

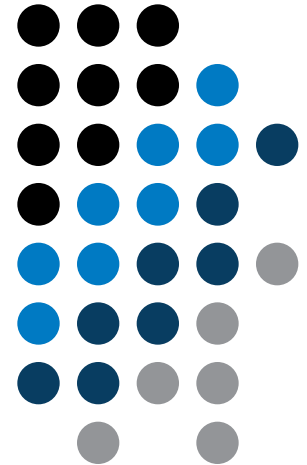
A0B17MTB – Matlab

Part #1



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You will learn ...

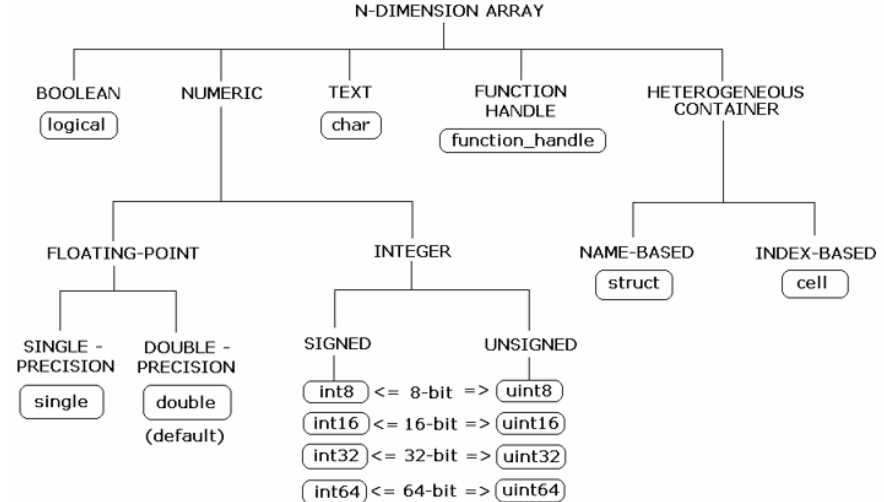
Scalars, vectors, matrices (class numeric)

Matrix operations

Command Window, Command History

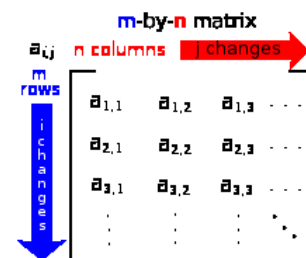
Saving and loading variables

Exercises



Matrices in Matlab

- matrix is a basic data structure in Matlab
- there are following types depending on size :
 - 1×1 (scalar)
 - $M \times 1$, or $1 \times N$ (vector)
 - $M \times N$ (matrix)
 - array (multidimensional matrices) $M \times N \times P \times Q \times R \times \dots$
- can be complex
- can contain text as well (beware the length)



Matrix creation

- following techniques are available:
 - element-by-element entering (suitable for small matrices only)
 - colon notation „:“ to define elements of a series
 - generation by built-in functions
 - generation of matrices in m-files
 - import and export from/to external files (.mat, .txt, .xls)

Matrix construction element-by-element

- test following commands to construct matrices by element enumeration
 - suitable for small matrices only

```
>> a1 = -1
>> a2 = [-1]    % brackets are redundant
```

$$a_1 = a_2 = -1$$

$$\mathbf{v}_1 = (-1 \ 0 \ 1)$$

$$\mathbf{v}_2 = \begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix}$$

```
>> v1 = [-1 0 1]
>> v2 = [-1; 0; 1]
```

```
>> M1 = [-1 0 1; -2 0 2]
>> M2 = [-1 -2; 0 0; 1 2]
>> M3 = [[-1 -2]; [0 0]] % inner brackets are redundant
```

$$\mathbf{M}_1 = \begin{pmatrix} -1 & 0 & 1 \\ -2 & 0 & 2 \end{pmatrix}, \quad \mathbf{M}_2 = \begin{pmatrix} -1 & -2 \\ 0 & 0 \\ 1 & 2 \end{pmatrix}, \quad \mathbf{M}_3 = \begin{pmatrix} -1 & -2 \\ 0 & 0 \end{pmatrix}$$

Matrix construction element-by-element

90 s ↑

- construct following matrices:
 - matrix values are defined inside square brackets []
 - semicolon „;“ separates individual rows of a matrix

$$\mathbf{A} = \begin{pmatrix} -1 & 1 \\ 1 & -2 \end{pmatrix} \quad \mathbf{B} = \begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{pmatrix}$$

Matrix construction

120 s ↑

- semicolon placed at the end of a command suppresses display of the output in Command Window

```
>> a = 1
>> b = 5;
```

- when more than one command on the same line, coma is used to separate each command

```
>> a = 1, b = 5
>> a = 1; b = 5;
```

- note: it is possible to copy and paste code including ">>"

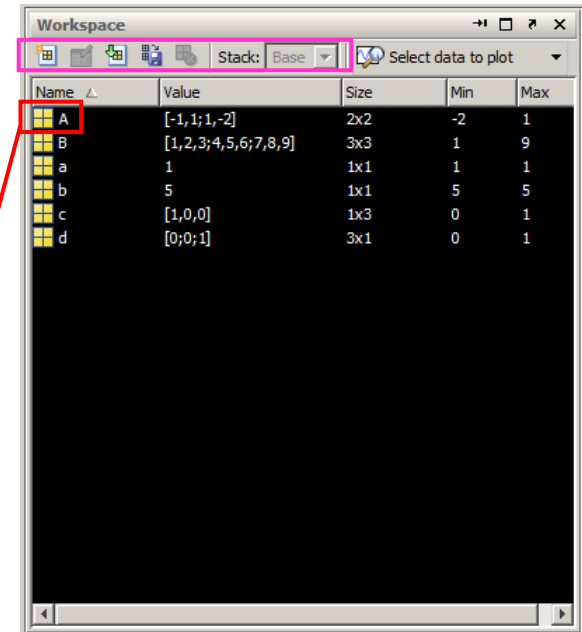
```
>> c = [1 0 0]
>> d = [0; 0; 1]
```

$$\mathbf{c} = \begin{pmatrix} 1 & 0 & 0 \end{pmatrix} \quad \mathbf{d} = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

- "row" vs. "column" vector

Workspace browser

- new variables
- deleting / modification of existing variables
- saving / loading
- size, elements of variables
 - other information can be added
- fast data plotting option
- all operations can be carried out using Matlab functions that we learn later, e.g.
min, max, size, length



Variable Editor - A

A <2x2 double>

	1	2	3	4	5	6	7
1	-1	1					
2	1	-2					
3							
4							
5							
6							
7							

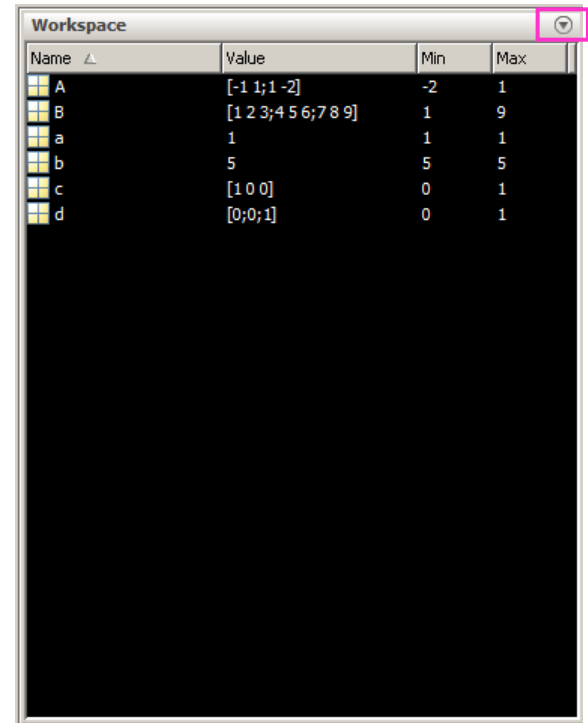
Workspace browser

- Workspace now contains variables A, B, a, b, c, d (from previous slides)
 - all variables in the base workspace are displayed

- variable `ans` contains the last result
 - can be used for calculation
 - overwritten by each command input!

```
>> 2*2, ans^2
```

- try to edit variables A, a
 - by a Matlab command directly
 - by change of value in `Workspace` browser
- try to delete variables B, c



Name	Value	Min	Max
A	[-1 1;1 -2]	-2	1
B	[1 2 3;4 5 6;7 8 9]	1	9
a	1	1	1
b	5	5	5
c	[1 0 0]	0	1
d	[0;0;1]	0	1

Basic math operators

- of several types:
 - arithmetic
 - matrix
 - vector
 - relational
 - logical
 - and other (to be mentioned later...)

+	addition
-	subtraction
*	multiplication
^	power
'	transpose
\	left matrix division
/	right matrix division
.	dot notation

- other operations using Matlab functions
 - complex conjugate,
 - sum, determinant, square root
 - and hundreds of other functions ...

Operator Precedence in Matlab

- according to the following table
 - see Matlab → Language Fundamentals → Operators and Elementary Operations → Arithmetic

higher priority ↑	1	parentheses	()					
	2	transpose, power	'	.'	^	.^		
	3	unary plus, unary minus, logical negation	+	-	~			
	4	multiplication, division	*	.*	/	\	./	.\
lower priority ↓	5	addition, subtraction	+	-				
	6	colon operator	:					
	7	relational operators	<	>	<=	>=	==	~=
	8	logical AND (element-wise)	&					
	9	logical OR (element-wise)						
	10	logical AND (short-circuit)	&&					
	11	logical OR (short-circuit)						

Basic math operators

200 s



- type in following commands
 - zero can be omitted with a decimal number beginning with zero (not recommended!)

```
>> a3 = -2/4  
>> a4 = -0.5  
>> a5 = -.5
```

- what is the difference between a3, a4, a5?
- beware the precedence of operators (we see in the next slides):

```
>> 3*5*6  
>> a1 = 15  
>> a2 = 10;  
>> a2/a3  
>> a2/a3*a4  
>> a2/(a3*a4)
```

- explain the difference between $a2/a3*a4$ and $a2/(a3*a4)$
- verify the rules of operator precedence from the previous slide

Lengthy commands in Matlab

120 s ↑

- it is suitable to structure command blocks for clarity:
 - next line: SHIFT+ENTER

```
>> A = [1 1 1]; B = [2 2 2]; % SHIFT+ENTER  
C = [2 3 2];
```

- three dots notation
 - for continuation of the same command on the next line
 - compare results:

```
>> A = [1 1 ...  
2 3]
```

```
>> A = [1 1  
2 3]
```

Basic math functions

- math functions in Matlab are generally divided in three groups:
 - scalar
 - function operates over individual elements of a matrix
 - e.g.: sin, sqrt, log, factorial
 - vector
 - Function operates over individual rows/columns of a matrix
 - e.g.: sum, max
 - matrix
 - function operates over whole matrix
 - e.g.: det, trace

Basic math functions #1

600 s ↑

- using Matlab help, calculate following expression: $a \sin^2(\alpha) + a \cos^2(\alpha) - a$
 - use numerical values you choose

- verify following logarithmic identity: $\log_{10}(a) + \log_{10}(b) - \log_{10}(ab) = 0$

- find sum of all elements in individual rows of the following matrix

$$\mathbf{T} = \begin{pmatrix} \frac{1}{2} & \frac{1}{3} & \frac{1}{4} & \frac{1}{5} \\ 6 & 7 & 8 & 9 \\ 0.2 & 0.3 & 0.4 & 0.5 \end{pmatrix}$$

Basic math functions #2

600 s ↑

- assume following vectors \mathbf{u} , \mathbf{v} :

$$\mathbf{u} = (1 \ 2 \ 3), \quad \mathbf{v} = (3 \ 2 \ 1)$$

- calculate

$$\mathbf{u}\mathbf{v}^T, \quad \mathbf{v}\mathbf{u}^T,$$

$$\mathbf{v}^T\mathbf{u}, \quad \mathbf{u}^T\mathbf{v},$$

$$\mathbf{u} \cdot \mathbf{v}, \quad \mathbf{u} \times \mathbf{v},$$

A

$$\begin{bmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 6 \end{bmatrix}$$

wikipedia.org

- following functions are needed:
 - transpose (.') of a matrix
 - dot scalar product
 - cross product
- what is the result of the above mentioned operations?

Basic math functions #3

420 s ↑

- use following code and round the resulting number to:

```
>> r = 1 + 10*rand(1)
```

- (a) nearest integer
 - (b) nearest integer greater than r
 - (c) nearest integer lower than r
 - (d) zero
 - (e) zero with precision of 2 decimal digits
- find remainder after r is divided by 0.1
 - *modulus vs. remainder after division*

note: one of the functions is called round

Matrix division in Matlab

- matrix operation
- two cases are distinguished: left division („\“ - `mldivide`) and right division („/“ - `mrdivide`)
 - A is invertible (regular), b is row (column) vector

$$\mathbf{Ax} = \mathbf{b}$$

solution to linear
system of equations

$$\mathbf{x} = \mathbf{A}^{-1}\mathbf{b}$$

```
>> x = A \ b
```

$$\mathbf{xA} = \mathbf{b}$$

$$\mathbf{x} = \mathbf{bA}^{-1}$$

```
>> x = b / A
```

Basic math functions #4

500 s ↑

- find the sum of diagonal elements (trace of a matrix) of the matrix **T** with elements coming from normal distribution with mean equal to 10 and standard deviation equal to 4

```
>> T = 10 + 4*randn(7, 7);
```

- find determinant of matrix **U**

$$\mathbf{U} = \begin{pmatrix} 1 & 2 & \frac{17}{81} \\ 0 & 2 & 0 \\ 0 & -2 & -1 \end{pmatrix}$$

- solve the linear system of equations

$$x_1 + 2x_2 + 3x_3 = 6$$

$$4x_1 + 5x_2 + 6x_3 = 15$$

$$7x_1 + 8x_2 + x_3 = 16$$

$$\mathbf{Ax} = \mathbf{b}$$

$$\mathbf{x} = \mathbf{A}^{-1}\mathbf{b}$$

Matlab commands

- Matlab is **cAsE sEnSiTiVe**
 - almost entirely, with certain exceptions (properties of graphic objects, ...)
 - pay attention to typos and variable names (see later)
 - new versions of Matlab offer certain options

```
>> AA = [1 1 1]  
>> Aa
```

- beware of different syntax in Mathematica
 - following syntax is incorrect both in Matlab and Mathematica:

```
>> Sin(pi/2) % function names start with lower case  
>> cos[pi/3] % function input is in parentheses ()
```

Predefined values in Matlab

- Matlab contains several predefined values
 - `eps` – precision of single/double numbers
 - `eps` determines the shortest distance between two single/double numbers
 - `ans` – most recent answer
 - `NaN` – *not a number* (every expression containing `NaN` results is `NaN`)
 - `NaN` can be used advantageously in some cases
 - `Inf` – *infinite number* (variable `Inf` can be used in calculation :))
 - pay attention to `Inf` propagation throughout your code (use allowed operations only)
 - `i`, `j` – complex unit
 - they are all basically functions (without input parameters)
- check results of the following expressions:

```
>> t1 = 10/0      % t1 = Inf
>> t2 = 0/0       % t2 = NaN
>> t3 = t1*5       % t3 = Inf
>> t4 = t1 + t2    % t4 = NaN
```
- `pi`, `intmin`, `intmax`, `realmin`, `realmax`, ... (functions)

Workspace – output deletion #1

- to clean (erase) the command window:

```
>> home % cursor (>>) is shifted to the top-left position  
>> clc % Command Window is erased
```

- try and compare

```
Command Window  
>> a2 = 10  
a3 = -2/4  
a2/a3  
  
a2 =  
    10  
  
a3 =  
   -0.5000  
  
ans =  
   -20  
fx >> |
```

```
>> home
```

```
>> clc
```

```
Command Window  
fx >>
```

Workspace – output deletion #2

- to clean one (or more) variable, use `clear`

```
>> clear           % whole Workspace is deleted
>> clear XX        % variable XX is deleted
>> clear XX YY      % variables XX and YY are deleted
>> clear z*         % everything starting with 'z' is deleted
```

- `clear` has a number other options (graphics, I/O)
- **try** to delete selected variables in workspace

Workspace – output deletion #3

- to delete all variables except for one (or several):

```
>> clearvars -except a3 % clears everything except a3
```

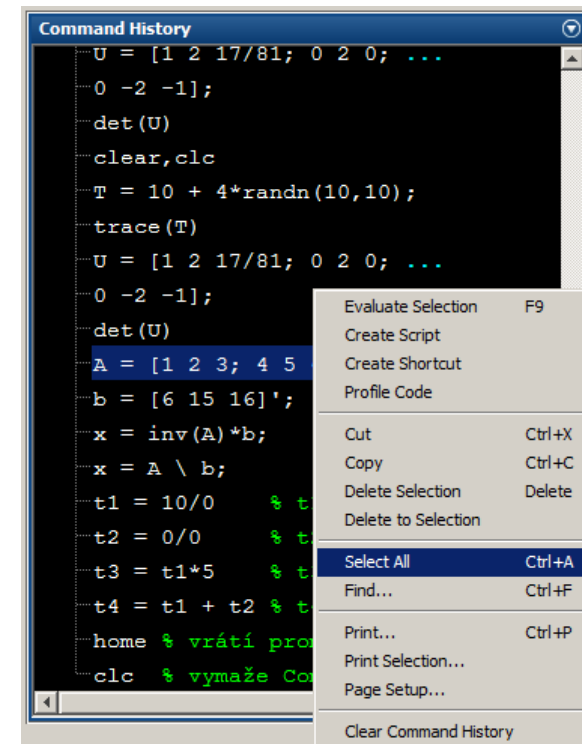
- further information in doc `clear`, doc `clearvars`

Command History window

- Command History window stores all commands from the Command Window
- Command History accessible though (↑ or ↓)
- it is possible to filter out past commands by

- e.g. `>> A = [` + ↑

- It is possible to copy-and-paste entire Command History
 - SHIFT / CTRL / CTRL+A → CTRL+C



- later on, we will work with scripts and functions to store all the commands/code

Variables storing and loading

- existing variables in Matlab Workspace can be stored on disk

```
>> save % stores all variables in matlab.mat in current folder  
>> save task1 % stores all variables in task1.mat  
>> save task1 a b c % stores variables „a“, „b“ and „c“ in task1.mat
```

- CTRL+S in Command Window / Command History
- loading variables is analogical

```
>> load % loads matlab.mat in current folder  
>> load task1 % loads all variables from task1.mat  
>> load task1 a b c % loads variables „a“, „b“ and „c“ from task1.mat
```

- alternatively, drag & drop the file from Current Folder in Command Window

Storing history and variables

180 s ↑

- save today's Command History
 - use *.txt file
- store all variables from Workspace in Data.mat
- try to store selected variables only
- clear Workspace and load above mentioned files
- both storing and loading can be carried out using mouse!!

.mat file structure

- .mat files of the 7.3 version have the HDF5 format
 - HDF = Hierarchical Data Format
 - enable to store variables exceeding 2GB (64-bit system)
 - scientific format for data storing
- advantages of accessing HDF directly for certain applications:
 - speed
 - it is possible to define structure of the file and the stored data
 - Matlab *High-Level* functions and HDF *Low-Level* functions are available
- for more detailed information see:
 - MATLAB → Data and File Management → Data Import and Export → Scientific Data

Variable names #1

- max. 63 characters starting with a letter (`>> namelengthmax`)
 - underscore is allowed in the variable name „_“ (not at the beginning!)
 - characters not allowed are colon „:“, hyphen „-“ and others
- lowercase letters in the names of scalars and variables (`a = 17.59;`)
- matrix names usually start with a capital letter (`A = [...]`)
 - clear huge matrices after they are used (`clear ..., memory'`)
- iteration variables, variables used in `for` cycles usually named `m`, `n`, `k` etc.
 - it is advisable to avoid `i`, `j` (complex unit)
- chose the names to correspond to the purpose of the variable
- avoid, if possible, standalone letter '1' (to be confused with 1) and predefined variables in Matlab environment

Variable names #2

- exceeding the maximum variable's name length :

```
>> a012345678901234567890123456789012345678901234567890123456789012345678901234567890123456789 = 10
Warning: 'a012345678901234567890123456789012345678901234567890123456789012345678901234567890123456789'
exceeds the MATLAB maximum name length of 63 characters and will be truncated to
'a0123456789012345678901234567890123456789012345678901'.

a01234567890123456789012345678901234567890123456789012345678901 =

    10
```

Variable names #3

- samples of valid variable names

```
a, A, b, c, x1, x2, M_12, test1, matrix_A, fx, fX
```

- samples of invalid variable names

```
1var      % starts with a number (not possible in Matlabu)
matrix A  % contains space
coef.a    % possible only if coef is of type 'struct'
Test-1    % algebraic expression: ans = Test - 1
f(y)      % makes sense when using symbolic expressions
```

- samples of valid numbers in Matlab
 - pay attention to miss inserted spaces after exponent and imaginary unit

```
3, -66, +0.0015, .015, 1.6025e-10, 3i, 3.17e5i, -3.51j
```

Discussed functions

<code>sin, cos</code>	trigonometric functions	
<code>sqrt</code>	square root	
<code>max</code>	largest element of column of a matrix; largest element of a vector	•
<code>sum</code>	sum of elements of column of a matrix; sum of elements of a vector	•
<code>log, log10</code>	natural logarithm, logarithm with base 10	
<code>factorial</code>	factorial	
<code>det, trace</code>	determinant of a (square) matrix, trace of a (square) matrix	
<code>transpose</code>	transpose	
<code>dot, cross</code>	scalar product, vector product	•
<code>inv</code>	invers of a matrix	
<code>round, ceil, floor, fix</code>	rounding	
<code>rem</code>	remainder after division	
<code>rand, randn</code>	random number generation	
<code>save, load</code>	storing, loading of variables	•
<code>clear, clearvars</code>	deleting variables and functions, deleting variables only	•
<code>home, clc</code>	command prompt shift, clears output	
<code>ans, eps</code>	returns last answer, numerical accuracy of Matlab	•

Exercise #1

180 s ↑

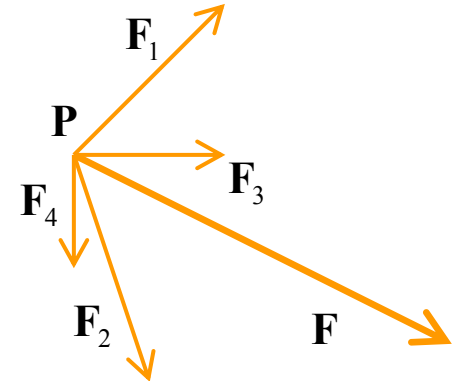
- forces were localized at point **P** in $(x-y)$ plane:

$$\mathbf{F}_1 = (2 \quad 2)$$

$$\mathbf{F}_3 = (2 \quad 0)$$

$$\mathbf{F}_2 = (1 \quad -3)$$

$$\mathbf{F}_4 = (0 \quad -1.5)$$



- what is the direction of the resultant force **F**?
- normalize the resulting vector

$$\mathbf{n}_F = \frac{\mathbf{F}}{|\mathbf{F}|} = \frac{\mathbf{F}}{\sqrt{F_x^2 + F_y^2 + F_z^2}}$$

Exercise #2

240 s



- type-in following commands:

```
>> clear, clc;  
>> w1 = [1 2 3], w2 = [-2 -3 -4].'  
>> w3 = [-2; -3; -4]  
>> w4 = w2 - w3, w5 = w2 - w1
```

- compare differences
- the error of calculating w5 resides in what?

- try also

```
>> w1*3, w1 - 3,  
>> w1 + [5 5 5],  
>> w6 = 5*w1 - [3 5 6] - w2,
```

- calculate the norm (magnitude) of vector w1
- try more options

$$\hat{\mathbf{w}}_1 = \frac{\mathbf{w}_1}{|\mathbf{w}_1|}$$

- how to modify the calculation in the case of a complex vector?

Exercise #3

180 s



- calculate roots of the quadratic function $-2x^2 - 5x = 3$
 - rearrange the terms of the function first

$$2x^2 + 5x + 3 = 0, \Rightarrow a = 2, b = 5, c = 3$$

$$x_{1,2} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-5 \pm \sqrt{25 - 24}}{4}$$

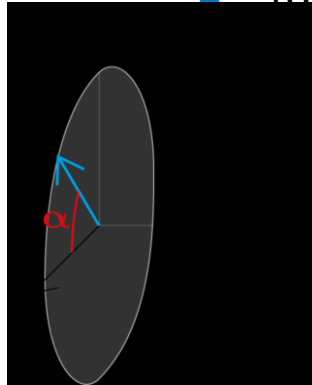
$$x_1 = -1, \quad x_2 = -\frac{3}{2}$$

- Matlab provides particular function for calculation of roots of a function, try to search it out

Exercise #4

300 s ↑

- consider matrices (prepare matrices for later use)
- rotating by angle α in x - z plane



$$\mathbf{R} = \begin{pmatrix} \cos(\alpha) & 0 & -\sin(\alpha) \\ 0 & 1 & 0 \\ \sin(\alpha) & 0 & \cos(\alpha) \end{pmatrix}$$

- mirroring across plane

$$1x + 2y + 0z = 0$$

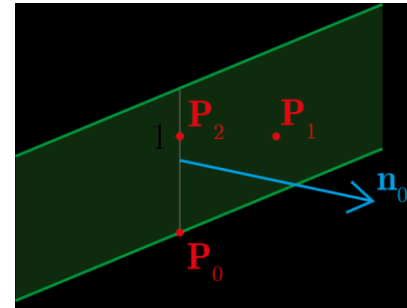
- use Householder's transform

$$\mathbf{P} = \mathbf{I} - 2\mathbf{n}_0\mathbf{n}_0^T$$

$$\mathbf{n}_0 = \frac{\mathbf{v}_1 \times \mathbf{v}_2}{|\mathbf{v}_1 \times \mathbf{v}_2|} \quad \mathbf{P}_1 = [-2; 1; 0]$$

$$\mathbf{P}_2 = [0; 0; 1]$$

$$\mathbf{v}_k = (\mathbf{P}_k - \mathbf{0}) \begin{pmatrix} \mathbf{x}_0 \\ \mathbf{y}_0 \\ \mathbf{z}_0 \end{pmatrix}, \quad k \in \{1, 2\}$$



Exercise #5

180 s



- use rotation matrix \mathbf{R} to rotate vector $\mathbf{k} = [1; 0; 0]$ by angle $\alpha = \pi/2$

$$\mathbf{m} = \mathbf{R}\mathbf{k} = (0 \quad 0 \quad 1)^T$$

- use reflection matrix \mathbf{P} across plane: $1x + 2y + 0z = 0$
 - to mirror vectors:

$$\mathbf{u}_1 = \mathbf{n}_0, \quad \mathbf{u}_2 = \left(\frac{5}{2} \quad 0 \quad 3 \right)^T$$

$$\mathbf{m}_1 = \mathbf{P}\mathbf{u}_1 = -\mathbf{n}_0, \quad \mathbf{m}_2 = \mathbf{P}\mathbf{u}_2 = \left(\frac{3}{2} \quad -2 \quad 3 \right)^T$$

- calculate the determinant of matrices \mathbf{R} and \mathbf{P}
 - can you interpret the results?

Thank you!



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