$$0 = \mathbf{p}_{ij}^{\top}\mathbf{p}_{ik} = \underline{\mathbf{n}}_{ij}^{\top}\mathbf{Q}\mathbf{Q}^{\top}\underline{\mathbf{n}}_{ik} = \underline{\mathbf{n}}_{ij}^{\top}\boldsymbol{\omega}^{-1}\underline{\mathbf{n}}_{ik}$$

3. orthogonal ray and plane $\mathbf{d}_k \parallel \mathbf{p}_{ij}$, $k \neq i,j$ — normal parallel to optical ray

$$\mathbf{p}_{ij} \simeq \mathbf{d}_k \quad \Rightarrow \quad \mathbf{Q}^{\top}\underline{\mathbf{n}}_{ij} = \lambda\mathbf{Q}^{-1}\underline{\mathbf{v}}_k \quad \Rightarrow \quad \underline{\mathbf{n}}_{ij} = \lambda\mathbf{Q}^{-\top}\mathbf{Q}^{-1}\underline{\mathbf{v}}_k = \lambda\boldsymbol{\omega}\,\underline{\mathbf{v}}_k, \qquad \lambda \neq 0$$

- n_{ij} may be constructed from non-orthogonal v_i and v_j, e.g. using the cross-ratio
- $\boldsymbol{\omega}$ is a symmetric, positive definite 3×3 matrix — IAC = Image of Absolute Conic

►cont'd

	configuration	equation	# constraints
(3)	orthogonal v.p.	$\underline{\mathbf{v}}_i^\top \boldsymbol{\omega}\, \underline{\mathbf{v}}_j = 0$	1
(4)	orthogonal v.l.	$\underline{\mathbf{n}}_{ij}^\top \boldsymbol{\omega}^{-1} \underline{\mathbf{n}}_{ik} = 0$	1
(5)	v.p. orthogonal to v.l.	$\underline{\mathbf{n}}_{ij} = \lambda \boldsymbol{\omega}\, \underline{\mathbf{v}}_k$	2
(6)	orthogonal raster $\theta = \pi/2$	$\omega_{12} = \omega_{21} = 0$	1
(7)	unit aspect $a = 1$ when $\theta = \pi/2$	$\omega_{11} - \omega_{22} = 0$	1
(8)	known principal point $u_0 = v_0 = 0$	$\omega_{13} = \omega_{31} = \omega_{23} = \omega_{32} = 0$	2

- these are homogeneous linear equations for the 5 parameters in $\boldsymbol{\omega}$ in the form $\mathbf{D}\mathbf{w} = \mathbf{0}$
  λ can be eliminated from (5)
- we need at least 5 constraints for full $\boldsymbol{\omega}$ symmetric 3×3
- we get $\mathbf{K}$ from $\boldsymbol{\omega}^{-1} = \mathbf{K}\mathbf{K}^\top$ by Choleski decomposition
  the decomposition returns a positive definite upper triangular matrix
  one avoids solving an explicit set of quadratic equations for the parameters in $\mathbf{K}$
- unlike in the naive method (2), we can introduce constraints on $\mathbf{K}$, e.g. (6)–(8)

Examples

Assuming orthogonal raster, unit aspect (ORUA): $\theta = \pi/2$, $a = 1$

$$\boldsymbol{\omega} \simeq \begin{bmatrix} 1 & 0 & -u_0 \\ 0 & 1 & -v_0 \\ -u_0 & -v_0 & f^2 + u_0^2 + v_0^2 \end{bmatrix}$$

Ex 1:

Assuming ORUA and known $m_0 = (u_0, v_0)$, two finite orthogonal vanishing points give f

$$\underline{\mathbf{v}}_1^\top \boldsymbol{\omega}\, \underline{\mathbf{v}}_2 = 0 \quad \Rightarrow \quad f^2 = \left|(\mathbf{v}_1 - \mathbf{m}_0)^\top (\mathbf{v}_2 - \mathbf{m}_0)\right|$$

in this formula, $\mathbf{v}_i$, $\mathbf{m}_0$ are Cartesian (not homogeneous)!

Ex 2:

Non-orthogonal vanishing points $\mathbf{v}_i$, $\mathbf{v}_j$, known angle ϕ: $\cos\phi = \dfrac{\underline{\mathbf{v}}_i^\top \boldsymbol{\omega} \underline{\mathbf{v}}_j}{\sqrt{\underline{\mathbf{v}}_i^\top \boldsymbol{\omega} \underline{\mathbf{v}}_i}\sqrt{\underline{\mathbf{v}}_j^\top \boldsymbol{\omega} \underline{\mathbf{v}}_j}}$

- leads to polynomial equations
- e.g. ORUA and $u_0 = v_0 = 0$ gives

$$(f^2 + \mathbf{v}_i^\top \mathbf{v}_j)^2 = (f^2 + \|\mathbf{v}_i\|^2) \cdot (f^2 + \|\mathbf{v}_j\|^2) \cdot \cos^2\phi$$

Image of Absolute Conic

This is the **K** matrix:

```
K = {{f, s, u0}, {0, a * f, v0}, {0, 0, 1}}
```

$$\begin{pmatrix} f & s & u_0 \\ 0 & a\,f & v_0 \\ 0 & 0 & 1 \end{pmatrix}$$

The ω matrix:

```
ω = Inverse[K.Transpose[K]] * Det[K]^2 // Simplify
```

$$\begin{pmatrix} a^2 f^2 & -a\,f\,s & a\,f\,(s\,v_0 - a\,f\,u_0) \\ -a\,f\,s & f^2 + s^2 & a\,f\,s\,u_0 - (f^2 + s^2)\,v_0 \\ a\,f\,(s\,v_0 - a\,f\,u_0) & a\,f\,s\,u_0 - (f^2 + s^2)\,v_0 & a^2 f^4 + a^2 u_0^2 f^2 - 2\,a\,s\,u_0\,v_0\,f + (f^2 + s^2)\,v_0^2 \end{pmatrix}$$

The ω matrix with no skew:

```
ω / f^2 /. s -> 0 // Simplify // MatrixForm
```

$$\begin{pmatrix} a^2 & 0 & -a^2 u_0 \\ 0 & 1 & -v_0 \\ -a^2 u_0 & -v_0 & a^2 f^2 + a^2 u_0^2 + v_0^2 \end{pmatrix}$$

ORUA

```
ω / f^2 /. {a -> 1, s -> 0} // Simplify
```

$$\begin{pmatrix} 1 & 0 & -u_0 \\ 0 & 1 & -v_0 \\ -u_0 & -v_0 & f^2 + u_0^2 + v_0^2 \end{pmatrix}$$

▶Camera Orientation from Two Finite Vanishing Points

Problem: Given $\mathbf{K}$ and two vanishing points corresponding to two known orthogonal directions $\mathbf{d}_1$, $\mathbf{d}_2$, compute camera orientation $\mathbf{R}$ with respect to the plane.

- 3D coordinate system <u>choice</u>, e.g.:

$$\mathbf{d}_1 = (1, 0, 0), \quad \mathbf{d}_2 = (0, 1, 0)$$

- we know that

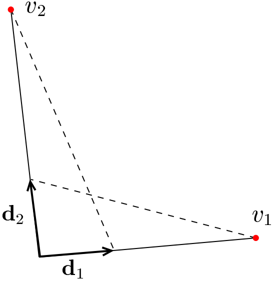
$$\mathbf{d}_i \simeq \mathbf{Q}^{-1}\underline{\mathbf{v}}_i = (\mathbf{K}\mathbf{R})^{-1}\underline{\mathbf{v}}_i = \mathbf{R}^{-1}\underbrace{\mathbf{K}^{-1}\underline{\mathbf{v}}_i}_{\underline{\mathbf{w}}_i}$$

$$\mathbf{R}\mathbf{d}_i \simeq \underline{\mathbf{w}}_i$$

- knowing $\mathbf{d}_{1,2}$ we conclude that $\underline{\mathbf{w}}_i / \|\underline{\mathbf{w}}_i\|$ is the i-th column $\mathbf{r}_i$ of $\mathbf{R}$

- the third column is orthogonal: $\mathbf{r}_3 \simeq \mathbf{r}_1 \times \mathbf{r}_2$

$$\mathbf{R} = \begin{bmatrix} \frac{\underline{\mathbf{w}}_1}{\|\underline{\mathbf{w}}_1\|} & \frac{\underline{\mathbf{w}}_2}{\|\underline{\mathbf{w}}_2\|} & \frac{\underline{\mathbf{w}}_1 \times \underline{\mathbf{w}}_2}{\|\underline{\mathbf{w}}_1 \times \underline{\mathbf{w}}_2\|} \end{bmatrix}$$



some suitable scenes

Application: Planar Rectification

Principle: Rotate camera parallel to the plane of interest.

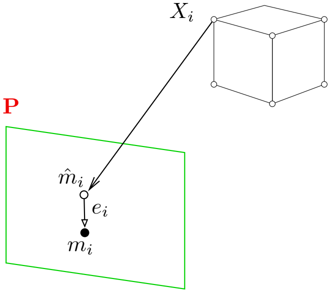
$$\underline{\mathbf{m}} \simeq \mathbf{KR}\begin{bmatrix}\mathbf{I} & -\mathbf{C}\end{bmatrix}\underline{\mathbf{X}} \qquad \underline{\mathbf{m}}' \simeq \mathbf{K}\begin{bmatrix}\mathbf{I} & -\mathbf{C}\end{bmatrix}\underline{\mathbf{X}}$$

$$\underline{\mathbf{m}}' \simeq \mathbf{K}(\mathbf{KR})^{-1}\,\underline{\mathbf{m}} = \mathbf{KR}^\top\mathbf{K}^{-1}\,\underline{\mathbf{m}} = \mathbf{H}\,\underline{\mathbf{m}}$$

- $\mathbf{H}$ is the rectifying homography
- both $\mathbf{K}$ and $\mathbf{R}$ can be calibrated from two finite vanishing points assuming ORUA →56
- not possible when one (or both) of them are infinite
- without ORUA we would need 4 additional views to calibrate $\mathbf{K}$ as on →53

▸Camera Resection

Camera calibration and orientation from a known set of $k \geq 6$ reference points and their images $\{(X_i, m_i)\}_{i=1}^{6}$.

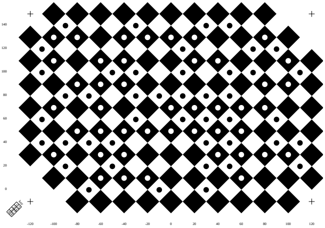


- X_i are considered exact
- m_i is a measurement subject to detection error

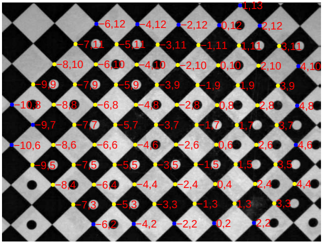
$$\mathbf{m}_i = \hat{\mathbf{m}}_i + \mathbf{e}_i \qquad \text{Cartesian}$$

- where $\underline{\hat{\mathbf{m}}}_i \simeq \mathbf{P}\underline{\mathbf{X}}_i$

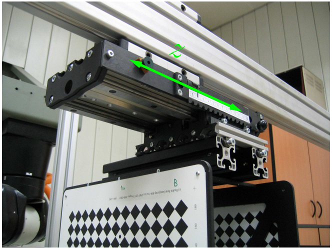
Resection Targets



calibration chart



automatic calibration point detection



resection target with translation stage

- target translated at least once
- by a calibrated (known) translation
- X_i point locations looked up in a table based on their code

►The Minimal Problem for Camera Resection

Problem: Given $k = 6$ corresponding pairs $\{(X_i, m_i)\}_{i=1}^{k}$, find $\mathbf{P}$

$$\lambda_i \underline{\mathbf{m}}_i = \mathbf{P}\underline{\mathbf{X}}_i, \qquad \mathbf{P} = \begin{bmatrix} \mathbf{q}_1^\top & q_{14} \\ \mathbf{q}_2^\top & q_{24} \\ \mathbf{q}_3^\top & q_{34} \end{bmatrix} \qquad \begin{array}{l} \underline{\mathbf{X}}_i = (x_i, y_i, z_i, 1), \quad i = 1, 2, \ldots, k, \; k = 6 \\ \underline{\mathbf{m}}_i = (u_i, v_i, 1), \quad \lambda_i \in \mathbb{R}, \; \lambda_i \neq 0 \end{array}$$

easy to modify for infinite points X_i but be aware of →64

expanded: $\lambda_i u_i = \mathbf{q}_1^\top \mathbf{X}_i + q_{14}, \quad \lambda_i v_i = \mathbf{q}_2^\top \mathbf{X}_i + q_{24}, \quad \lambda_i = \mathbf{q}_3^\top \mathbf{X}_i + q_{34}$

after elimination of λ_i: $(\mathbf{q}_3^\top \mathbf{X}_i + q_{34})u_i = \mathbf{q}_1^\top \mathbf{X}_i + q_{14}, \quad (\mathbf{q}_3^\top \mathbf{X}_i + q_{34})v_i = \mathbf{q}_2^\top \mathbf{X}_i + q_{24}$

Then

$$\mathbf{A}\,\mathbf{q} = \begin{bmatrix} \mathbf{X}_1^\top & 1 & \mathbf{0}^\top & 0 & -u_1\mathbf{X}_1^\top & -u_1 \\ \mathbf{0}^\top & 0 & \mathbf{X}_1^\top & 1 & -v_1\mathbf{X}_1^\top & -v_1 \\ \vdots & & & & & \vdots \\ \mathbf{X}_k^\top & 1 & \mathbf{0}^\top & 0 & -u_k\mathbf{X}_k^\top & -u_k \\ \mathbf{0}^\top & 0 & \mathbf{X}_k^\top & 1 & -v_k\mathbf{X}_k^\top & -v_k \end{bmatrix} \cdot \begin{bmatrix} \mathbf{q}_1 \\ q_{14} \\ \mathbf{q}_2 \\ q_{24} \\ \mathbf{q}_3 \\ q_{34} \end{bmatrix} = \mathbf{0} \tag{9}$$

- we need 11 indepedent parameters for $\mathbf{P}$
- $\mathbf{A} \in \mathbb{R}^{2k,12}, \; \mathbf{q} \in \mathbb{R}^{12}$
- 6 points in a general position give $\operatorname{rank} \mathbf{A} = 12$ and there is no non-trivial null space
- drop one row to get rank 11 matrix, then the basis vector of the null space of $\mathbf{A}$ gives $\mathbf{q}$

▶The Jack-Knife Solution for $k = 6$

- given the 6 correspondences, we have 12 equations for the 11 parameters
- can we use all the information present in the 6 points?

Jack-knife estimation

1. $n := 0$
2. for $i = 1, 2, \ldots, 2k$ do
   a) delete i-th row from $\mathbf{A}$, this gives $\mathbf{A}_i$
   b) if $\dim \operatorname{null} \mathbf{A}_i > 1$ continue with the next i
   c) $n := n + 1$
   d) compute the right null-space $\mathbf{q}_i$ of $\mathbf{A}_i$ — e.g. by 'economy-size' SVD
   e) $\hat{\mathbf{q}}_i := \mathbf{q}_i$ normalized to $q_{34} = 1$ and dimension-reduced — assuming finite cam. with $P_{3,4} = 1$
3. from all n vectors $\hat{\mathbf{q}}_i$ collected in Step 1d compute

$$\mathbf{q} = \frac{1}{n}\sum_{i=1}^{n} \hat{\mathbf{q}}_i, \qquad \operatorname{var}[\mathbf{q}] = \frac{n-1}{n} \operatorname{diag} \sum_{i-1}^{n} (\hat{\mathbf{q}}_i - \mathbf{q})(\hat{\mathbf{q}}_i - \mathbf{q})^\top$$

regular for $n \geq 11$

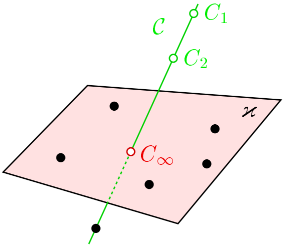
- have a solution + an error estimate, per individual elements of $\mathbf{P}$ (except P_{34})
- at least 5 points must be in a general position (→64)
- large error indicates near degeneracy
- computation not efficient with $k > 6$ points, needs $\binom{2k}{11}$ draws, e.g. $k = 7 \Rightarrow 364$ draws
- better error estimation method: decompose $\mathbf{P}_i$ to $\mathbf{K}_i$, $\mathbf{R}_i$, $\mathbf{t}_i$ (→32), represent $\mathbf{R}_i$ with 3 parameters (e.g. Euler angles, or in Cayley representation →139) and compute the errors for the parameters

▶Degenerate (Critical) Configurations for Camera Resection

Let $\mathcal{X} = \{X_i;\, i = 1, \ldots\}$ be a set of points and $\mathbf{P}_1 \not\simeq \mathbf{P}_j$ be two regular (rank-3) cameras. Then two configurations $(\mathbf{P}_1, \mathcal{X})$ and $(\mathbf{P}_j, \mathcal{X})$ are image-equivalent if

$$\mathbf{P}_1 \underline{\mathbf{X}}_i \simeq \mathbf{P}_j \underline{\mathbf{X}}_i \quad \text{for all} \quad X_i \in \mathcal{X}$$

there is a non-trivial set of other cameras that see the same image



Case 4

- **importantly**: If all calibration points $X_i \in \mathcal{X}$ lie on a plane $\varkappa$ then camera resection is non-unique and all image-equivalent camera centers lie on a spatial line $\mathcal{C}$ with the $C_\infty = \varkappa \cap \mathcal{C}$ excluded

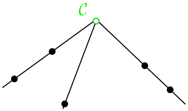
  this also means we cannot resect if all X_i are infinite
- by adding points $X_i \in \mathcal{X}$ to $\mathcal{C}$ we gain nothing
- there are additional image-equivalent configurations, see next

proof sketch in [H&Z, Sec. 22.1.2]

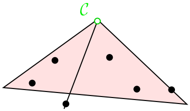
Note that if $\mathbf{Q}$, $\mathbf{T}$ are suitable homographies then $\mathbf{P}_1 \simeq \mathbf{Q}\mathbf{P}_0\mathbf{T}$, where $\mathbf{P}_0$ is canonical and the analysis can be made with $\hat{\mathbf{P}}_j \simeq \mathbf{Q}^{-1}\mathbf{P}_j$

$$\mathbf{P}_0 \underbrace{\mathbf{T}\underline{\mathbf{X}}_i}_{\underline{\mathbf{Y}}_i} \simeq \hat{\mathbf{P}}_j \underbrace{\mathbf{T}\underline{\mathbf{X}}_i}_{\underline{\mathbf{Y}}_i} \quad \text{for all} \quad Y_i \in \mathcal{Y}$$

cont'd (all cases)



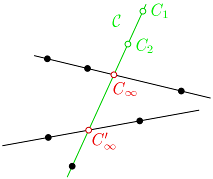
Case 5



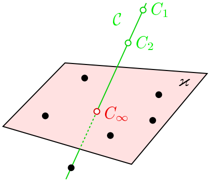
Case 6

- cameras C_1, C_2 co-located at point $\mathcal{C}$
- points on three optical rays or one optical ray and one optical plane

- Case 5: camera sees 3 isolated point images
- Case 6: cam. sees a line of points and an isolated point



Case 3

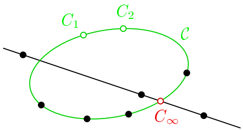


Case 4

- cameras lie on a line $\mathcal{C} \setminus \{C_\infty, C'_\infty\}$
- points lie on $\mathcal{C}$ and
  1. on two lines meeting $\mathcal{C}$ at C_∞, C'_∞
  2. or on a plane meeting $\mathcal{C}$ at C_∞

- Case 3: camera sees 2 lines of points

Case 2

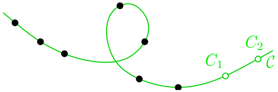


- cameras lie on a planar conic $\mathcal{C} \setminus \{C_\infty\}$

  not necessarily an ellipse
- points lie on $\mathcal{C}$ and an additional line meeting the conic at C_∞

- Case 2: camera sees 2 lines of points

Case 1



- cameras and points all lie on a twisted cubic $\mathcal{C}$

- Case 1: camera sees a conic

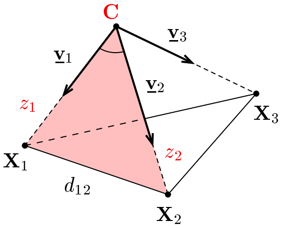
►Three-Point Exterior Orientation Problem (P3P)

Calibrated camera rotation and translation from Perspective images of 3 reference Points.

Problem: Given $\mathbf{K}$ and three corresponding pairs $\{(m_i, X_i)\}_{i=1}^3$, find $\mathbf{R}$, $\mathbf{C}$ by solving

$$\lambda_i \underline{\mathbf{m}}_i = \mathbf{K}\mathbf{R}\,(\mathbf{X}_i - \mathbf{C}), \qquad i = 1, 2, 3$$

configuration w/o rotation in (11)



1. Transform $\underline{\mathbf{v}}_i \stackrel{\text{def}}{=} \mathbf{K}^{-1}\underline{\mathbf{m}}_i$. Then

$$\lambda_i \underline{\mathbf{v}}_i = \mathbf{R}\,(\mathbf{X}_i - \mathbf{C}). \qquad (10)$$

2. Eliminate $\mathbf{R}$ by taking — rotation preserves length: $\|\mathbf{R}\mathbf{x}\| = \|\mathbf{x}\|$

$$|\lambda_i| \cdot \|\underline{\mathbf{v}}_i\| = \|\mathbf{X}_i - \mathbf{C}\| \stackrel{\text{def}}{=} z_i \qquad (11)$$

3. Consider only angles among $\underline{\mathbf{v}}_i$ and apply Cosine Law per triangle $(\mathbf{C}, \mathbf{X}_i, \mathbf{X}_j)$ $i, j = 1, 2, 3,\ i \neq j$

$$d_{ij}^2 = z_i^2 + z_j^2 - 2\, z_i\, z_j\, c_{ij},$$

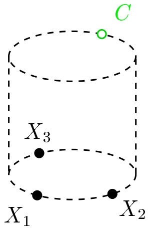
$$z_i = \|\mathbf{X}_i - \mathbf{C}\|,\ \ d_{ij} = \|\mathbf{X}_j - \mathbf{X}_i\|,\ \ c_{ij} = \cos(\angle \underline{\mathbf{v}}_i\, \underline{\mathbf{v}}_j)$$

4. Solve system of 3 quadratic eqs in 3 unknowns z_i [Fischler & Bolles, 1981]
   there may be no real root; there are up to 4 solutions that cannot be ignored (verify on additional points)

5. Compute $\mathbf{C}$ by trilateration (3-sphere intersection) from $\mathbf{X}_i$ and z_i; then λ_i from (11) and $\mathbf{R}$ from (10)

Similar problems (P4P with unknown f) at http://cmp.felk.cvut.cz/minimal/ (with code)

Degenerate (Critical) Configurations for Exterior Orientation



unstable solution

- center of projection C located on the orthogonal circular cylinder with base circumscribing the three points X_i

unstable: a small change of X_i results in a large change of C
can be detected by error propagation

degenerate

- camera C is coplanar with points (X_1, X_2, X_3) but is not on the circumscribed circle of (X_1, X_2, X_3) camera sees a line

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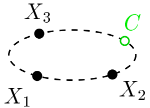


no solution

1. C cocyclic with (X_1, X_2, X_3) camera sees a line

- additional critical configurations depend on the method to solve the quadratic equations

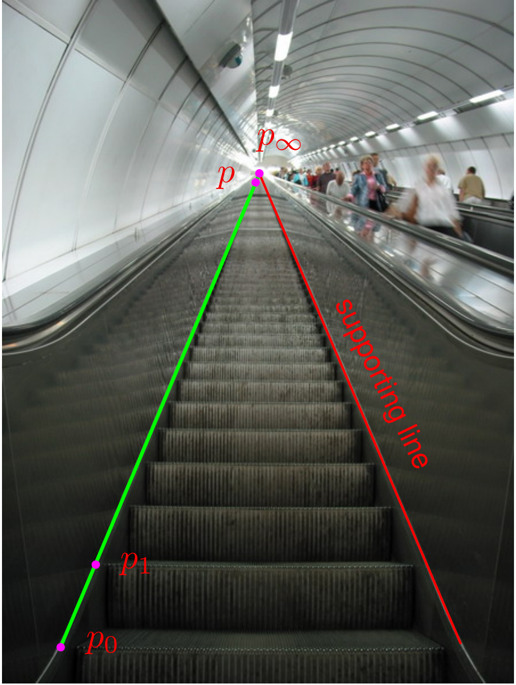
[Haralick et al. IJCV 1994]

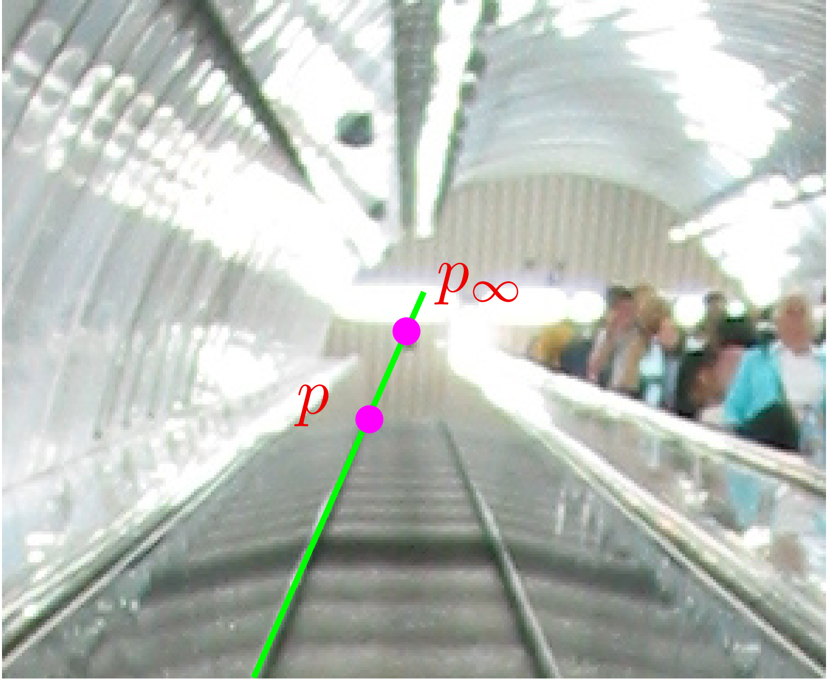
►Populating A Little ZOO of Minimal Geometric Problems in CV

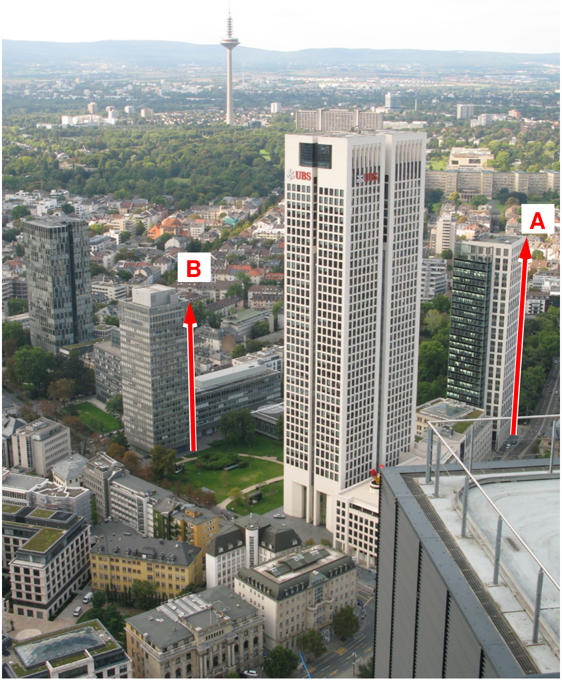
problem	given	unknown	slide
camera resection	6 world–img correspondences $\left\{(X_i,\, m_i)\right\}_{i=1}^{6}$	$\mathbf{P}$	62
exterior orientation	$\mathbf{K}$, 3 world–img correspondences $\left\{(X_i,\, m_i)\right\}_{i=1}^{3}$	$\mathbf{R}$, $\mathbf{C}$	66

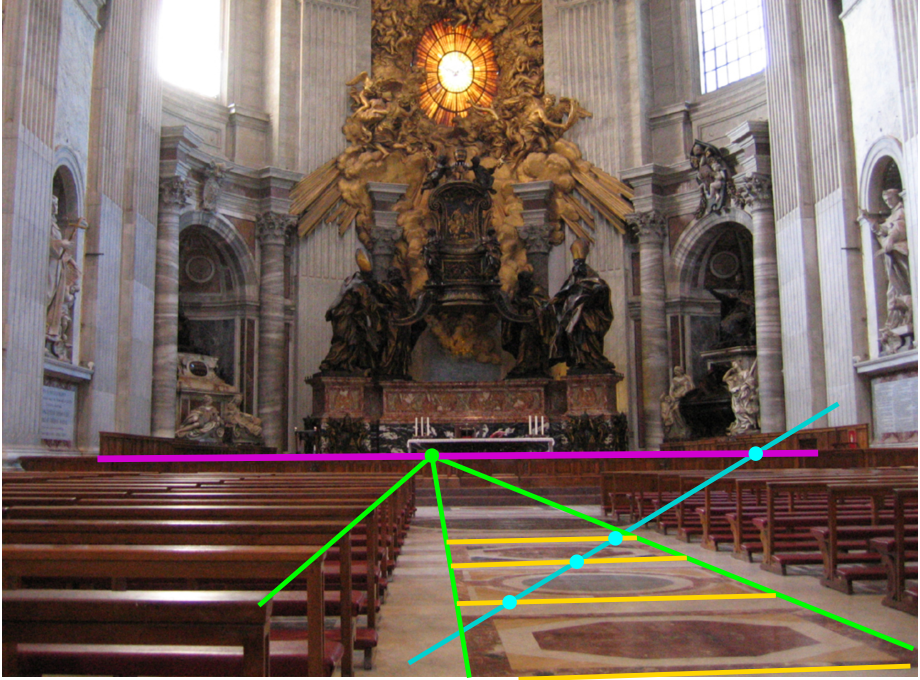
- camera resection and exterior orientation are similar problems in a sense:
  - we do resectioning when our camera is uncalibrated
  - we do orientation when our camera is calibrated
- more problems to come

Thank You


p_∞
p
supporting line
p_1
p_0


p_∞
p


UBS
UBS
A
B

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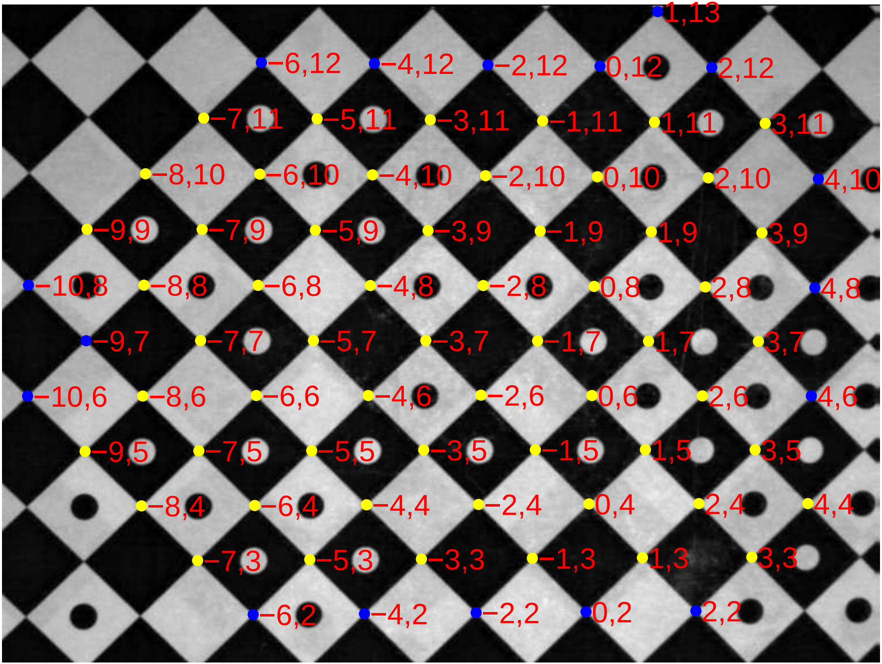
BOOKS
FILMS
TICKETS


1,13
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−7,11 −5,11 −3,11 −1,11 1,11 3,11
−8,10 −6,10 −4,10 −2,10 0,10 2,10 4,10
−9,9 −7,9 −5,9 −3,9 −1,9 1,9 3,9
−10,8 −8,8 −6,8 −4,8 −2,8 0,8 2,8 4,8
−9,7 −7,7 −5,7 −3,7 −1,7 1,7 3,7
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−7,3 −5,3 −3,3 −1,3 1,3 3,3
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