

Entrance test - B4M36SMU

1 Do not play with the matches

There are 4 matches lying on a table. The goal of an automated robot is to gradually remove them such that there is no match remaining on the table. The robot can remove 1 or 2 matches in one step. The problem is that the robot's arm is unreliable, it can remove more matches than the robot planned. To be precise, in half the attempts the arm removes one more match than planned. If the robot tries to remove more matches than actually available, the task becomes cyclic (-1 turns into 4, -2 turns into 3). Propose the optimal robot control strategy, the goal is to minimize the number of steps.

- (a) Propose a task formalization based on Markov Decision Process (MDP).
- (b) Formally derive the optimal strategy. If the derivation turns out difficult, show a few steps only and define the termination conditions.
- (c) Use the derivation ad b and for each state select one out of two available actions.
- (d) How many steps the robot with the optimal control strategy needs to reach zero matches?

2 CNF

Convert to CNF:

$$\exists x \forall y \forall z (\text{person}(x) \wedge ((\text{likes}(x, y) \wedge \neg \text{equal}(y, z)) \Rightarrow \neg \text{likes}(x, z)))$$

3 Bayesian Decision Making

Imagine you are a shepherd, whose sheep are constantly attacked by wild beasts. You care about your beloved flock and recall the machine learning course you took at the farming university during childhood.

Three species of predators live in the surrounding forests: bear, wolf and lynx. You know from experience that – from eight cases there are four predator attacks by wolves, three attacks by bears and one by lynx. The wolves are known to attack in pack and usually kill more sheep at once (two or more sheep are killed in 8 out of 10 attacks). Bears are solitary, but may also kill more sheep (two or more sheep are killed in 5 of 10 attacks). Lynx always kills one sheep. Wolves never return to the same place for a new attack next night, a lynx returns only in one out of ten cases, bear returns in half of cases. Assume that the event of predator returning to the same place and the number of sheep eaten are independent.

Answer the following questions:

- (a) What is the probability of an attack, in which a lynx killed more than one sheep?

For the following questions, suppose exactly one of your sheep was killed by a wild animal during the night.

- (b) What is the probability that sheep was attacked by bear?
- (c) Which predator killed the sheep? Decide merely on the basis of maximum likelihood (with the assumption of uniform a priori probability). Explain the decision.
- (d) Which predator killed the sheep? Use the method of maximum a posterior probability. Explain the decision.
- (e) What is the probability that a predator will return the next night? Assume the MAP hypothesis about the killer from the previous question and provide a simplified approximation.
- (f) What is the probability that a predator will return the next night? Consider full Bayesian learning.

4 Resolution principle

Using resolution, prove from

$$\forall x \exists y : \text{human}(x) \rightarrow \text{mother}(x, y) \wedge \text{human}(y)$$

that

$$\forall x : \text{human}(x) \rightarrow \text{mother}(\text{mother_of}(x), \text{mother_of}(\text{mother_of}(x)))$$

i.e. that every human has a grandmother.