

Examples

Assuming orthogonal raster, unit aspect (ORUA): $\theta = \pi/2$, $a = 1$

$$\mathbf{K} = \begin{bmatrix} f & 0 & m_0 \\ 0 & f & v_0 \\ 0 & 0 & 1 \end{bmatrix} \quad \boldsymbol{\omega} \simeq \begin{bmatrix} 1 & 0 & -u_0 \\ 0 & 1 & -v_0 \\ -u_0 & -v_0 & f^2 + u_0^2 + v_0^2 \end{bmatrix}$$

Ex 1:

Assuming ORUA and known $m_0 = (u_0, v_0)$, two finite orthogonal vanishing points give f

$$\underline{\mathbf{v}}_1^\top \boldsymbol{\omega} \underline{\mathbf{v}}_2 = 0 \quad \Rightarrow \quad f^2 = |(\mathbf{v}_1 - \mathbf{m}_0)^\top (\mathbf{v}_2 - \mathbf{m}_0)|$$

in this formula, \mathbf{v}_i , \mathbf{m}_0 are Cartesian (not homogeneous)!

Ex 2:

Non-orthogonal vanishing points \mathbf{v}_i , \mathbf{v}_j , known angle ϕ : $\cos \phi = \frac{\underline{\mathbf{v}}_i^\top \boldsymbol{\omega} \underline{\mathbf{v}}_j}{\sqrt{\underline{\mathbf{v}}_i^\top \boldsymbol{\omega} \underline{\mathbf{v}}_i} \sqrt{\underline{\mathbf{v}}_j^\top \boldsymbol{\omega} \underline{\mathbf{v}}_j}}$

- leads to polynomial equations
- e.g. ORUA and $u_0 = v_0 = 0$ gives

$$(f^2 + \mathbf{v}_i^\top \mathbf{v}_j)^2 = (f^2 + \|\mathbf{v}_i\|^2) \cdot (f^2 + \|\mathbf{v}_j\|^2) \cdot \cos^2 \phi$$

Image of Absolute Conic

This is the \mathbf{K} matrix:

$$\mathbf{K} = \{ \{f, s, u_0\}, \{0, a \cdot f, v_0\}, \{0, 0, 1\} \}$$

$$\begin{pmatrix} f & s & u_0 \\ 0 & a f & v_0 \\ 0 & 0 & 1 \end{pmatrix}$$

The ω matrix:

$$\omega = \text{Inverse}[\mathbf{K}.\text{Transpose}[\mathbf{K}]] * \text{Det}[\mathbf{K}]^2 // \text{Simplify}$$

$$\begin{pmatrix} a^2 f^2 & -a f s & a f (s v_0 - a f u_0) \\ -a f s & f^2 + s^2 & a f s u_0 - (f^2 + s^2) v_0 \\ a f (s v_0 - a f u_0) & a f s u_0 - (f^2 + s^2) v_0 & a^2 f^4 + a^2 u_0^2 f^2 - 2 a s u_0 v_0 f + (f^2 + s^2) v_0^2 \end{pmatrix}$$

The ω matrix with no skew:

$$\omega / f^2 /. s \rightarrow 0 // \text{Simplify} // \text{MatrixForm}$$

$$\begin{pmatrix} a^2 & 0 & -a^2 u_0 \\ 0 & 1 & -v_0 \\ -a^2 u_0 & -v_0 & a^2 f^2 + a^2 u_0^2 + v_0^2 \end{pmatrix}$$

ORUA

$$\omega / f^2 /. \{a \rightarrow 1, s \rightarrow 0\} // \text{Simplify}$$

$$\begin{pmatrix} 1 & 0 & -u_0 \\ 0 & 1 & -v_0 \\ -u_0 & -v_0 & f^2 + u_0^2 + v_0^2 \end{pmatrix}$$

► Camera Orientation from Two Finite Vanishing Points

Problem: Given \mathbf{K} and two vanishing points corresponding to two known orthogonal directions $\mathbf{d}_1, \mathbf{d}_2$, compute camera orientation \mathbf{R} with respect to the plane.

- 3D coordinate system choice, e.g.:

$$\mathbf{d}_1 = (1, 0, 0), \quad \mathbf{d}_2 = (0, 1, 0)$$

- we know that

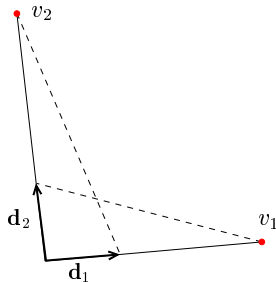
$$\mathbf{d}_i \simeq \mathbf{Q}^{-1} \mathbf{v}_i = (\mathbf{K}\mathbf{R})^{-1} \mathbf{v}_i = \mathbf{R}^{-1} \underbrace{\mathbf{K}^{-1} \mathbf{v}_i}_{\mathbf{w}_i}$$

$$\mathbf{R} \mathbf{d}_i \simeq \mathbf{w}_i \quad \mathbf{R}^T \mathbf{R} = \mathbf{I}$$

- knowing $\mathbf{d}_{1,2}$ we conclude that $\mathbf{w}_i / \|\mathbf{w}_i\|$ is the i -th column \mathbf{r}_i of \mathbf{R}
- the third column is orthogonal:

$$\mathbf{r}_3 \simeq \mathbf{r}_1 \times \mathbf{r}_2$$

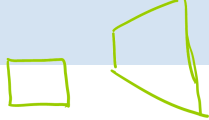
$$\mathbf{R} = \begin{bmatrix} \frac{\mathbf{w}_1}{\|\mathbf{w}_1\|} & \frac{\mathbf{w}_2}{\|\mathbf{w}_2\|} & \frac{\mathbf{w}_1 \times \mathbf{w}_2}{\|\mathbf{w}_1 \times \mathbf{w}_2\|} \end{bmatrix}$$



some suitable scenes



Application: Planar Rectification



Principle: Rotate camera parallel to the plane of interest.



$K (KR)^{-1}$

$$\underline{m} \simeq KR [I \quad -C] \underline{X}$$

$$\underline{m}' \simeq K [I \quad -C] \underline{X}$$



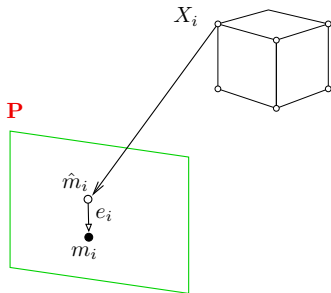
$$\underline{m}' \simeq K(KR)^{-1} \underline{m} = KR^T K^{-1} \underline{m} = H \underline{m}$$

interp2()

- H is the rectifying homography
- both K and R can be calibrated from two finite vanishing points **assuming ORUA** →56
- not possible when one (or both) of them are infinite
- without ORUA we would need 4 additional views to calibrate K as on →53

► Camera Resection

Camera calibration and orientation from a known set of $k \geq 6$ reference points and their images $\{(X_i, m_i)\}_{i=1}^6$.



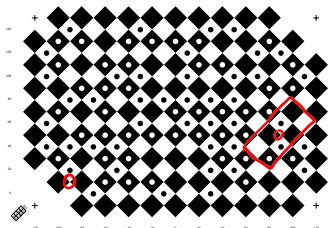
- X_i are considered exact
- m_i is a measurement subject to detection error

$$\mathbf{m}_i = \hat{\mathbf{m}}_i + \mathbf{e}_i \quad \text{Cartesian}$$

- where $\hat{\mathbf{m}}_i \simeq \mathbf{P}\mathbf{X}_i$

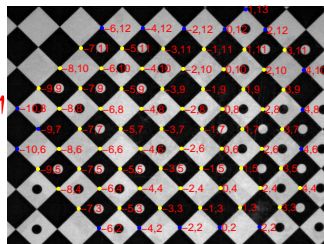
$$\lambda_i \hat{\mathbf{m}}_i = \mathbf{P}\mathbf{X}_i$$

Resection Targets

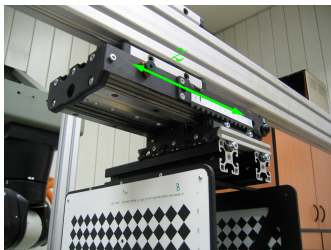


calibration chart

00100101



automatic calibration point detection



resection target with translation stage

- target translated at least once
- by a calibrated (known) translation
- X_i point locations looked up in a table based on their code

► The Minimal Problem for Camera Resection

Problem: Given $k = 6$ corresponding pairs $\{(X_i, m_i)\}_{i=1}^k$, find \mathbf{P}

$$\lambda_i \underline{m}_i = \mathbf{P} \underline{X}_i, \quad \mathbf{P} = \begin{bmatrix} \mathbf{q}_1^\top & q_{14} \\ \mathbf{q}_2^\top & q_{24} \\ \mathbf{q}_3^\top & q_{34} \end{bmatrix} \quad \begin{array}{l} \underline{X}_i = (x_i, y_i, z_i, 1), \quad i = 1, 2, \dots, k, \quad k = 6 \\ \underline{m}_i = (u_i, v_i, 1), \quad \lambda_i \in \mathbb{R}, \quad \lambda_i \neq 0 \end{array}$$

easy to modify for infinite points X_i but be aware of $\rightarrow 64$

expanded:

$$\lambda_i u_i = \mathbf{q}_1^\top \mathbf{X}_i + q_{14}, \quad \lambda_i v_i = \mathbf{q}_2^\top \mathbf{X}_i + q_{24}, \quad \lambda_i = \mathbf{q}_3^\top \mathbf{X}_i + q_{34}$$

after elimination of λ_i : $(\mathbf{q}_3^\top \mathbf{X}_i + q_{34})u_i = \mathbf{q}_1^\top \mathbf{X}_i + q_{14}, \quad (\mathbf{q}_3^\top \mathbf{X}_i + q_{34})v_i = \mathbf{q}_2^\top \mathbf{X}_i + q_{24}$

Then

$$\mathbf{A} \mathbf{q} = \begin{bmatrix} \mathbf{X}_1^\top & 1 & \mathbf{0}^\top & 0 & -u_1 \mathbf{X}_1^\top & -u_1 \\ \mathbf{0}^\top & 0 & \mathbf{X}_1^\top & 1 & -v_1 \mathbf{X}_1^\top & -v_1 \\ \vdots & & & & & \\ \mathbf{X}_k^\top & 1 & \mathbf{0}^\top & 0 & -u_k \mathbf{X}_k^\top & -u_k \\ \mathbf{0}^\top & 0 & \mathbf{X}_k^\top & 1 & -v_k \mathbf{X}_k^\top & -v_k \end{bmatrix} \cdot \begin{bmatrix} \mathbf{q}_1 \\ q_{14} \\ \mathbf{q}_2 \\ q_{24} \\ \mathbf{q}_3 \\ q_{34} \end{bmatrix} = \mathbf{0} \quad (9)$$

- we need 11 independent parameters for \mathbf{P}
- $\mathbf{A} \in \mathbb{R}^{2k, 12}$, $\mathbf{q} \in \mathbb{R}^{12}$
- 6 points in a general position give rank $\mathbf{A} = 12$ and there is no non-trivial null space
- drop one row to get rank 11 matrix, then the basis vector of the null space of \mathbf{A} gives \mathbf{q}

► The Jack-Knife Solution for $k = 6$

- given the 6 correspondences, we have 12 equations for the 11 parameters
- can we use all the information present in the 6 points?

Jack-knife estimation

1. $n := 0$
2. for $i = 1, 2, \dots, 2k$ do
 - a) delete i -th row from \mathbf{A} , this gives \mathbf{A}_i
 - b) if $\dim \text{null } \mathbf{A}_i > 1$ continue with the next i
 - c) $n := n + 1$
 - d) compute the right null-space \mathbf{q}_i of \mathbf{A}_i
 - e) $\hat{\mathbf{q}}_i := \mathbf{q}_i$ normalized to $q_{34} = 1$ and dimension-reduced
3. from all n vectors $\hat{\mathbf{q}}_i$ collected in Step 1d compute

$$\mathbf{q} = \frac{1}{n} \sum_{i=1}^n \hat{\mathbf{q}}_i, \quad \text{var}[\mathbf{q}] = \frac{n-1}{n} \text{diag} \sum_{i=1}^n (\hat{\mathbf{q}}_i - \mathbf{q})(\hat{\mathbf{q}}_i - \mathbf{q})^\top \quad \text{regular for } n \geq 11$$

- have a solution + an error estimate, per individual elements of \mathbf{P} (except P_{34})
- at least 5 points must be in a general position ($\rightarrow 64$)
- large error indicates near degeneracy
- computation not efficient with $k > 6$ points, needs $\binom{2k}{11}$ draws, e.g. $k = 7 \Rightarrow 364$ draws
- better error estimation method: decompose \mathbf{P}_i to $\mathbf{K}_i, \mathbf{R}_i, \mathbf{t}_i$ ($\rightarrow 32$), represent \mathbf{R}_i with 3 parameters (e.g. Euler angles, or in Cayley representation $\rightarrow 139$) and compute the errors for the parameters



e.g. by 'economy-size' SVD
assuming finite cam. with $P_{3,4} = 1$

► Degenerate (Critical) Configurations for Camera Resection

Let $\mathcal{X} = \{X_i; i = 1, \dots\}$ be a set of points and $\mathbf{P}_1 \neq \mathbf{P}_j$ be two regular (rank-3) cameras. Then two configurations $(\mathbf{P}_1, \mathcal{X})$ and $(\mathbf{P}_j, \mathcal{X})$ are image-equivalent if

$$\mathbf{P}_1 \underline{\mathbf{X}}_i \simeq \mathbf{P}_j \underline{\mathbf{X}}_i \quad \text{for all } X_i \in \mathcal{X}$$

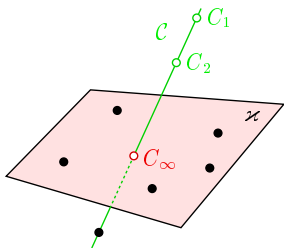
there is a non-trivial set of other cameras that see the same image

- **importantly:** If all calibration points $X_i \in \mathcal{X}$ lie on a plane \varkappa then camera resection is non-unique and all image-equivalent camera centers lie on a spatial line \mathcal{C} with the $C_\infty = \varkappa \cap \mathcal{C}$ excluded

this also means we cannot resect if all X_i are infinite

- by adding points $X_i \in \mathcal{X}$ to \mathcal{C} we gain nothing
- there are additional image-equivalent configurations, see next

proof sketch in [H&Z, Sec. 22.1.2]

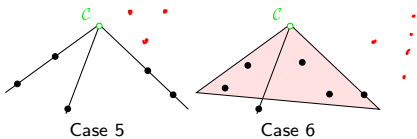


Case 4

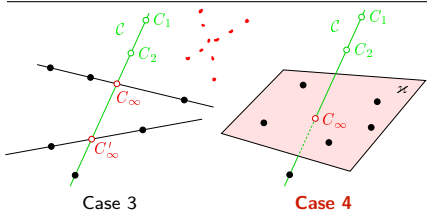
Note that if \mathbf{Q}, \mathbf{T} are suitable homographies then $\mathbf{P}_1 \simeq \mathbf{Q}\mathbf{P}_0\mathbf{T}$, where \mathbf{P}_0 is canonical and the analysis can be made with $\hat{\mathbf{P}}_j \simeq \mathbf{Q}^{-1}\mathbf{P}_j$

$$\underbrace{\mathbf{P}_0}_{\underline{\mathbf{Y}}_i} \underline{\mathbf{T}\mathbf{X}}_i \simeq \underbrace{\hat{\mathbf{P}}_j}_{\underline{\mathbf{Y}}_i} \underline{\mathbf{T}\mathbf{X}}_i \quad \text{for all } Y_i \in \mathcal{Y}$$

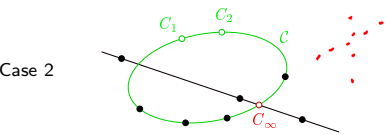
cont'd (all cases)



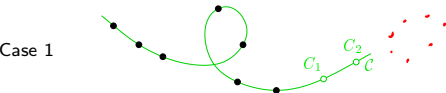
- cameras C_1, C_2 co-located at point C
- points on three optical rays or one optical ray and one optical plane
- Case 5: camera sees 3 isolated point images
- Case 6: cam. sees a line of points and an isolated point



- cameras lie on a line $C \setminus \{C_\infty, C'_\infty\}$
- points lie on C and
 1. on two lines meeting C at C_∞, C'_∞
 2. or on a plane meeting C at C_∞
- Case 3: camera sees 2 lines of points



- cameras lie on a planar conic $C \setminus \{C_\infty\}$
not necessarily an ellipse
- points lie on C and an additional line meeting the conic at C_∞
- Case 2: camera sees 2 lines of points



- cameras and points all lie on a twisted cubic C
- Case 1: camera sees points on a conic

► Three-Point Exterior Orientation Problem (P3P)

Calibrated camera rotation and translation from Perspective images of 3 reference Points.

Problem: Given \mathbf{K} and three corresponding pairs $\{(m_i, X_i)\}_{i=1}^3$, find \mathbf{R} , \mathbf{C} by solving

$$\lambda_i \underline{\mathbf{m}}_i = \mathbf{K}\mathbf{R}(\mathbf{X}_i - \mathbf{C}), \quad i = 1, 2, 3$$

1. Transform $\underline{\mathbf{v}}_i \stackrel{\text{def}}{=} \mathbf{K}^{-1}\underline{\mathbf{m}}_i$. Then

$$\lambda_i \underline{\mathbf{v}}_i = \mathbf{R}(\mathbf{X}_i - \mathbf{C}). \quad (10)$$

2. Eliminate \mathbf{R} by taking rotation preserves length: $\|\mathbf{R}\mathbf{x}\| = \|\mathbf{x}\|$

$$|\lambda_i| \cdot \|\underline{\mathbf{v}}_i\| = \|\mathbf{X}_i - \mathbf{C}\| \stackrel{\text{def}}{=} z_i \quad (11)$$

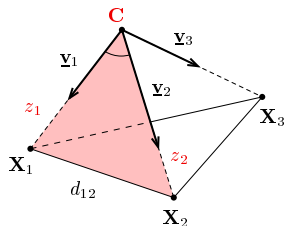
3. Consider only angles among $\underline{\mathbf{v}}_i$ and apply Cosine Law per triangle $(\mathbf{C}, \mathbf{X}_i, \mathbf{X}_j)$ $i, j = 1, 2, 3, i \neq j$

$$d_{ij}^2 = z_i^2 + z_j^2 - 2 z_i z_j c_{ij},$$

$$z_i = \|\mathbf{X}_i - \mathbf{C}\|, \quad d_{ij} = \|\mathbf{X}_j - \mathbf{X}_i\|, \quad c_{ij} = \cos(\angle \underline{\mathbf{v}}_i \underline{\mathbf{v}}_j)$$

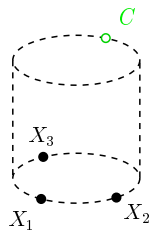
4. Solve system of 3 quadratic eqs in 3 unknowns z_i [Fischler & Bolles, 1981]
there may be no real root; there are up to 4 solutions that cannot be ignored (verify on additional points)
5. Compute \mathbf{C} by trilateration (3-sphere intersection) from \mathbf{X}_i and z_i ; then λ_i from (11) and \mathbf{R} from (10)

configuration w/o rotation in (11)



Similar problems (P4P with unknown f) at <http://cmp.felk.cvut.cz/minimal/> (with code)

Degenerate (Critical) Configurations for Exterior Orientation



unstable solution

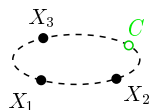
- center of projection C located on the orthogonal circular cylinder with base circumscribing the three points X_i

unstable: a small change of X_i results in a large change of C
can be detected by error propagation

degenerate

- camera C is coplanar with points (X_1, X_2, X_3) but is not on the circumscribed circle of (X_1, X_2, X_3)

camera sees point on a line



no solution

- C cocyclic with (X_1, X_2, X_3) camera sees points on a line

- additional critical configurations depend on the method to solve the quadratic equations

[Haralick et al. IJCV 1994]

Thank You



