

Stereovision

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mostly covered by

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additional references



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J. Gluckman and S. K. Nayar. Rectifying transformations that minimize resampling effects. In *Proc IEEE CS Conf on Computer Vision and Pattern Recognition*, vol. 1:111–117. 2001.



M. Pollefeys, R. Koch, and L. V. Gool. A simple and efficient rectification method for general motion. In *Proc Int Conf on Computer Vision*, vol. 1:496–501, 1999.

What Are The Relative Distances?



- monocular vision already gives a rough 3D sketch because we understand the scene

What Are The Relative Distances?



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- left: we have no help from image interpretation
- right: ambiguous interpretation due to a combination of missing texture and occlusion

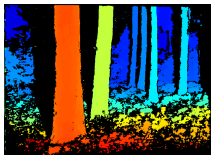
► How Difficult Is Stereo?



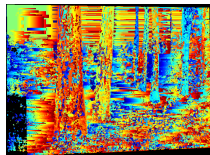
- when we do not recognize the scene and cannot use high-level constraints the problem seems difficult (right, less so in the center)
- most stereo matching algorithms do not require scene understanding prior to matching
- the success of a model-free stereo matching algorithm is unlikely:



left image



a good disparity map



disparity map from WTA

WTA Matching:

for every left-image pixel
find the most similar
right-image pixel
along the
corresponding epipolar
line [Marroquin 83]

A Summary of Our Observations and an Outlook

1. simple matching algorithms do not work
2. stereopsis requires image interpretation in sufficiently complex scenes
or another-modality measurement

we have a tradeoff: model strength \leftrightarrow universality

Outlook:

1. represent the occlusion constraint: correspondences are not independent due to occlusions
 - epipolar rectification
 - disparity
 - uniqueness as an occlusion constraint
2. represent piecewise continuity the weakest of interpretations; piecewise: object boundaries
 - ordering as a weak continuity model
3. use a consistent framework
 - looking for the most probable solution (MAP)

► Linear Epipolar Rectification for Easier Correspondence Search

Obs:

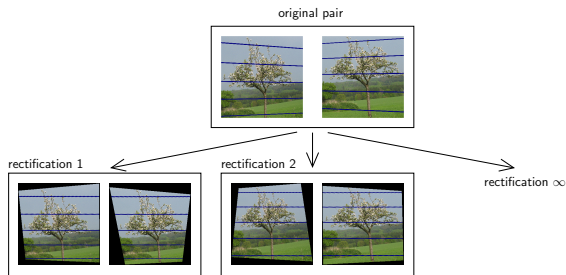
- if we map epipoles to infinity, epipolar lines become parallel
- we then rotate them to become horizontal
- we then scale to make corresponding epipolar lines colinear
- this can be achieved by a pair of homographies applied to the images

Problem: Given fundamental matrix \mathbf{F} or camera matrices \mathbf{P}_1 , \mathbf{P}_2 , compute a pair of homographies that maps epipolar lines to horizontal with the same row coordinate.

Procedure:

1. find a pair of rectification homographies \mathbf{H}_1 and \mathbf{H}_2 .
2. warp images using \mathbf{H}_1 and \mathbf{H}_2 and transform the fundamental matrix

$$\mathbf{F} \mapsto \mathbf{H}_2^{-\top} \mathbf{F} \mathbf{H}_1^{-1} \quad \text{or the cameras } \mathbf{P}_1 \mapsto \mathbf{H}_1 \mathbf{P}_1, \quad \mathbf{P}_2 \mapsto \mathbf{H}_2 \mathbf{P}_2.$$

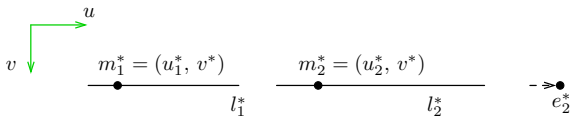


► Rectification Homographies

Assumption: Cameras $(\mathbf{P}_1, \mathbf{P}_2)$ are rectified by a homography pair $(\mathbf{H}_1, \mathbf{H}_2)$:

$$\mathbf{P}_i^* = \mathbf{H}_i \mathbf{P}_i = \mathbf{H}_i \mathbf{K}_i \mathbf{R}_i \begin{bmatrix} \mathbf{I} & -\mathbf{C}_i \end{bmatrix}, \quad i = 1, 2$$

rectified entities: \mathbf{F}^* , \mathbf{l}_2^* , \mathbf{l}_1^* , etc:



- the rectified location difference $d = u_1^* - u_2^*$ is called disparity

corresponding epipolar lines must be:

1. parallel to image rows \Rightarrow epipoles become $e_1^* = e_2^* = (1, 0, 0)$
2. equivalent $l_2^* = l_1^* \Rightarrow$ (a) $\mathbf{l}_2^* \simeq \mathbf{l}_1^* \simeq \mathbf{e}_1^* \times \underline{\mathbf{m}}_1 = [\mathbf{e}_1^*]_{\times} \underline{\mathbf{m}}_1$, (b) $\mathbf{l}_2^* \simeq \mathbf{F}^* \underline{\mathbf{m}}_1$

- therefore the canonical fundamental matrix is

$$\mathbf{F}^* \simeq \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{bmatrix}$$

A two-step rectification procedure

1. find some pair of primitive rectification homographies $\hat{\mathbf{H}}_1, \hat{\mathbf{H}}_2$
2. upgrade to a pair of optimal rectification homographies while preserving \mathbf{F}^*

► Geometric Interpretation of Linear Rectification

What pair of physical cameras is compatible with \mathbf{F}^* ?

- we know that $\mathbf{F} = (\mathbf{Q}_1 \mathbf{Q}_2^{-1})^\top [\mathbf{e}_1]_\times$ →78
- we choose $\mathbf{Q}_1^* = \mathbf{K}_1^*$, $\mathbf{Q}_2^* = \mathbf{K}_2^* \mathbf{R}^*$; then

$$(\mathbf{Q}_1^* \mathbf{Q}_2^{*-1})^\top [\mathbf{e}_1^*]_\times = (\mathbf{K}_1^* \mathbf{R}^{*\top} \mathbf{K}_2^{*-1})^\top \mathbf{F}^*$$

- we look for \mathbf{R}^* , \mathbf{K}_1^* , \mathbf{K}_2^* compatible with

$$(\mathbf{K}_1^* \mathbf{R}^{*\top} \mathbf{K}_2^{*-1})^\top \mathbf{F}^* = \lambda \mathbf{F}^*, \quad \mathbf{R}^* \mathbf{R}^{*\top} = \mathbf{I}, \quad \mathbf{K}_1^*, \mathbf{K}_2^* \text{ upper triangular}$$

- we also want \mathbf{b}^* from $\mathbf{e}_1^* \simeq \mathbf{P}_1^* \mathbf{C}_2^* = \mathbf{K}_1^* \mathbf{b}^*$ \mathbf{b}^* in cam. 1 frame
- result:

$$\mathbf{R}^* = \mathbf{I}, \quad \mathbf{b}^* = \begin{bmatrix} b \\ 0 \\ 0 \end{bmatrix}, \quad \mathbf{K}_1^* = \begin{bmatrix} k_{11} & k_{12} & k_{13} \\ 0 & f & v_0 \\ 0 & 0 & 1 \end{bmatrix}, \quad \mathbf{K}_2^* = \begin{bmatrix} k_{21} & k_{22} & k_{23} \\ 0 & f & v_0 \\ 0 & 0 & 1 \end{bmatrix} \quad (33)$$

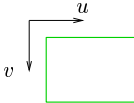
- rectified cameras are in canonical relative pose not rotated, canonical baseline
- rectified calibration matrices can differ in the first row only
- when $\mathbf{K}_1^* = \mathbf{K}_2^*$ then the rectified pair is called the standard stereo pair and the homographies standard rectification homographies
- standard rectification homographies: points at infinity have zero disparity

$$\mathbf{P}_i^* \mathbf{X}_\infty = \mathbf{K} [\mathbf{I} \quad -\mathbf{C}_i] \mathbf{X}_\infty = \mathbf{K} \mathbf{X}_\infty \quad i = 1, 2$$





- this does not mean that the images are not distorted after rectification

► The Degrees of Freedom in Epipolar Rectification

Proposition 1 Homographies \mathbf{A}_1 and \mathbf{A}_2 are rectification-preserving if the images stay rectified, i.e. if $\mathbf{A}_2^{-\top} \mathbf{F}^* \mathbf{A}_1^{-1} \simeq \mathbf{F}^*$, which gives

$$\mathbf{A}_1 = \begin{bmatrix} l_1 & l_2 & l_3 \\ 0 & s_v & t_v \\ 0 & q & 1 \end{bmatrix}, \quad \mathbf{A}_2 = \begin{bmatrix} r_1 & r_2 & r_3 \\ 0 & s_v & t_v \\ 0 & q & 1 \end{bmatrix},$$


where $s_v \neq 0$, t_v , $l_1 \neq 0$, l_2 , l_3 , $r_1 \neq 0$, r_2 , r_3 , q are 9 free parameters.

general	transformation		standard
l_1, r_1	horizontal scales		$l_1 = r_1$
l_2, r_2	horizontal shears		$l_2 = r_2$
l_3, r_3	horizontal shifts		$l_3 = r_3$
q	common special projective		
s_v	common vertical scale		
t_v	common vertical shift		
9 DoF			$9 - 3 = 6$ DoF

- q is rotation about the baseline
- s_v changes the focal length

proof: find a rotation \mathbf{G} that brings \mathbf{K} to upper triangular form via RQ decomposition: $\mathbf{A}_1 \mathbf{K}_1^* = \hat{\mathbf{K}}_1 \mathbf{G}$ and $\mathbf{A}_2 \mathbf{K}_2^* = \hat{\mathbf{K}}_2 \mathbf{G}$

Corollary for Proposition 1 Let $\bar{\mathbf{H}}_1$ and $\bar{\mathbf{H}}_2$ be (primitive or other) rectification homographies. Then $\mathbf{H}_1 = \mathbf{A}_1 \bar{\mathbf{H}}_1$, $\mathbf{H}_2 = \mathbf{A}_2 \bar{\mathbf{H}}_2$ are also rectification homographies.

Proposition 2 Pairs of rectification-preserving homographies $(\mathbf{A}_1, \mathbf{A}_2)$ form a group with group operation $(\mathbf{A}'_1, \mathbf{A}'_2) \circ (\mathbf{A}_1, \mathbf{A}_2) = (\mathbf{A}'_1 \mathbf{A}_1, \mathbf{A}'_2 \mathbf{A}_2)$.

Proof:

- closure by Proposition 1
- associativity by matrix multiplication
- identity belongs to the set
- inverse element belongs to the set by $\mathbf{A}_2^\top \mathbf{F}^* \mathbf{A}_1 \simeq \mathbf{F}^* \Leftrightarrow \mathbf{F}^* \simeq \mathbf{A}_2^{-\top} \mathbf{F}^* \mathbf{A}_1^{-1}$

► Primitive Rectification

Goal: Given fundamental matrix \mathbf{F} , derive some simple rectification homographies $\mathbf{H}_1, \mathbf{H}_2$

1. Let the SVD of \mathbf{F} be $\mathbf{UDV}^\top = \mathbf{F}$, where $\mathbf{D} = \text{diag}(1, d^2, 0)$, $1 \geq d^2 > 0$
2. Write \mathbf{D} as $\mathbf{D} = \mathbf{A}^\top \mathbf{F}^* \mathbf{B}$ for some regular \mathbf{A}, \mathbf{B} . For instance (\mathbf{F}^* is given $\rightarrow 152$)

$$\mathbf{A} = \begin{bmatrix} 0 & 0 & 1 \\ 0 & -d & 0 \\ 1 & 0 & 0 \end{bmatrix}, \quad \mathbf{B} = \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & d & 0 \end{bmatrix}$$

3. Then

$$\mathbf{F} = \mathbf{UDV}^\top = \underbrace{\mathbf{UA}^\top}_{\hat{\mathbf{H}}_2^\top} \mathbf{F}^* \underbrace{\mathbf{BV}^\top}_{\hat{\mathbf{H}}_1}$$

and the primitive rectification homographies are

$$\hat{\mathbf{H}}_2 = \mathbf{AU}^\top, \quad \hat{\mathbf{H}}_1 = \mathbf{BV}^\top$$

⊛ P1; 1pt: derive some other admissible \mathbf{A}, \mathbf{B}

- rectification homographies do exist $\rightarrow 152$
- there are other primitive rectification homographies, these suggested are just simple to obtain

► Primitive Rectification Suffices for Calibrated Cameras

Obs: calibrated cameras: $d = 1 \Rightarrow \hat{\mathbf{H}}_1, \hat{\mathbf{H}}_2$ are orthogonal

1. determine primitive rectification homographies ($\hat{\mathbf{H}}_1, \hat{\mathbf{H}}_2$) from the essential matrix
2. choose a suitable common calibration matrix \mathbf{K} , e.g.

$$\mathbf{K} = \begin{bmatrix} f & 0 & u_0 \\ 0 & f & v_0 \\ 0 & 0 & 1 \end{bmatrix}, \quad f = \frac{1}{2}(f^1 + f^2), \quad u_0 = \frac{1}{2}(u_0^1 + u_0^2), \quad \text{etc.}$$

3. the final rectification homographies applied as $\mathbf{P}_i \mapsto \mathbf{H}_i \mathbf{P}_i$ are

$$\mathbf{H}_1 = \mathbf{K} \hat{\mathbf{H}}_1 \mathbf{K}_1^{-1}, \quad \mathbf{H}_2 = \mathbf{K} \hat{\mathbf{H}}_2 \mathbf{K}_2^{-1}$$

- we got a standard stereo pair ($\rightarrow 153$) and non-negative disparity

$$\text{let } \mathbf{K}_i^{-1} \mathbf{P}_i = \mathbf{R}_i [\mathbf{I} \quad -\mathbf{C}_i], \quad i = 1, 2 \quad \text{note we started from } \mathbf{E}, \text{ not } \mathbf{F}$$

$$\mathbf{H}_1 \mathbf{P}_1 = \mathbf{K} \hat{\mathbf{H}}_1 \mathbf{K}_1^{-1} \mathbf{P}_1 = \mathbf{K} \underbrace{\mathbf{B} \mathbf{V}^\top \mathbf{R}_1}_{\mathbf{R}^*} [\mathbf{I} \quad -\mathbf{C}_1] = \mathbf{K} \mathbf{R}^* [\mathbf{I} \quad -\mathbf{C}_1]$$

$$\mathbf{H}_2 \mathbf{P}_2 = \mathbf{K} \hat{\mathbf{H}}_2 \mathbf{K}_2^{-1} \mathbf{P}_2 = \mathbf{K} \underbrace{\mathbf{A} \mathbf{U}^\top \mathbf{R}_2}_{\mathbf{R}^*} [\mathbf{I} \quad -\mathbf{C}_2] = \mathbf{K} \mathbf{R}^* [\mathbf{I} \quad -\mathbf{C}_2]$$

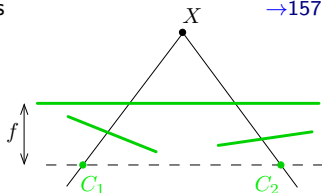
- one can prove that $\mathbf{B} \mathbf{V}^\top \mathbf{R}_1 = \mathbf{A} \mathbf{U}^\top \mathbf{R}_2$ with the help of essential matrix decomposition (13)
- points at infinity project to $\mathbf{K} \mathbf{R}^*$ in both images \Rightarrow they have zero disparity

$\rightarrow 160$

► Summary & Remarks: Linear Rectification

- rectification is done with a pair of homographies (one per image) →151
 - ⇒ rectified camera centers are equal to the original ones
 - binocular rectification: a 9-parameter family of rectification homographies
 - trinocular rectification: has 9 or 6 free parameters (depending on additional constrains)
 - in general, linear rectification is not possible for more than three cameras
- rectified cameras are in canonical orientation →153
 - ⇒ rectified image projection planes are coplanar
- equal rectified calibration matrices give standard rectification →153
 - ⇒ rectified image projection planes are equal
- primitive rectification is standard in calibrated cameras →157

standard rectification homographies reproject onto a common image plane parallel to the baseline



Corollary

- standard rectified pair: disparity vanishes when corresponding 3D points are at infinity
 - known \mathbf{F} used alone gives no constraints on standard rectification homographies
 - for that we need either of these:
 1. projection matrices, or calibrated cameras, or
 2. a few points at infinity calibrating $k_{1i}, k_{2i}, i = 1, 2, 3$ in (33)

Optimal choice for the free parameters

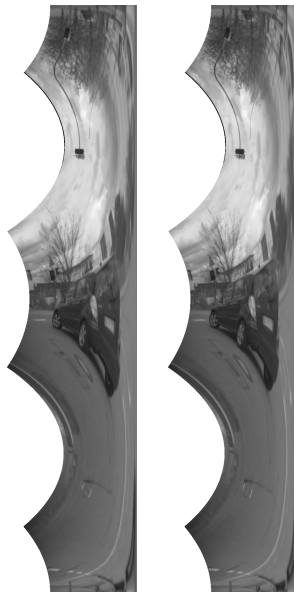
- by minimization of residual image distortion, eg. [Gluckman & Nayar 2001]

$$\mathbf{A}_1^* = \arg \min_{\mathbf{A}_1} \iint_{\Omega} (\det J(\mathbf{A}_1 \hat{\mathbf{H}}_1 \mathbf{x}) - 1)^2 d\mathbf{x}$$

- by minimization of image information loss [Matoušek, ICIG 2004]
- non-linear rectification suitable for forward motion
non-parametric: [Pollefeys et al. 1999]
analytic: [Geyer & Daniilidis 2003]

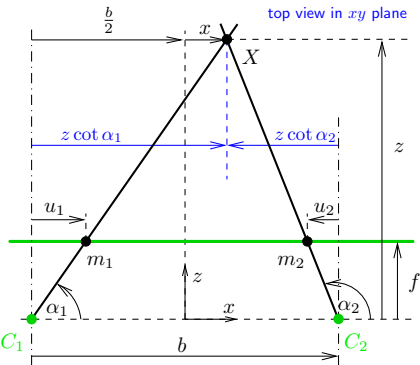


forward egomotion



rectified images, Pollefeys' method

► Binocular Disparity in Standard Stereo Pair



- Assumptions: single image line, standard camera pair

$$b = z \cot \alpha_1 - z \cot \alpha_2$$

$$u_1 = f \cot \alpha_1$$

$$u_2 = f \cot \alpha_2$$

$$b = \frac{b}{2} + x - z \cot \alpha_2$$

$X = (x, z)$ from **disparity** $d = u_1 - u_2$:

$$z = \frac{bf}{d}, \quad x = \frac{b}{d} \frac{u_1 + u_2}{2}, \quad y = \frac{bv}{d}$$

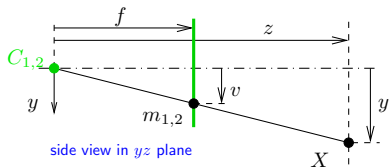
f, d, u, v in pixels, b, x, y, z in meters

Observations

- constant disparity surface is a frontoparallel plane
- distant points have small disparity
- relative error in z is large for small disparity

$$\frac{1}{z} \frac{dz}{dd} = -\frac{1}{d}$$

- increasing the baseline or the focal length increases disparity and reduces the error



Structural Ambiguity in Stereovision

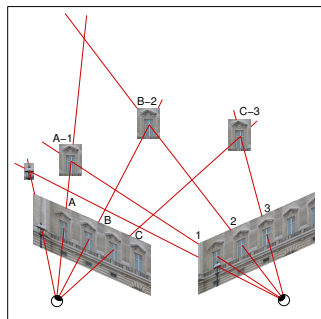
- we can recognize matches but have no scene model
- lack of an occlusion model \Rightarrow structural ambiguity in the presence of repetitions (or lack of texture)
- lack of a continuity model



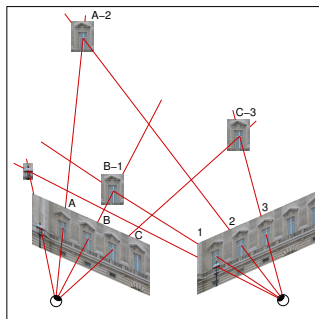
left image



right image



interpretation 1



interpretation 2

Thank You



